Quiz #3, Quality control 2015 Fall

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- **Not only answer but also derivations**

+1

$p<0.1$, $n>100$
0.08 150

1. Let $X \sim \text{Bin}(n=150,\ \ p=0.08)$. For a = 2.03205 and b = 21.96795, the exact value is $\alpha = \Pr(a \le X \le b)$ =0.9954. Find the approximate value $\alpha^* = \Pr(a \le X \le b)$ by the normal approximation
[Exactly calculate up to 5 digits: 0.00000]

$np = 150 \times 0.08 = 12$

$\sqrt{np(1-p)} = \sqrt{150 \times 0.08 \times 0.92} = \sqrt{11.04} = 3.32265$

$\alpha^* = P_r(a \le X \le b)$

$= P(2.03205 \le X \le 21.96795)$

$= \Phi\left(\frac{21.96795-12}{3.32265}\right) - \Phi\left(\frac{2.03205-12}{3.32265}\right)$

$= \Phi(3) - \Phi(-3) = \Phi(3) - (1-\Phi(3)) = 2 \times 0.99865 - 1 = \underline{\underline{0.9973}}$ ※

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2. Chips of an engineering plastic have average size 45mm and standard deviation 2mm. Upper and lower specification limits for the chips are 48mm and 41mm, respectively.

1) What is the probability that a chip gets outside the specification?
[up to 4 digits: 0.0000]
2) We obtained sizes of chips (43, 44, 47, 46, 45). Find an estimate of the standard deviation by "range method". [up to 2 digits: 0.00]

$X \sim N(\mu = 45, \sigma^2 = 4)$ $[41. 48]$

1.) $P(\text{a chip gets outside the specification})$

$= P(X < 41) + P(X > 48)$

$= \Phi\left(\frac{41-45}{2}\right) + 1 - \Phi\left(\frac{48-45}{2}\right)$

$= \Phi(-2) + 1 - \Phi(1.5) =$

$= 1 - \Phi(2) + 1 - \Phi(1.5)$

$= 1 - 0.9772 + 1 - 0.9332 =$

$= \underline{\underline{0.0896}}$ ※

2.) range method : $\hat{\sigma} = \frac{R}{d_2}$

$R = 47 - 43 = 4$

$(n=5)$, $d_2 = 2.326$

$\therefore \hat{\sigma} = \frac{4}{2.326} = \underline{\underline{1.72}}$ ※

$|Z_0| > Z_{\alpha/2} =$

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3. Suppose $X_i, i = 1,..,n \overset{iid}{\sim} N(\mu, \sigma^2)$. An engineer considers a test $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$ with level $\alpha = 0.0027$. Let $\delta = \mu_1 - \mu_0$ be the shift of mean.

+1 1) Derive the producer's risk (Type I error)

+1 2) Under $H_1 : \mu_1 = \mu_0 + \delta$, derive the consumer's risk (Type II error) in terms of δ, σ, and Φ.

+2 3) Draw OC curve for $n = 4$

(Plot must be clear and contain detailed information such that I understand it)

1.) producer's risk : type I error $= \alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$

0.0027

$= P(|Z_0| > Z_{0.00135} \mid \mu = \mu_0)$

$= P(|Z_0| > 3 \mid \mu = \mu_0)$

$= \Phi(-3) + 1 - \Phi(3)$

$= 1 - \Phi(3) + 1 - \Phi(3) = 2 - 2 \times 0.99865 = 0.0027$ #

2.) $\delta = \mu_1 - \mu_0$, $Z_{0.00135} = 3$

consumer's risk : type II error $= \beta = P(\text{Accept } H_0 \mid H_0 \text{ is false})$

$= P(|Z_0| \leq 3 \mid \mu_1 = \mu_0 + \delta)$

$= P\left(-3 \leq \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \leq 3 \mid \mu_1 = \mu_0 + \delta\right)$

$= P\left(-3 \leq \frac{\bar{X} - \mu_1}{\sigma/\sqrt{n}} + \frac{\overbrace{\mu_1 - \mu_0}^{=\delta}}{\sigma/\sqrt{n}} \leq 3 \mid \mu_1 = \mu_0 + \delta\right)$

$= P\left(-3 - \frac{\sqrt{n}\delta}{\sigma} \leq Z \leq 3 - \frac{\sqrt{n}\delta}{\sigma} \mid \mu_1 = \mu_0 + \delta\right)$

$= \Phi\left(3 - \frac{\sqrt{n}\delta}{\sigma}\right) - \Phi\left(-3 - \frac{\sqrt{n}\delta}{\sigma}\right)$ #

3.) $n = 4$, let $d = \frac{\delta}{\sigma}$

$\beta(d) = \Phi(3 - 2d) - \Phi(-3 - 2d)$

$\beta(0) = \Phi(3) - \Phi(-3) = 2 \times 0.99865 - 1 = 0.9973$

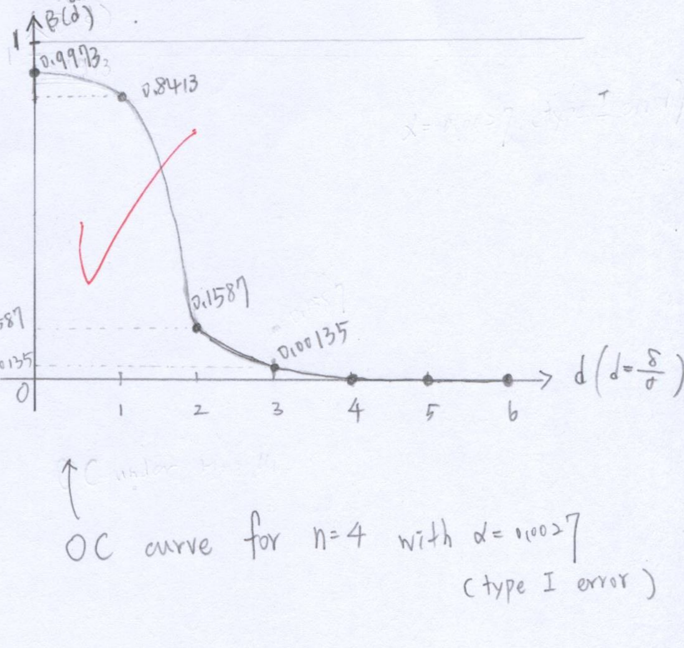
$\beta(1) = \Phi(1) - \underbrace{\Phi(-5)}_{\approx 0} \approx 0.8413$

$\beta(2) \approx \Phi(-1) = 1 - \Phi(1) = 0.1587$

$\beta(3) \approx \Phi(-3) = 1 - \Phi(3) = 1 - 0.99865 = 0.00135$

$\beta(4) \approx \Phi(-5) \approx 0$

$\beta(5) \approx 0$.



OC curve for $n = 4$ with $\alpha = 0.0027$ (type I error)

Tables

Table: the normal distribution $p = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$

z	0	0.5	1	1.5	2	2.5	3	3.5	4
P	0.5	0.6915	0.8413	0.9332	0.9772	0.9938	0.99865	0.99977	0.99997

Table: Conversion of range to standard deviation

n	2	3	4	5	6
d_2	1.128	1.683	2.059	2.326	2.534

Table: Square, square-root table

number	square	square root
3.141593	9.869604	1.772454
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3.000
10	100	3.162
11	121	3.317
11.04	121.8816	3.32265
12	144	3.464