- **1 :**

  The following data are generated by using R program with seed 1000.

  - **Model 1 ($\mu = 5, \sigma_w = 1$):**

$$x_{ij} = \mu + \sigma_w \varepsilon_{ij}, i = 1,2,\ldots,20; j = 1,2,\ldots 5$$

$$\varepsilon_{ij} \sim normal(0,1)$$

Group (i)	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	$\bar{X}_{i\bullet}$
1	4.554222	3.794143	5.041126	5.639388	4.213446	4.648465
2	4.614511	4.524132	5.719751	4.981494	3.626882	4.693354
3	4.017572	4.445511	5.121381	4.879128	3.663959	4.425510
4	5.170057	5.155079	5.024932	2.953415	5.213154	4.703327
5	7.670072	3.772984	5.834247	5.532572	4.353175	5.432610
6	5.603161	3.216156	5.334942	5.560976	6.220936	5.187234
7	4.788546	5.699430	4.293563	4.534849	3.233801	4.510038
8	5.189289	4.633819	6.057601	4.258379	3.651641	4.758146
9	4.482694	6.411736	5.185465	4.956309	4.784087	5.164058
10	6.463775	5.229667	5.107624	3.621897	4.031817	4.890956
11	5.251711	3.905306	5.397643	4.003698	5.100578	4.731787
12	5.953680	3.209677	5.311701	7.553988	4.139162	5.233642
13	5.543928	4.607662	6.235442	6.196086	4.504253	5.417474
14	4.705659	4.426503	6.619209	4.043072	5.041237	4.967136
15	3.501690	5.660959	5.285458	6.388866	4.840656	5.135526
16	4.539081	5.168438	6.395493	5.728426	5.335090	5.433306
17	6.169276	5.247967	4.641851	6.383493	5.412069	5.570931
18	4.876992	4.933771	2.677509	3.954343	7.057875	4.700098
19	6.971532	3.079005	5.462126	4.839276	4.895788	5.049546
20	5.467839	5.443921	5.828553	4.612950	7.018938	5.674440

- **Model 2 ($\mu = 5, \sigma_B = 0.5, \sigma_w = 1$):**

$$x_{ij} = \mu + \sigma_B \omega_i + \sigma_w \varepsilon_{ij}, i = 1,2,\ldots,20; j = 1,2,\ldots 5$$

$$\varepsilon_{ij} \sim normal(0,1); \omega_i \sim normal(0,1)$$

Group (i)	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	$\bar{X}_{i\bullet}$
1	4.884458	3.993059	4.773532	5.383147	4.639781	4.734795
2	4.409352	4.247851	3.747666	5.304714	4.405890	4.423095
3	4.432590	5.738276	5.982648	5.124387	4.936076	5.242795
4	4.916164	6.549170	3.946011	4.039151	5.526762	4.995451
5	5.321651	5.555885	4.250995	5.961483	3.881700	4.994343
6	4.743361	6.319757	6.799351	5.129820	5.807677	5.759993
7	4.242360	5.992219	5.295910	6.531243	5.166016	5.445550
8	2.065207	2.887680	4.196393	4.017300	2.807315	3.194779
9	5.699279	6.437563	7.509910	5.215444	5.082408	5.988921
10	5.295805	6.010172	6.065763	4.976307	5.604828	5.590575
11	5.736996	4.608594	5.197830	4.558380	4.074746	4.835309
12	3.656535	6.760157	6.078518	5.087129	4.668696	5.250207
13	6.570281	5.913282	6.382530	5.998608	5.388063	6.050553
14	4.983247	4.414903	5.757359	6.037060	4.543601	5.147234
15	5.545699	4.745326	5.195877	6.256265	6.334660	5.615565
16	6.413839	5.334606	4.896458	5.096295	3.821501	5.112540
17	6.062135	5.874254	3.461577	4.910589	4.484323	4.958576
18	4.175903	4.408097	4.157622	5.211966	3.617642	4.314246
19	5.633567	6.613473	5.672301	3.748557	5.093693	5.352318
20	3.488085	5.073371	4.613394	4.712177	4.652583	4.507922

- **Model 3(** $\mu = 5, \sigma_B = 0.5, \sigma_w = 1, \rho = 0.3$ **) :**

$$x_{ij} = \mu + W_i + \sigma_w \varepsilon_{ij}, i = 1,2,\ldots,20;\ j = 1,2,\ldots 5$$
$$where$$
$$W_i = \rho W_{i-1} + \sigma_B \omega_i$$
$$and$$
$$W_0 = 0$$
$$\varepsilon_{ij} \sim normal(0,1); \omega_i \sim normal(0,1)$$

Group (i)	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	$\overline{X}_{i\bullet}$
1	5.487341	4.559299	5.022075	4.568612	5.030033	4.933472
2	4.944559	4.756679	4.765088	4.944346	5.537157	4.989566
3	4.517332	5.093143	4.528666	4.257059	5.292468	4.737734
4	5.017799	4.981661	5.006866	4.946724	5.031146	4.996839
5	4.640710	4.258222	4.801851	6.152139	6.044380	5.179460
6	5.022369	5.027563	5.126835	5.288733	5.049887	5.103077
7	5.768936	4.601219	4.524366	5.255006	5.120015	5.053908
8	4.972723	4.990932	4.983278	4.977369	5.036822	4.992225
9	5.158329	5.370061	4.709943	4.932691	5.386069	5.111419
10	4.812349	4.823376	5.128690	4.785637	5.030073	4.916025
11	5.395632	5.187932	5.188117	5.667299	5.552015	5.398199
12	5.083087	5.135706	4.970993	5.143301	5.212035	5.109024
13	5.419851	4.295871	5.030562	4.918669	5.686552	5.070301
14	5.316610	6.012962	5.623861	5.499797	5.552126	5.601071
15	4.995852	4.459541	6.110687	4.938734	4.989878	5.098938
16	4.997310	4.994206	4.998930	4.988557	5.012709	4.998342
17	4.927663	5.109915	5.222868	4.767768	5.098294	5.025301
18	4.714467	4.811075	4.964654	4.488815	5.147538	4.825310
19	3.646588	6.581917	5.343577	4.474809	5.638300	5.137038
20	5.076920	4.913621	5.010453	4.848895	4.872633	4.944504

■ **Model 4 (** $\mu=5,\sigma_B=0.5,\sigma_w=1,\rho=0.7$ **):**

$$x_{ij}=\mu+W_i+\sigma_w\varepsilon_{ij}, i=1,2,\ldots,20;\ j=1,2,\ldots 5$$
$$where$$
$$W_i=\rho W_{i-1}+\sigma_B\omega_i$$
$$and$$
$$W_0=0$$
$$\varepsilon_{ij}\sim normal(0,1);\omega_i\sim normal(0,1)$$

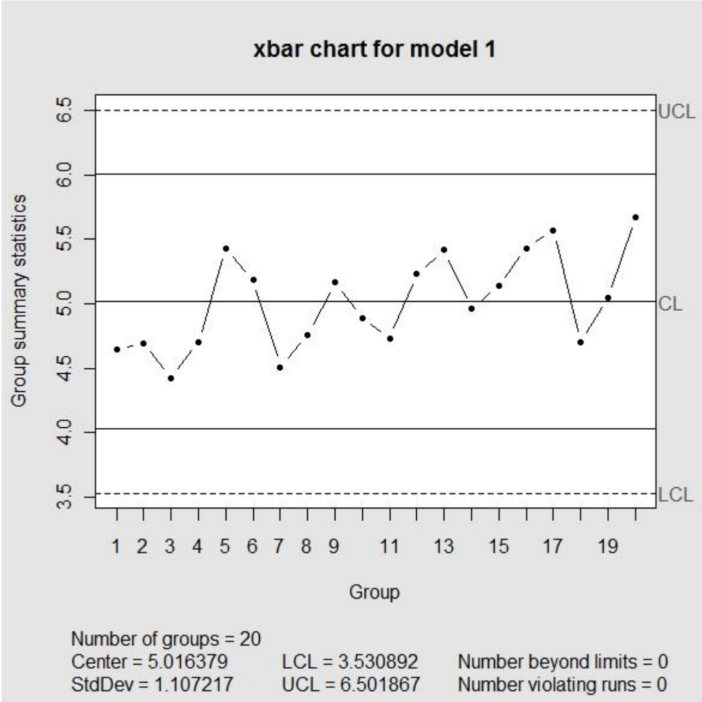
Group (i)	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	$\overline{X}_{i\bullet}$
1	5.075752	5.057924	5.140108	5.021519	5.243833	5.107827
2	5.379592	5.311999	4.696614	5.291032	4.671180	5.070083
3	4.882123	5.125858	4.834064	5.133317	5.183257	5.031724
4	4.994319	4.995737	4.991742	5.002208	5.005740	4.997949
5	5.235881	5.318053	5.505534	5.012952	4.569988	5.128482
6	5.858052	4.653725	4.695446	4.926322	5.608402	5.148389
7	5.841066	4.136100	5.041714	4.358691	5.400584	4.955631
8	4.771886	4.859224	4.976178	4.863013	5.789497	5.051960
9	4.937166	4.983229	4.995191	5.006615	5.050224	4.994485
10	4.958852	5.004160	5.035276	4.974598	5.006001	4.995777
11	4.994094	4.950768	5.321380	5.026626	5.193332	5.097240
12	5.223393	4.607918	5.360498	5.604016	5.258636	5.210892
13	7.230278	6.093693	4.782537	5.947688	5.199249	5.850689
14	4.930252	4.797881	5.207956	5.483842	5.263724	5.136731
15	5.407964	4.602265	4.740606	4.968977	4.970147	4.937992
16	4.859654	4.905138	4.696297	4.822177	5.098281	4.876310
17	4.765400	5.120552	4.514176	5.294420	4.669034	4.872716
18	4.993846	5.016594	5.001760	5.017756	4.991342	5.004260
19	4.535201	4.843544	4.901321	4.903630	5.020605	4.840860
20	5.005116	5.023708	4.994223	5.005175	4.983104	5.002265

- **2 :**

  We apply the qcc package,

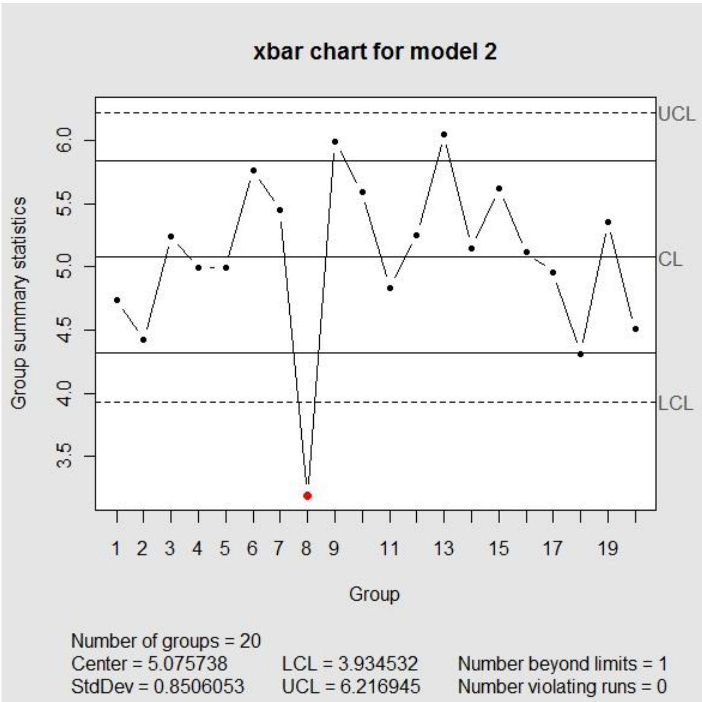
  and then add the upper warning limit and lower warning limit.

  - **Model 1:**



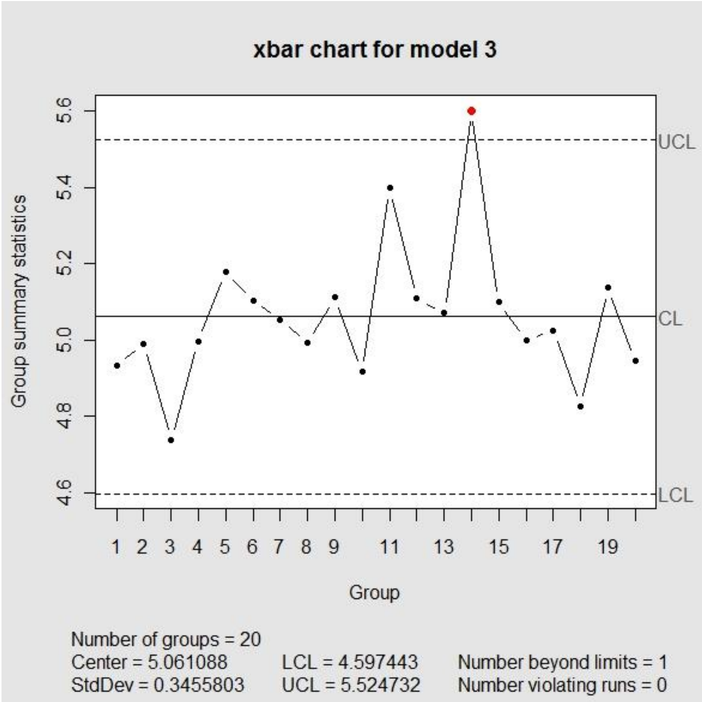
Short comment : the process is in-control.

  - **Model 2:**



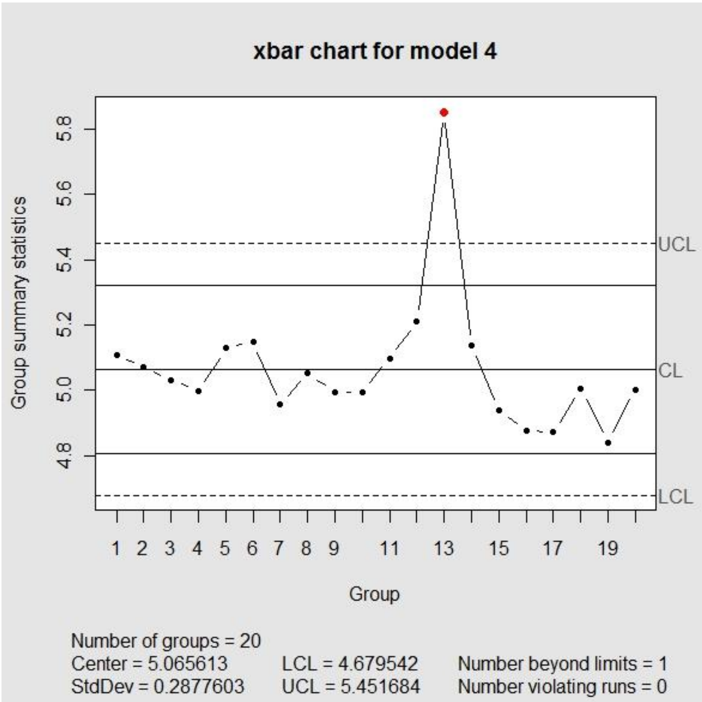
Short comment : the process is out-of-control at group 8.

- **Model 3:**



Short comment : the process is out-of-control at group 14.

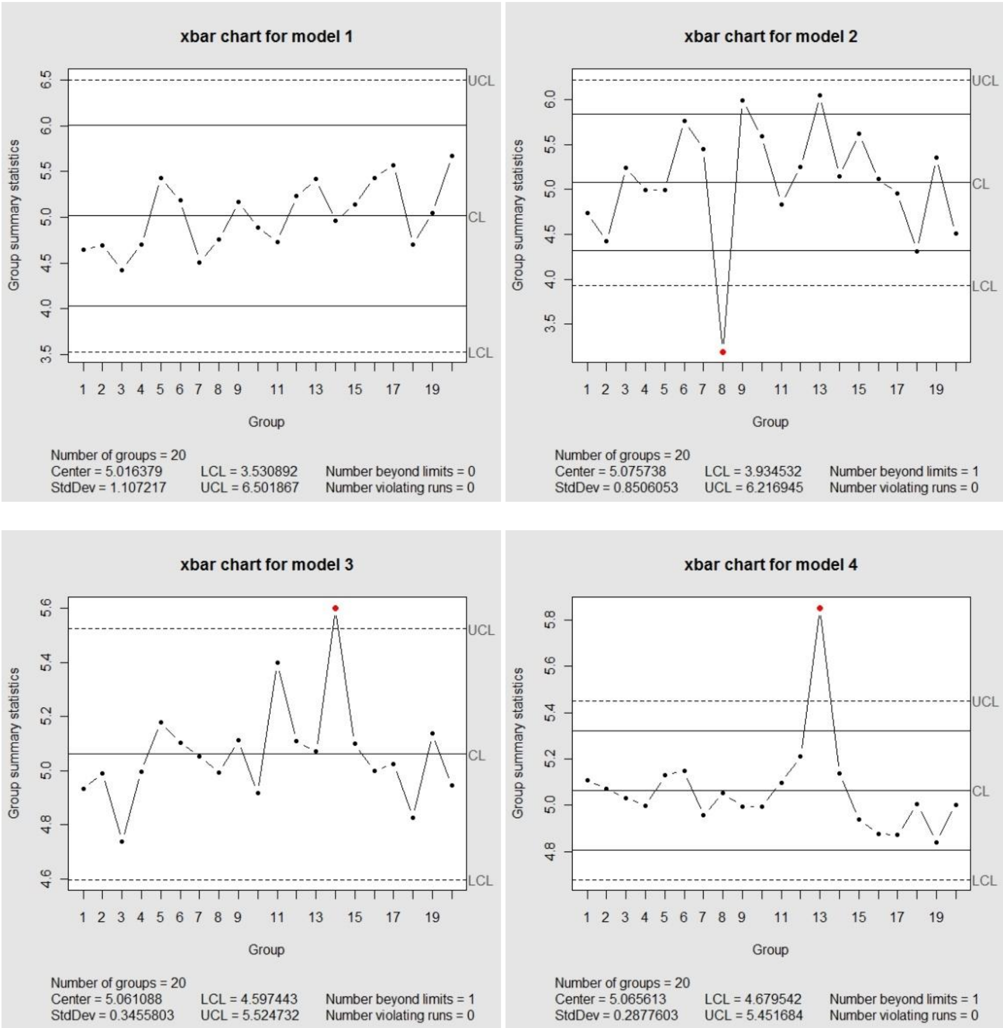
- **Model 4:**



Short comment : the process is out-of-control at group 13.

- **3 :**

From top to right (model 1, model 2, model 3, model 4)



Model 1 is the simplest model, it only involve the within-group variance.
Therefore, the variation of between group doesn't have clear discrepancy.
However, the model 2 add the between-group variance into the model, which results the group mean have pretty distance.
Then, we see the model 3 put the correlation coefficient into the model, which means that the group will be effected by the previous group.
Clearly ,the xbar chart for model 3 shows that the jump from group to group are not much frequently.
Moreover, the model 4 put the high correlation coefficient($\rho = 0.7$) into the model, which leads the jump from group to group are more stable than model 3.

- **4 :**
  - **By monogram :**

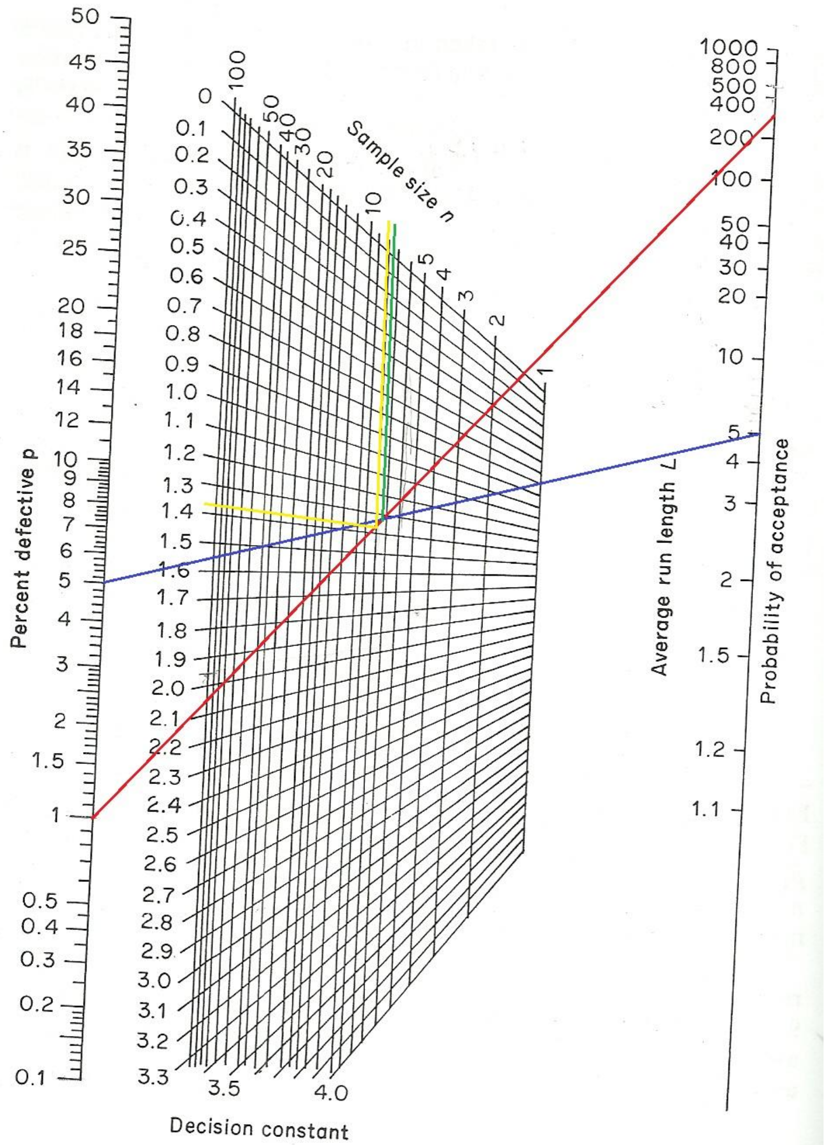


Figure 9.3 Nomogram for single specification limit, action line only.

n=8

the modify $k_A = 1.37$

- **By formula :**

$$q_a = \frac{1}{L_a} = \frac{1}{300} = 0.0033$$

$$q_r = \frac{1}{L_r} = \frac{1}{5} = 0.2$$

$$Z_{q_a} = 2.72$$

$$Z_{q_r} = 0.84$$

$$Z_{p_a} = 2.33$$

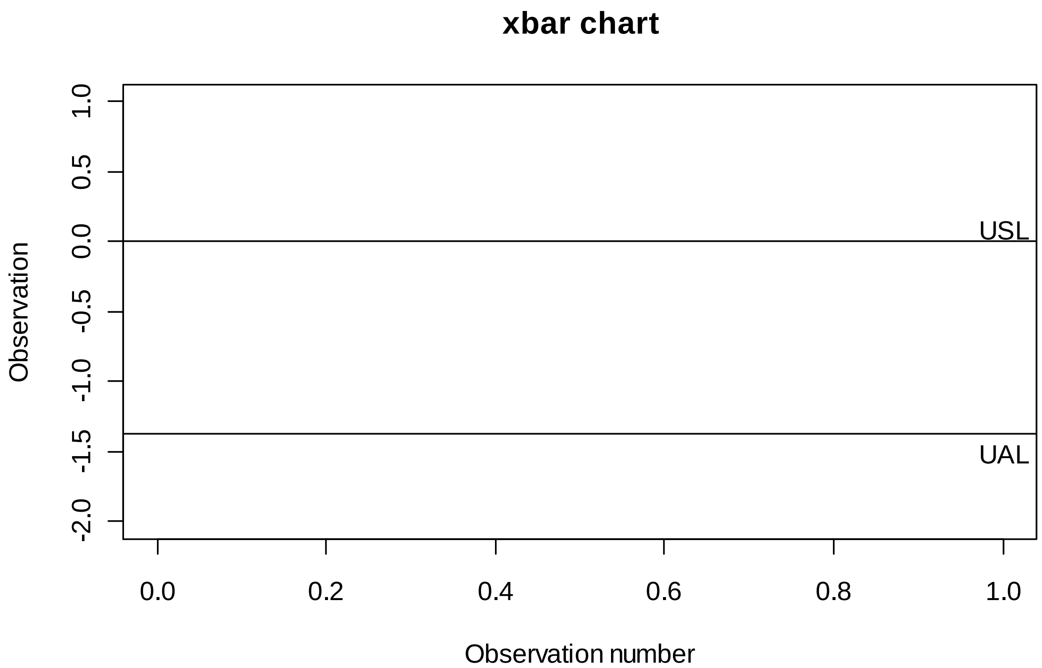
$$Z_{p_r} = 1.645$$

$$n = \frac{\left(Z_{q_a} - Z_{q_r}\right)^2}{\left(Z_{p_a} - Z_{p_r}\right)^2} = \frac{(2.72 - 0.84)^2}{(2.33 - 1.645)^2} = 7.5324$$

we take $n = 8$

$$k_A = Z_{p_a} - \frac{Z_{q_a}}{\sqrt{n}} = 2.33 - \frac{2.72}{\sqrt{8}} = 1.368335$$

The values of both methods are quite close.



(Figure made by 廖哲瑋)

R codes

```
library(MASS)
library(qcc)

### model 1 ###
A<-matrix(nrow=20,ncol=5)
e<-matrix(nrow=20,ncol=5)

for(i in 1:20){
e[i,]<-rnorm(5,0,1)
}


for(i in 1:20){
for(j in 1:5){
A[i,j]<-5+1*e[i,j]
}
}

a<-qcc(A[1:20,],type="xbar")
summary(qcc(A[1:20,],type="xbar"))

abline(h=a$center+2*a$std.dev/sqrt(5),lty=1)
abline(h=a$center-2*a$std.dev/sqrt(5),lty=1)



### model 2 ###

B<-matrix(nrow=20,ncol=5)
e<-matrix(nrow=20,ncol=5)
w<-rnorm(20,0,1)
for(i in 1:20){
e[i,]<-rnorm(5,0,1)
}

for(i in 1:20){
```

```
for(j in 1:5){
B[i,j]<-5+0.5*w[i]+1*e[i,j]
}
}



b<-qcc(B[1:20,],type="xbar")
summary(qcc(B[1:20,],type="xbar"))

abline(h=b$center+2*b$std.dev/sqrt(5),lty=1)
abline(h=b$center-2*b$std.dev/sqrt(5),lty=1)

### model 3 ###
rho<-0.3
sigmaB<-0.4
C<-matrix(nrow=20,ncol=5)
e<-matrix(nrow=20,ncol=5)
w<-rnorm(20,0,1)
for(i in 1:20){
e[i,]<-rnorm(5,0,1)
}
W<-numeric()
W[1]<-rho*0+sigmaB*w[1]

for(i in 2:20){
W[i]<-rho*W[i-1]+sigmaB*w[i]
}

for(i in 1:20){
for(j in 1:5){
C[i,j]<-5+W[i]*1*e[i,j]
}
}

c<-qcc(C[1:20,],type="xbar")
summary(qcc(C[1:20,],type="xbar"))
abline(h=c$center+2*c$std.dev/sqrt(5),lty=1)
abline(h=c$center-2*c$std.dev/sqrt(5),lty=1)
```

```
### model 4 ###
rho<-0.7
sigmaB<-0.4
D<-matrix(nrow=20,ncol=5)
e<-matrix(nrow=20,ncol=5)
w<-rnorm(20,0,1)
for(i in 1:20){
e[i,]<-rnorm(5,0,1)
}
W<-numeric()
W[1]<-rho*0+sigmaB*w[1]

for(i in 2:20){
W[i]<-rho*W[i-1]+sigmaB*w[i]
}

for(i in 1:20){
for(j in 1:5){
D[i,j]<-5+W[i]*1*e[i,j]
}
}



d<-qcc(D[1:20,],type="xbar")
summary(qcc(D[1:20,],type="xbar"))
abline(h=d$center+2*d$std.dev/sqrt(5),lty=1)
abline(h=d$center-2*d$std.dev/sqrt(5),lty=1)
```