Quality control 2013 Spring, Final Exam [+50 points]

Not only answer but also calculation

Proof and calculation must be clear

1. Basic control charts [+10]

2. Control ellipse [+12]

3. Multivariate chart for μ [+8]

4. LR-Chart for Σ [+10]

5. $|S|$-Chart for Σ [+10]

Your Name:

1. Basic control charts [+10]

1) [+2] An engineer designs a 3-sigma $\overline{X}$ -Chart with the consumer's risk below 0.20. Using some approximation, find the sample size n under $\mu-\mu_0=1$ and $\sigma=1$. Here, μ_0 and μ represent the in-control and out-of-control means.

2) [+2] An engineer designs a 3-sigma p-Chart with the consumer's risk below 0.50. Using some approximation, find the sample size n under $p_0=0.01$ and $p=0.04$. Here, p_0 and p represent the in-control and out-of-control fraction nonconforming.

3) [+2] A manufacture conducts a waterproof testing for 5 electric boards. The number of defective circuits on the board is recorded as follows:

Board ID	1	2	3	4	5
The number of defectives circuits	20	20	23	25	12
The number of circuits	200	200	200	200	200
Defect rates					

Draw a p-control chart.

4) [+2] Remember that the p-Chart uses an approximation $\dfrac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}\approx N(0,1)$, where $\hat{p}=D/n,\quad D\sim Bin(n,p_0)$. Explain the reason why this approximation holds (with mathematical proof).

5) [+2] Discuss whether the normal approximation in the data of **3)** is suitable or not (with some theoretical or numerical explanations).

2. Control ellipse [+12]

Consider data $\mathbf{X}_1,\ldots,\mathbf{X}_{16} \overset{iid}{\sim} N_2(\boldsymbol{\mu},\Sigma)$, where $\boldsymbol{\mu}=\begin{pmatrix}1\\1\end{pmatrix}$, $\Sigma=\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}$.

(1) [+2] Derive the distribution of $\overline{\mathbf{X}}=\frac{1}{16}\sum_{i=1}^{16}\mathbf{X}_i$

(Hint: use the moment generating function or characteristic function)

(2) [+2] Define a control ellipse for monitoring $\boldsymbol{\mu}=\begin{pmatrix}1\\1\end{pmatrix}$.

(3) [+6] Draw a control ellipse for $\alpha = 0.010$.

(4) [+2] Is $\overline{X}=\begin{pmatrix}\overline{X}_1\\ \overline{X}_2\end{pmatrix}=\begin{pmatrix}1/4\\ -1/4\end{pmatrix}$ in-control? Verify your answer.

3. Multivariate chart for $\boldsymbol{\mu}$ [+8]

The data below are generated from the bivariate normal distribution with $N_2\left(\boldsymbol{\mu} = \begin{bmatrix} 50 \\ 30 \end{bmatrix}, \Sigma = \begin{bmatrix} 200 & 100 \\ 100 & 100 \end{bmatrix}\right)$, n = 25 and m = 15.

K	$\bar{\mathbf{X}}_k$	X_0^2	T^2
1	(58, 32)		
2	(60, 33)		
3	(50, 27)		
. .	. . .	. . .	
14	(75, 45)		
15	(55, 27)		

From the above data, we obtained

$$\bar{\bar{\mathbf{X}}} = \begin{bmatrix} 55 \\ 30 \end{bmatrix}, \; S = \begin{bmatrix} 200 & 130 \\ 130 & 120 \end{bmatrix}.$$

1) [+4] Draw χ^2-chart with $\alpha = 0.01$.

2) [+4] Draw Hotelling T^2-chart with $\alpha = 0.01$ (you can assume that m is large enough).

4. LR-Chart for Σ [+10]

The data $\mathbf{X}_{1k},\ldots,\mathbf{X}_{nk}$, $k=1,\ldots,m$ are generated from the bivariate normal distribution with $N_2\left(\boldsymbol{\mu}=\begin{bmatrix}55\\30\end{bmatrix}, \Sigma=\begin{bmatrix}200 & 130\\130 & 120\end{bmatrix}\right)$, n = 25 and m = 15.

Let $\overline{\mathbf{X}}_k=\frac{1}{n}\sum_{i=1}^{n}\mathbf{X}_{ik}$ and $S_k=\frac{1}{n-1}\sum_{i=1}^{n}(\mathbf{X}_{ik}-\overline{\mathbf{X}}_k)(\mathbf{X}_{ik}-\overline{\mathbf{X}}_k)'$.

k	$\overline{\mathbf{X}}_k$	S_k	W_k	$\lvert S_k \rvert$
1	(58, 32)	$\begin{bmatrix}200 & 100\\100 & 100\end{bmatrix}$		
. .	. .	. .		.
7	(42, 20)	$\begin{bmatrix}150 & 100\\100 & 130\end{bmatrix}$		
. .				
15	(55, 27)	$\begin{bmatrix}180 & 130\\130 & 140\end{bmatrix}$		

Based on group k^{th} data, engineer wish to test $H_0:\Sigma=\Sigma_0$ vs. $H_0:\Sigma\neq\Sigma_0$ with $\alpha=0.01$.

1) [+3] Derive the **formula** of the maximized likelihoods $L_k(\hat{\boldsymbol{\mu}}_0,\Sigma_0)$ under H_0 and $L_k(\hat{\boldsymbol{\mu}},\hat{\Sigma})$ under $H_0\cup H_0$ in terms of $A_k=(n-1)S_k$ and Σ_0.

2) [+3] Derive the formula of the LR statistics W_k in terms of $A_k=(n-1)S_k$ and Σ_0. Then, describe the "convergence" of W_k (no need proof, but clearly state).

3) [+4] Draw the likelihood ratio LR-chart.

5. $|S|$-Chart for Σ [+10]

Consider the same data structure as Question 4.

1) [+4] Derive the mean and variance of $|S_k|$.

2) [+4] Draw $|S|$-chart with $\alpha = 0.01$.

3) [+ 2] Which is better between LR-chart and $|S|$-chart? Give theoretical or practical reason [your reason must be scientific; don't use words such as "easy", "convenient", etc.]

Answer 1 (basic chart):

1) Note that $\delta = \mu - \mu_0 = 1$ and $\sigma = 1$. Then, $k = \delta / \sigma = 1$.

$0.2 = \Phi(3 - \sqrt{n}k) \Rightarrow n = \{3 - \Phi^{-1}(0.2)\}^2 = \{3 + 0.84\}^2 = 14.45$

$\therefore$ n = 15

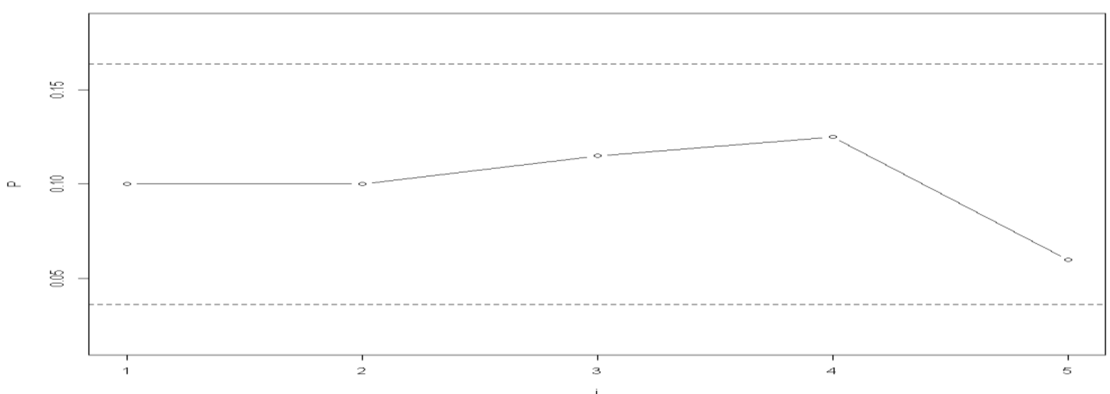
2) If $p = UCL = p_0 + 3\sqrt{\frac{p_0(1-p_0)}{n}}$, then $\beta = 0.5$. Hence, $0.04 = 0.01 + 3\sqrt{\frac{0.01 * 0.99}{n}}$

$\therefore$ n = 99

3) $\bar{p} = \frac{20+20+23+25+12}{200 \times 5} = 0.1$.

UCL= $\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1 + 3\sqrt{\frac{0.09}{200}}$ =0.164,

LCL= $\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1 - 3\sqrt{\frac{0.09}{200}}$ =0.0364



4) One can write $D = \sum_{i=1}^{n} Y_i$, where $Y_1, ..., Y_n \overset{iid}{\sim} Bin(1, p)$. By the CLT,

$$\frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \sum_{i=1}^{n} \frac{Y_i - p_0}{\sqrt{p_0(1-p_0)}} = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{Y_i - p_0}{\sqrt{p_0(1-p_0)}} \xrightarrow{d} N(0,1), \quad n \to \infty .$$

5) For instance, the normal approximation is good if $np_0 > 5$ & $n(1-p_0) > 5$ (p.105 of Casella and Berger, 2002). Other criterion is that the normal approximation is good if $p \geq 0.1 \,\&\, np > 10$ holds (Montgomery, 2009). In this example, $n\bar{p} = 200 \times 0.1 = 20 > 5$. Hence, both conditions are satisfied. Hence, it is appropriate to use the normal approximation.

Answer 2 (Control ellipse):

(1) Note that the m.g.f. of $\mathbf{X}_i$ is $E[\exp(\mathbf{t}'\mathbf{X}_i)] = \exp\left[\mathbf{t}'\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}'\Sigma\mathbf{t}\right]$.

The m.g.f. of $\overline{\mathbf{X}}$ is

$$E[\exp(\mathbf{t}'\overline{\mathbf{X}})] = E[\exp(\frac{1}{n}\sum_{i=1}^{n}\mathbf{t}'\mathbf{X}_i)] = \prod_{i=1}^{n} E\left[\exp\left\{\left(\frac{\mathbf{t}}{n}\right)'\mathbf{X}_i\right\}\right]$$

$$= \left(\exp\left[\left(\frac{\mathbf{t}}{n}\right)'\boldsymbol{\mu} + \frac{1}{2}\left(\frac{\mathbf{t}}{n}\right)'\Sigma\left(\frac{\mathbf{t}}{n}\right)\right]\right)^n = \exp\left[\mathbf{t}'\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}'\left(\frac{\Sigma}{n}\right)\mathbf{t}\right]$$

This is the m.g.f. of $N_2[\boldsymbol{\mu}, \Sigma/n]$. Therefore,

$$\overline{\mathbf{X}} \sim N_2\left[\begin{pmatrix}1\\1\end{pmatrix}, \frac{1}{16}\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}\right].$$

(2) Note that $(\overline{X}_1 - 1, \overline{X}_2 - 1)\left[\frac{1}{16}\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}\right]^{-1}\begin{pmatrix}\overline{X}_1 - 1\\ \overline{X}_2 - 1\end{pmatrix} \sim \chi^2_{df=2}$,

and $\chi^2_{\alpha=0.010, df=2} = 9.21$. Therefore, the control ellipse is

$$C = \left\{(\overline{X}_1, \overline{X}_2) \middle| (\overline{X}_1 - 1, \overline{X}_2 - 1)\left[\frac{1}{16}\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}\right]^{-1}\begin{pmatrix}\overline{X}_1 - 1\\ \overline{X}_2 - 1\end{pmatrix} \le \chi^2_{\alpha=0.010, df=2}\right\}.$$

$$= \left\{(\overline{X}_1, \overline{X}_2) \middle| \quad 5(\overline{X}_1 - 1)^2 + 8(\overline{X}_1 - 1)(\overline{X}_2 - 1) + 5(\overline{X}_2 - 1)^2 \le \frac{9 \times 9.21}{16}\right\}$$

(3) Eigenvalues and vectors of $\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}$ are

$\lambda_1 = 9$, $\mathbf{e}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ and $\lambda_2 = 1$, $\mathbf{e}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$.

Radius of principal axis is $\sqrt{\lambda_1 \chi^2_{\alpha=0.010, df=2}/n} \approx \sqrt{9 \times 9.21/16} = 2.28$,

Radius of its orthogonal axis is $\sqrt{\lambda_2 \chi^2_{\alpha=0.010, df=2}/n} \approx \sqrt{9.21/16}$ =0.76.

$\sigma_1^2\sigma_2^2 - \sigma_{12}^2 = 9$

- On principal axis:

$$\boldsymbol{\mu}+\sqrt{\lambda_1\chi^2_{\alpha=0.010,df=2}/n}\,\mathbf{e}_1=\begin{bmatrix}1\\1\end{bmatrix}+2.28\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}2.61\\-0.61\end{bmatrix}.$$

$$\boldsymbol{\mu}-\sqrt{\lambda_1\chi^2_{\alpha=0.010,df=2}/n}\,\mathbf{e}_1=\begin{bmatrix}1\\1\end{bmatrix}-2.28\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-0.61\\2.61\end{bmatrix}.$$

- On its orthogonal axis:

$$\boldsymbol{\mu}+\sqrt{\lambda_2\chi^2_{\alpha=0.010,df=2}/n}\,\mathbf{e}_2=\begin{bmatrix}1\\1\end{bmatrix}+0.76\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}1.54\\1.54\end{bmatrix}.$$

$$\boldsymbol{\mu}-\sqrt{\lambda_2\chi^2_{\alpha=0.010,df=2}/n}\,\mathbf{e}_2=\begin{bmatrix}1\\1\end{bmatrix}-0.76\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}0.46\\0.46\end{bmatrix}.$$

- If $\bar{X}_1=1$,

$$UCL_{\bar{x}_2}=\mu_2+\sqrt{\frac{\sigma_1^2\sigma_2^2-\sigma_{12}^2}{\sigma_1^2}\frac{\chi^2_{\alpha=0.010,df=2}}{n}}=1+\sqrt{\frac{9}{5}\frac{9.21}{16}}=2.02.$$

$$LCL_{\bar{x}_2}=\mu_2+\sqrt{\frac{\sigma_1^2\sigma_2^2-\sigma_{12}^2}{\sigma_1^2}\frac{\chi^2_{\alpha=0.010,df=2}}{n}}=1-\sqrt{\frac{9}{5}\frac{9.21}{16}}=-0.02.$$

- If $\bar{X}_2=1$,

$$UCL_{\bar{x}_1}=\mu_1+\sqrt{\frac{\sigma_1^2\sigma_2^2-\sigma_{12}^2}{\sigma_2^2}\frac{\chi^2_{\alpha=0.010,df=2}}{n}}=1+\sqrt{\frac{9}{5}\frac{9.21}{16}}=2.02.$$

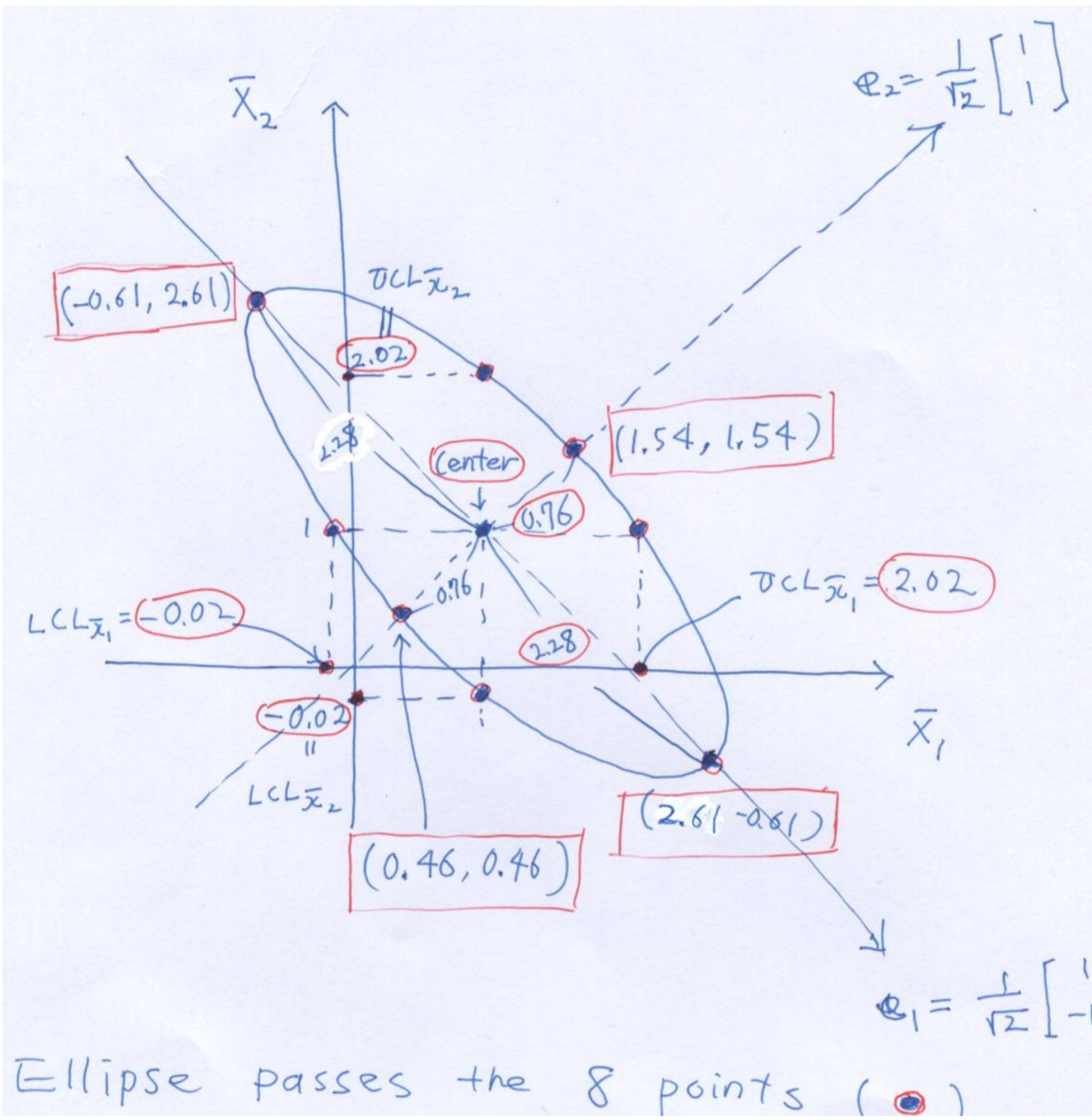
$$LCL_{\bar{x}_1}=\mu_1+\sqrt{\frac{\sigma_1^2\sigma_2^2-\sigma_{12}^2}{\sigma_2^2}\frac{\chi^2_{\alpha=0.010,df=2}}{n}}=1-\sqrt{\frac{9}{5}\frac{9.21}{16}}=-0.02.$$

(4) Let $(\bar{X}_1,\bar{X}_2)=(1/4,-1/4)$. The chi-square statistics is

$$(\bar{X}_1-1,\bar{X}_2-1)\left[\frac{1}{16}\begin{pmatrix}5&-4\\-4&5\end{pmatrix}\right]^{-1}\begin{pmatrix}\bar{X}_1-1\\\bar{X}_2-1\end{pmatrix}=(-3/4,-5/4)\left[\frac{1}{16}\begin{pmatrix}5&-4\\-4&5\end{pmatrix}\right]^{-1}\begin{pmatrix}-3/4\\-5/4\end{pmatrix}$$

$$=(-3,-5)\begin{pmatrix}5&-4\\-4&5\end{pmatrix}^{-1}\begin{pmatrix}-3\\-5\end{pmatrix}=\frac{1}{9}(-3,-5)\begin{pmatrix}5&4\\4&5\end{pmatrix}\begin{pmatrix}-3\\-5\end{pmatrix}=290/9=32.22>\chi^2_{\alpha=0.010,df=2}=9.21$$

$\therefore$ out-of-control.


$\bar{X}_2$
$e_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$
(-0.61, 2.61)
UCL$_{\bar{x}_2}$
2.02
2.28
Center
(1.54, 1.54)
0.76
1
LCL$_{\bar{x}_1}$ = -0.02
0.76
UCL$_{\bar{x}_1}$ = 2.02
2.28
-0.02
LCL$_{\bar{x}_2}$
$\bar{X}_1$
(2.61 -0.61)
(0.46, 0.46)
Ellipse passes the 8 points

Answer 3 (Multivariate Chart):

UCL= $\chi^2_{\alpha=0.010,df=2} = 9.21$ for (1) and (2)

Since $N_2\left(\boldsymbol{\mu}=\begin{bmatrix}50\\30\end{bmatrix}, \Sigma=\begin{bmatrix}200 & 100\\100 & 100\end{bmatrix}\right)$, $\Sigma^{-1}=\frac{1}{10000}\begin{bmatrix}100 & -100\\-100 & 200\end{bmatrix}=\frac{1}{100}\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix}$.

Since $\overline{\overline{\mathbf{X}}}=\begin{bmatrix}55\\30\end{bmatrix}$, $S=\begin{bmatrix}200 & 130\\130 & 120\end{bmatrix}$, $S^{-1}=\frac{1}{7100}\begin{bmatrix}120 & -130\\-130 & 200\end{bmatrix}=\frac{1}{710}\begin{bmatrix}12 & -13\\-13 & 20\end{bmatrix}$.

For k=1,

$$X_0^2=\frac{1}{4}(8,2)\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix}\begin{bmatrix}8\\2\end{bmatrix}=10, \quad T^2=\frac{25}{710}(3,2)\begin{bmatrix}12 & -13\\-13 & 20\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}=\frac{25\times 32}{710}=11.27.$$

Similarly, we obtain as below:

k	$\overline{X}_1$	$\overline{X}_2$	X_0^2	T^2
1	58	32	10.00 (out)	1.127
2	60	33	14.50 (out)	3.169
3	50	27	4.50	3.169
14	75	45	81.25 (out)	52.817 (out)
15	55	27	18.25 (out)	6.338

Codes:

```
n=25
x1=c(58,60,50,75,55);x2=c(32,33,27,45,27)

m=length(x1)
x_mat=cbind(x1,x2)
Mu=c(50,30)
Sigma=matrix(c(200,100,100,100),2,2)

X2=NULL
for(k in 1:m){
  x_vec=c(x1[k],x2[k])
  X2[k]=t(c(x_vec-Mu))%*%solve(Sigma/n)%*%c(x_vec-Mu)
}

qchisq(1-0.01,df=2) ### UCL

Mu=c(55,30)
Sigma=matrix(c(200,130,130,120),2,2)

T2=NULL
for(k in 1:m){
x_vec=c(x1[k],x2[k])
T2[k]=t(c(x_vec-Mu))%*%solve(Sigma/n)%*%c(x_vec-Mu)
}

qchisq(1-0.01,df=2) ### UCL
round(T2,3)
```

Answer 4 (LR-chart):

1) $L_k(\hat{\boldsymbol{\mu}}_0, \Sigma_0) = \dfrac{1}{(2\pi)^{25} |\Sigma_0|^{25/2}} \exp\left\{-\dfrac{1}{2}\text{tr}(\Sigma_0^{-1} A_k)\right\}$, $L_k(\hat{\boldsymbol{\mu}}, \hat{\Sigma}) = \dfrac{25^{25}}{(2\pi)^{25} |A_k|^{25/2}} \exp\{-25\}$.

2) $\dfrac{L_k(\hat{\boldsymbol{\mu}}_0, \Sigma_0)}{L_k(\hat{\boldsymbol{\mu}}, \hat{\Sigma})} = \dfrac{|A_k|^{25/2}}{25^{25} |\Sigma_0|^{25/2}} \exp\left\{-\dfrac{1}{2}\text{tr}(\Sigma_0^{-1} A_k) + 25\right\}$.

$$W_k = -2\log \frac{L_k(\hat{\boldsymbol{\mu}}_0, \Sigma_0)}{L_k(\hat{\boldsymbol{\mu}}, \hat{\Sigma})} = -25\log\left(\frac{|A_k|}{|\Sigma_0|}\right) + 50\log 25 + \text{tr}(\Sigma_0^{-1} A_k) - 50 \xrightarrow{d} \chi^2_{df=3} \quad (n \to \infty).$$

3) UCL= $\underline{\chi^2_{\alpha=0.01, df=3} = 11.34}$.

Since $\Sigma = \begin{bmatrix} 200 & 130 \\ 130 & 120 \end{bmatrix}$,

$|\Sigma| = 200 \times 120 - 130^2 = 24000 - 16900 = 7100$, $\Sigma^{-1} = \dfrac{1}{710}\begin{bmatrix} 12 & -13 \\ -13 & 20 \end{bmatrix}$

For k=1,

$S_1 = \begin{bmatrix} 200 & 100 \\ 100 & 100 \end{bmatrix}$, $A_1 = 24 \times \begin{bmatrix} 200 & 100 \\ 100 & 100 \end{bmatrix}$, $|A_1| = 24^2 \times \left|\begin{bmatrix} 200 & 100 \\ 100 & 100 \end{bmatrix}\right| = 24^2 \times 10000$.

$\dfrac{|A_1|}{|\Sigma|} = \dfrac{24^2 \times 10000}{7100} = \dfrac{24^2 \times 100}{71} = 811.27$,

$\Sigma^{-1} A_k = \dfrac{24}{710}\begin{bmatrix} 12 & -13 \\ -13 & 20 \end{bmatrix} \times \begin{bmatrix} 200 & 100 \\ 100 & 100 \end{bmatrix} = \dfrac{24}{710}\begin{bmatrix} 1100 & -100 \\ -600 & 700 \end{bmatrix}$.

$$W_1 = -25\log\left(\frac{|A_k|}{|\Sigma_0|}\right) + 50\log 25 + \text{tr}(\Sigma_0^{-1} A_k) - 50 = -25\log(811.27) + 50\log 25 + \frac{24}{710} \times 1800 - 50$$

$\underline{= 4.32 < 11.34 \quad (\text{in-control})}$

For k=7 and 15,

$\underline{W_7 = 5.61 < 11.34 \quad (\text{in-control})}$,

$\underline{W_{15} = 1.55 < 11.34 \quad (\text{in-control})}$

Answer 5 (S-chart):

1) Note that $|S_k| \sim \frac{|\Sigma|}{(n-1)^2}\prod_{i=1}^{p}\chi^2_{n-i}$.

$$E|S_k| = \frac{|\Sigma|}{(n-1)^p}\prod_{i=1}^{p}(n-i) = b_1|\Sigma|,$$

$$E|S_k|^2 = \frac{|\Sigma|^2}{(n-1)^{2p}}\prod_{i=1}^{p}E(\chi^2_{n-i})^2 = \frac{|\Sigma|^2}{(n-1)^{2p}}\prod_{i=1}^{p}\{2(n-i)+(n-i)^2\} = \frac{|\Sigma|^2}{(n-1)^{2p}}\prod_{i=1}^{p}(n-i)\prod_{i=1}^{p}(2+n-i)$$

.

$$Var(|S_k|) = \frac{|\Sigma|^2}{(n-1)^{2p}}\prod_{i=1}^{p}(n-i)\prod_{i=1}^{p}(2+n-i) - \left[\frac{|\Sigma|}{(n-1)^p}\prod_{i=1}^{p}(n-i)\right]^2$$

$$= \frac{|\Sigma|^2}{(n-1)^{2p}}\prod_{i=1}^{p}(n-i)\left[\prod_{i=1}^{p}(2+n-i) - \prod_{i=1}^{p}(n-i)\right] = b_2|\Sigma|^2$$

2) $z_{\alpha/2} = z_{0.01/2} = z_{0.005} = \Phi(0.995) = 2.58$ (by Table).

$$|\Sigma| = \left|\begin{bmatrix} 200 & 130 \\ 130 & 120 \end{bmatrix}\right| = 200\times120 - 130^2 = 24000 - 16900 = 7100 .$$

$$b_1 = \frac{(25-1)(25-2)}{(25-1)^2} = \frac{23}{24} = 0.958 ,$$

$$b_2 = \frac{(25-1)(25-2)}{(25-1)^4}\{(27-1)(27-2)-(25-1)(25-2)\} = \frac{23}{24^3}98 = 0.163 .$$

<u>Center= $b_1|\Sigma| = 7100\times0.958 = 6801.8$.</u>

<u>UCL= $b_1|\Sigma| + z_{\alpha/2}\times\sqrt{b_2}\,|\Sigma| = 6801.8 + 2.58\sqrt{0.163}\times7100 = 14197.37$.</u>

<u>LCL= $b_1|\Sigma| - z_{\alpha/2}\times\sqrt{b_2}\,|\Sigma| = 6801.8 - 3.29\sqrt{0.163}\times7100 =$ -593.77</u> .

<u>$|S_1| = 10000$ (in-control)</u>

<u>$|S_7| = 9500$ (in-control)</u>

<u>$|S_{15}| = 8300$ (in-control).</u>