+6

Quiz #2, Quality control, 2018 Fall [+6 points]

Name: 林祐頡 Lin-Yeu-Chieh

Let $X \sim Bin(n=22, p=0.5)$. Answer up to 3 digits 0.xxx

+1 (1) [+1] Approximate $\Pr(X \le 10)$ by the normal approximation with continuity correction.

$X \sim Bin(22, \frac{1}{2})$

$E(X) = 22 \times \frac{1}{2} = 11$

$Var(X) = 22 \times \frac{1}{2} \times (1-\frac{1}{2}) = 5.5$

$P(X \le 10) \approx P(X < 10+0.5)$

$= P\left(\frac{X-11}{\sqrt{5.5}} < \frac{10.5-11}{\sqrt{5.5}}\right)$

$= \Phi\left(\frac{-0.5}{2.3452}\right) \approx \Phi(-0.213)$

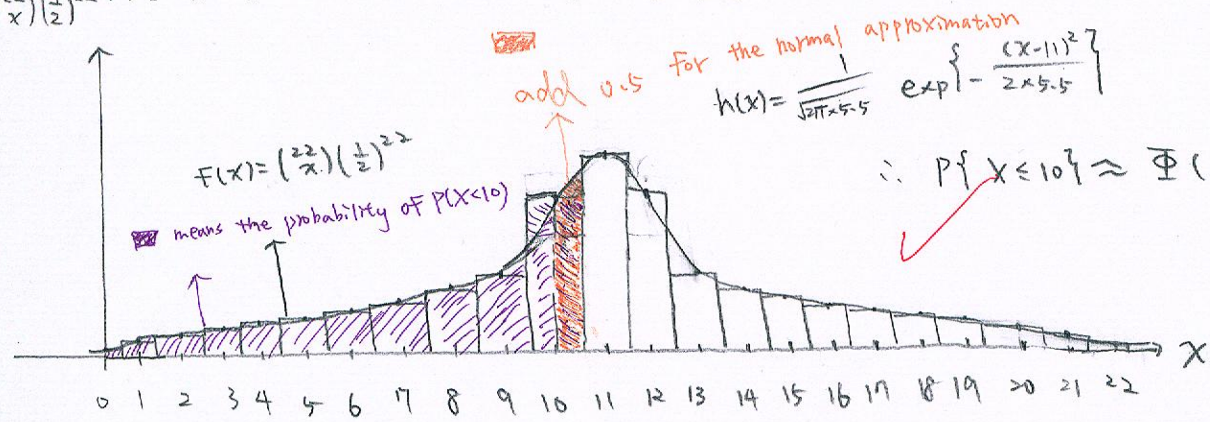
$= 1-\Phi(0.213) = 1-0.584$

$= 0.416$ ※

(where $\Phi(\cdot)$ is the c.d.f of $N(0,1)$)

+2 (2) [+2] Explain the above answer by using a graph of the pdf of the normal distribution.

$F(x) = \binom{22}{x}\left(\frac{1}{2}\right)^{22}$



$\therefore P\{X \le 10\} \approx \Phi(-0.213)$ ※

+1 (3) [+1] Approximate $\Pr(X = 10)$ by the normal approximation with continuity correction.

$P(X=10) \approx P(10-0.5 < X < 10+0.5)$

$= P\left(\frac{9.5-11}{\sqrt{5.5}} < \frac{X-11}{\sqrt{5.5}} < \frac{10.5-11}{\sqrt{5.5}}\right)$

$= \Phi(-0.213) - \Phi(-0.6396)$

$= [1-\Phi(0.213)] - [1-\Phi(0.6396)]$

$\approx \Phi(0.64) - \Phi(0.213)$

$= 0.739 - 0.584$

$= 0.155$ ※

(where $\Phi(\cdot)$ is the c.d.f of $N(0,1)$)

+1 (4) [+1] Calculate the exact value $\Pr(X=10)$ using using $\binom{22}{10} = 646646$

$P(X=10) = \binom{22}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{12}$

$= 646646 \times \left(\frac{1}{1024}\right) \times \frac{1}{4096}$

$= \frac{646646}{4194304} \approx 0.15417$ ※

+1 (5) [+1] What are the conditions of the normal approximation for n and p? Are they satisfied?

the conditions of the normal approximation for n and p are $\begin{cases} np \ge 10 \\ 0.1 \le p \le 0.9 \end{cases}$

$\because np = 22 \times \frac{1}{2} = 11$ and $0.1 \le p \le 0.9$ $\therefore$ they are satisfied ※

Table: the c.d.f. of N(0,1) $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$

z	0	0.213	0.5	0.64	1	1.5	1.96	2	2.5	3	3.5	4
P	0.5	0.584	0.6915	0.739	0.8413	0.9332	0.975	0.9772	0.9938	0.99865	0.99977	0.99997