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Spot and derivative pricing in the EEX power market ☆

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Abstract

Using spot and futures price data from the German EEX Power market, we test the adequacy of various one-factor and two-factor models for electricity spot prices. The models are compared along two different dimensions: (1) We assess their ability to explain the major data characteristics and (2) the forecasting accuracy for expected future spot prices is analyzed. We find that the regime-switching models clearly outperform its competitors in almost all respects. The best results are obtained using a two-regime model with a Gaussian distribution in the spike regime. Furthermore, for short and medium-term periods our results underpin the frequently stated hypothesis that electricity futures quotes are consistently greater than the expected future spot, a situation which is denoted as contango.

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[1] Views and opinions expressed in this article are my personal and do not necessarily reflect the views of my employer.

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1. Introduction

The last decade has witnessed radical changes in the structure of many power markets in the world. Electricity markets are being transformed from a highly regulated government controlled system into deregulated local markets. Energy exchanges like the European Energy Exchange (EEX) in Germany have been established as competitive wholesale markets where electricity spot prices as well as futures and forward contracts are traded. As pointed out by Pilipovic (1997), electricity trading has transformed from a primarily technical business, to one in which the product is treated quite the same way as any other commodity. However, one has to bear in mind that electricity is a very unique commodity that cannot be economically stored, while end user demand shows strong seasonality. Further, effects like power plant outages or imperfect transmission grid reliability may have extreme effects on electricity prices.

Since electricity markets switched from virtually fixed and regulated prices to the introduction of competitive pricing, both consumers as producers are exposed to significantly greater risks (for a detailed discussion see e.g. Kaminski, 1999). The typical characteristics of electricity spot prices have been studied by various authors (e.g., Pilipovic, 1997; Clewlow and Strickland, 2000) and include seasonality, mean-reverting behavior, high volatilities, and the occurrence of jumps and spikes. While stock market returns usually display daily standard deviations in the range of 1–2%, it is not uncommon for electricity price changes to have daily standard deviations of up to 40% (e.g., Mugele et al., 2005). As a consequence, for the modeling of electricity prices and the valuation of electricity derivatives, one has to adjust developed for financial or alternative commodity markets.

In recent years, a variety of models has been suggested to capture the above mentioned characteristics of electricity prices. The first critical step in defining a model for electricity prices consists of finding an appropriate description of the seasonal pattern. Bhanot (2000) uses dummy variables or piece-wise constant functions (see also Lucia and Schwartz, 2002; Mugele et al., 2005), an approximation by sinusoidal functions is applied by Pilipovic (1997) and Weron et al. (2004a), whereas Simonsen (2003) and Weron et al. (2004b) approximate the underlying periodical structure by a wavelet decomposition. The description of the remaining stochastic component is the second critical ingredient. Clewlow and Strickland (2000), Johnson and Barz (1999) and Eydeland and Geman (2000) suggest the use of jump diffusion models to account for the observed spikes. Kholodnyi (2004) models spikes as non-Markovian stochastic processes allowing for self-reversing jumps. Escribano et al. (2002) propagates the use of GARCH models to account for the volatility clustering. Finally, regime switching models have gained high popularity in modeling electricity prices. Initially suggested in the context of electricity markets by Huisman and Mahieu (2003), a variety of model extensions has been introduced in recent years (among others e.g. Huisman and De Jong, 2003; Weron et al., 2004a; Bierbrauer et al., 2004; Haldrup and Nielsen, 2004).

Our goal is to conduct a detailed and broad empirical analysis by examining the explanatory power and the goodness-of-fit of various different electricity price models. In particular, we consider the jump-diffusion model of Kluge (2004), the non-linear mean-reverting model suggested in Barlow (2002) and certain different specifications of regime-switching models. As data source we have chosen the German EEX power market that shows a steady increase both in number and volume of traded products since its opening in 1999. Our results indicate that all models achieve a similar accuracy in forecasting the conditional mean, however, regime-switching models have more explanatory power with respect to the extreme observations. The analysis of the predictive power leads to an intuitive test about the presence of contango. Given a market quote for the futures price and a consistent estimate for the expected future spot allows us to determine consistent estimates for the futures premium paid in electricity market. Our results are in accord with the findings of Longstaff and Wang (2004) and Botterud et al. (2002) who report a significant positive premium (which translates into a negative market price of risk).

The remainder of the article is organized as follows. Section 2 summarizes the stylized facts about spot electricity prices. Different models for electricity spot prices are treated in Section 3, Section 4 presents the empirical results and Section 5 concludes.

2. Stylized facts of electricity markets

From an economic point of view, electricity is a non-storable good which causes demand and supply to be balanced on a knife-edge. Relatively small changes in load or generation can cause large changes in price and all in a matter of hours. The special characteristics of electricity spot prices will be briefly summarized in this section.

2.1. Seasonality

Due to the real time balancing needs of electricity supply and demand and an exogenously given cyclical demand, resulting electricity prices are very cyclical as well. The seasonal component in electricity prices is more pronounced than in any other commodity and several different seasonal patterns can be found in electricity prices during the course of a day, week and year. They mainly arise due to changing level of business activities or climate conditions, such as temperature or the number of daylight hours. In some countries also the supply side shows seasonal variations in output. Hydro units, for example, are heavily dependent on precipitation and snow melting, which varies from season to season. Thus, the seasonal fluctuations in demand and supply translate into the seasonal behavior of spot electricity prices.

2.2. Volatility

Another stylized fact of electricity spot prices is the unusually high volatility of the that is unprecedented in any other financial or other commodity markets. It is not unusual to observe annualized volatilities of more than 1000% on hourly spot prices. The high volatility can be traced back to storage, capacity and transmission problems and the need for markets to be balanced in real time. Inventories cannot be used to smooth price fluctuations. Temporary demand and supply imbalances in the market are difficult to correct in the short-term. As a result price movements in electricity markets are more extreme than in other commodity markets.

2.3. Mean reversion

Besides seasonality, electricity spot prices – as well as other commodity prices – are in general regarded to be mean reverting, (e.g. Schwartz, 1997). The form of mean reversion observed in electricity markets is a critical difference to most other financial markets. Interest rate markets, for instance, exhibit mean reversion in a weak form – the actual rate of reversion appears to be related to economic cycles and is therefore slow. In electricity markets, however, the rate of reversion is very strong, what can be explained by the markets fundamentals. When there is an increase in demand generation assets with higher marginal costs will enter the market on the supply side, pushing prices higher. When demand returns to normal levels, these generations assets with relatively high marginal costs will be turned off and prices will fall. This rational operating policy for the employment of generation assets supports the assumption of mean reversion in electricity spot prices. Further, the determinants of demand like weather and climate are cyclical as well.

2.4. Jumps and spikes

In addition to mean reversion and strong seasonality, spot electricity prices exhibit infrequent, but large spikes or jumps. Price jumps tend to occur due to sudden outages or failures in the power grid and lead to a large increase in prices in a very short amount of time. From a modeling point of view, price jumps are unpredictable discontinuities in the price process. Spikes, however are typically interpreted as the result of a sudden increase in demand and when demand reaches the limit of available capacity, the electricity prices exhibit positive price spikes. In periods of lower demand, electricity prices fall. Due to the operating cost or constraints of generators, who cannot adjust to the new demand level, also negative price spikes can occur. From a modeling point of view, price spikes are short time intervals where the price process exhibits a non-Markovian behavior and where prices increase or decrease significantly in a continuous way. The typical explanation for these phenomena is a highly non-linear supply-demand curve in combination with the non-storability of electricity.

3. Spot price models for electricity prices

From the stylized facts presented in the previous section, we conclude that one cannot simply rely on models developed for financial or other commodity markets when modeling electricity prices or pricing electricity derivatives. The following subsections provide a brief overview of the most popular approaches suggested in the literature so far – including mean-reverting diffusions, jump-diffusions and regime-switching models. Following the literature (e.g., Pilipovic, 1997; Lucia and Schwartz, 2002), we assume – unless otherwise stated – that the models discussed below describe the stochastic component of the log-price process after applying an appropriate demeaning procedure. More precisely, if S_t denotes the spot price at time t, then we discuss stochastic models for $(Y_t)_{t\geqslant 0}$, with

$$Y_t = \log S_t - f_t, \quad t \geqslant 0, \tag{1}$$

where f_t is a deterministic function describing the seasonal pattern. The estimation of f_t will be further discussed in Section 4.

3.1. Mean-reversion models

One of the first models that has been examined in the context of electricity markets is the classical Vasiček-process. Lucia and Schwartz (2002) assume

$$\mathrm{d}Y_t = \alpha(\mu - Y_t)\mathrm{d}t + \sigma\,\mathrm{d}W_t, \quad t \geqslant 0, \tag{2}$$

where $(Y_t)_{t\geqslant 0}$ represents the deseasonalized log-price process, $(W_t)_{t\geqslant 0}$ a standard Brownian motion, and α, μ and σ are real constants. This process offers analytical tractability and straight-forward parameter estimation. A main drawback of model specification (2) is its lack to explain the observed price spikes.

A probabilistically related approach – although quite different from the underlying motivation – has been developed in Barlow (2002). By making certain assumptions on the functional form of the supply and demand curve, the author derives a non-linear Ornstein–Uhlenbeck process as appropriate model for electricity prices. In electricity markets, the price S_t at time t is determined by equating supply $u = u(t, S_t)$ and demand $d = d(t, S_t)$ – both a function of time and the current price S_t. Barlow (2002) assumes that the supply is independent of time $(u(t, S_t) = g(S_t))$ whereas the demand $d(t, S_t) = D_t$ can be modeled by a mean-reverting stochastic process $(D_t)_{t\geqslant 0}$ which is independent of the current price level S_t. Choosing g to be the Box–Cox-transformation, the author shows that the resulting model for the spot price process can be expressed as

$$S_t = \begin{cases} (1+\beta X_t)^{\frac{1}{\beta}}, & \beta \neq 0, \\ \exp(X_t), & \beta = 0, \end{cases} \quad t \geqslant 0 \tag{3}$$

where β denotes a real parameter. The process $(X_t)_{t\geqslant 0}$ is a function of the demand process $(D_t)_{t\geqslant 0}$ and assumed to have the following dynamic:

$$\mathrm{d}X_t = \alpha(\mu - X_t)\mathrm{d}t + \sigma\,\mathrm{d}W_t, \quad t \geqslant 0. \tag{4}$$

As we are not concerned with fitting demand data, the exact relationship between D and X is of no further importance. The behavior of the process depends crucially on the values of the parameter β. For $\beta = 0$, we obtain the classical mean-reverting specification for the log-prices whereas for $\beta = 1$ the spot price itself follows a mean-reverting diffusion. In general, the non-linear transform is designed to help explaining the observed price jumps and the smaller the values for β, the more pronounced the jumps will be.

3.2. Stochastic models with jumps

Early publications on models for electricity prices with a jump component include Deng (1999), Johnson and Barz (1999), Bhanot (2000), Clewlow and Strickland (2000), Knittel and Roberts (2001). All these models are typically based on a jump-diffusion as in Merton (1976). Thus, the process describing the stochastic part $(Y_t)_{t\geqslant 0}$ of the deseasonalized log-price process equals

$$\mathrm{d}Y_t = \alpha(\mu - Y_t)\mathrm{d}t + \sigma\,\mathrm{d}W_t + q\,\mathrm{d}N_t, \quad t \geqslant 0, \tag{5}$$

where $(W_t)_{t\geqslant 0}$ is a standard Brownian motion and $(N_t)_{t\geqslant 0}$ a homogeneous Poisson process with intensity λ. The jump size q is often assumed to be normally distributed with mean v and variance τ^2. Once a jump has been triggered by the jump part, the mean-reversion part is responsible to force the price to fall back to its normal level. However, due to the short-term lived nature of jumps or spikes in electricity prices this may be not fast enough and leads to an erroneous specification of the true mean-reverting process. Hence, recent publications suggest to capture the fast mean-reverting behavior of the jumps by considering two different mean-reversion rates for the normal and the jump part. Kluge (2004) suggests the following specification for $(Y_t)_{t\geqslant 0}$:

$$Y_t = X_t + Z_t, \quad t \geqslant 0, \tag{6}$$

where the dynamics of the two processes X and Z are described by a mean-reverting diffusion $\mathrm{d}X_t = \alpha(\mu - Y_t)\mathrm{d}t + \sigma\,\mathrm{d}W_t$ and mean-reverting jump-process $\mathrm{d}Z_t = -\beta Z_t\mathrm{d}t + q\,\mathrm{d}N_t$ where α, μ, σ and β are real parameters. The interpretation for $(W_t)_{t\geqslant 0}$, $(N_t)_{t\geqslant 0}$ and q remains the same as before.

3.3. Regime-switching models

Regime-switching models have been introduced in various different contexts by Quandt (1958), Goldfeld and Quandt (1973) and Hamilton (1989, 1994). The underlying idea is to model the observed stochastic behavior of a specific time series by two separate phases or regimes with different underlying processes. In our context, this means, that a sudden jump in electricity prices could be considered as a change to another regime. The switching mechanism is typically assumed to be governed by a time-homogeneous hidden Markov chain with $k \in \mathbb{N}$ different possible states representing the k different regimes. For the ease of exposition we restrict our description of regime-switching models to the discrete time case.

3.3.1. Regime-switching models with two independent states

The regime-switching model with two independent states distinguishes between a base regime ($R_t = 1$) and a spike regime ($R_t = 2$), where $(R_t)_{t\in I}$ represents a time-homogeneous hidden Markov chain. The observable stochastic process $(Y_t)_{t\in\mathbb{N}}$ – again the reader may think of the stochastic component of the deseasonalized log-price process for electricity – is now represented in the form $Y_t = Y_{t,R_t},\ t \in \mathbb{N}$ where the processes $(Y_{t,1})_{t\in\mathbb{N}}$ and $(Y_{t,2})_{t\in\mathbb{N}}$ are assumed to be independent from each other and also independent from $(R_t)_{t\in\mathbb{N}}$. Y_t equals $Y_{t,i}$ given that the current regime at time t equals i, i.e. given $R_t = i$. The transition matrix Π of the hidden Markov chain R contains the conditional probabilities p_{ij} of switching from regime i at time t to regime j at time $t+1$:

$$\Pi = (p_{ij})_{i,j=1,2} = (P(R_{t+1} = j \mid R_t = i))_{i,j=1,2} = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix}. \tag{7}$$

The probability of being in state j at time $t+m$ starting from state i at time t can be expressed as

$$(P(R_{t+m} = j \mid R_t = i))_{i,j=1,2} = (\Pi')^m \cdot e_i, \tag{8}$$

where Π' denotes the transpose of Π and e_i denotes the ith column of the 2×2 identity matrix.

Our remaining task is to specify the two stochastic processes $Y_{t,1}$ and $Y_{t,2}$. Considering the typical behavior of electricity spot prices described in the previous section, it seems reasonable to assume a mean-reverting process for the "base regime" ($R_t = 1$). The "spike regime" ($R_t = 2$) is more difficult to handle. A typical path will admit huge jumps from time to time but after the jump the path will come back to a normal level rather. This suggests that the series does usually not show consecutive observations from the 'spike regime', but rather a number of single isolated observations. As a result, the intertemporal dependence of the process $S_{t,2}$ is virtually unobservable. Therefore, the process $S_{t,2}$ describing the spike regime is modeled by independent and identically distributed realizations of a probability distribution F. The Gaussian (Huisman and De Jong, 2003), lognormal (Weron et al., 2004a) and Pareto (Bierbrauer et al., 2004) distribution have been suggested in the literature as appropriate candidates for F, but other distributions for modeling the spike regime are possible, too.[2]

In summary, we are considering the following two stochastic processes:

$$Y_{t,1} = c + \phi Y_{t-1,1} + \epsilon_t, \quad t \in \mathbb{N} \tag{9}$$

for the base regime and

$$Y_{t,2} \overset{\text{iid}}{\sim} F, \quad t \in \mathbb{N} \tag{10}$$

for the spike regime. The innovations ϵ_t in Eq. (9) are assumed to be iid centered normal, ϕ and c denote real constants and F is the chosen distribution function for the spike regime. Note that process (9) is the discrete version of a standard Vasiček model.

It has also been suggested to force the process directly back into the base regime after a jump into the spike regime has occurred. From a modeling perspective, this translates into considering the basic regime switching model with two independent states as specified in Eqs. (9) and (10) with the following restricted transition matrix:

$$\Pi^r = (p_{ij})_{i,j=1,2} = (P(R_{t+1} = j \mid R_t = i))_{i,j=1,2} = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 & 0 \end{pmatrix}. \tag{11}$$

This model will be denoted as two independent regime-switching model with single jumps.

3.4. A regime-switching model with three states

Huisman and Mahieu (2003) propose a regime-switching model with three possible regimes. The idea behind this specification differs explicitly from the previously described two-state models. The authors identify three possible regimes: (1) the 'base regime' ($R_t = 1$) modeling the 'normal' electricity price dynamics, (2) an 'initial jump regime' ($R_t = 2$) for a sudden increase or decrease in price, and (3) a 'reversing jump regime' ($R_t = 3$) describing how prices move back to the normal regime after the initial jump has occurred. Obviously, the underlying idea implies that the initial jump regime is immediately followed by the reversing regime and then moves back to the base regime, such that $p_{23} = p_{31} = 1$ and $p_{13} = 0$. The process will be described by a mean-reverting process in the base regime, a random walk in the initial and reversing jump regime where the direction of the innovation processes in the initial and reversing jump regime are opposed. We have

[2] When choosing the distribution for the spike regime, one should keep in mind that one is actually modeling the log-spot prices and therefore a heavy-tailed distribution might lead to an infinite mean for the price process itself. This in turn might create problems for derivative pricing.

$$Y_t = \begin{cases} \phi Y_{t-1} + c + \epsilon_t, & t \in \mathbb{N} \quad R_t = 1 \text{ (base)}, \\ Y_{t-1} + \xi_t & \phantom{t \in \mathbb{N}} \quad R_t = 2 \text{ (initial jump)}, \quad t \in \mathbb{N}, \\ Y_{t-1} - \xi_t & \phantom{t \in \mathbb{N}} \quad R_t = 3 \text{ (reverse jump)} \end{cases} \tag{12}$$

where $\epsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2)$ represent the innovations of the base regime and $\xi_t \overset{\text{iid}}{\sim} N(\nu, \tau^2)$ the innovations of the jump regimes. Obviously, the normal distribution in the initial and reversing jump regime could be replaced by alternative distributional assumptions.

The reader may notice that according to specification (12), the price processes in the different regime are no longer independent. In contrast to the two-regime models, the three-regime model does not allow for consecutive spikes or to remain at a different price level for two or more periods after a jump.

4. Empirical results

4.1. Data description

For our empirical analysis we use daily average spot prices from the European Electricity Exchange (EEX) in Leipzig. At the EEX, the spot market is a day-ahead market and the spot is an hourly contract with physical delivery on the next day. The 24 hourly spot prices are determined in a daily auction. Products range from predetermined hourly blocks for each of the 24 h of a day to special contracts for base load, peak load and weekend contracts (see EEX, 2004, for more details). The futures market offers derivative products that do not comprise physical settlement of electricity during the delivery period, but are primarily used as hedging instrument against market uncertainties. The underlying of the futures contract is the so-called Phelix Index (Physical Electricity Index) that is calculated from spot market prices on a daily basis. Depending on the corresponding products on the spot market, the index distinguishes between base load and peak load. The Phelix base *day* price is an equally weighted average of all 24 hourly spot prices for that particular day. The Phelix base *month* price is the mean of all Phelix base day prices of that month. These arithmetic averages over a specific period are the reference prices in all cash-settlement calculations at expiration of derivative contracts. EEX currently offers futures both on Phelix Base and Peak Index with different delivery periods of one month, three months and one year. The delivery period specifies the Phelix index that serves as underlying. For example, in the case of a Phelix base month futures with maturity December 2005, the reference price at maturity is the value of the Phelix base month index in December 2005 (see Fig. 1).

Our data comprises three years of Phelix base day prices from October 1, 2000 to September 30, 2003 totaling 1095 observations and a set of futures quotes from the last day of our price sample, i.e. of September 30, 2003. The dynamic of the spot and spot log-prices is visualized in Fig. 2. A summary statistics of the raw spot price data set and related series is presented in Table 1. The data display all the properties which we previously summarized in Section 2. Prices range between 3.12 and 240.26€. The minimum was reached on May 1, 2003, an official holiday, and the maximum on a cold December day in 2001. The standard deviation of daily logarithmic price changes equals 37.0% which translates into an annualized volatility of 707%. The spot prices admit a kurtosis of over 100 and a skewness of 7.5 implying a heavy-tailed and right skewed price distribution.

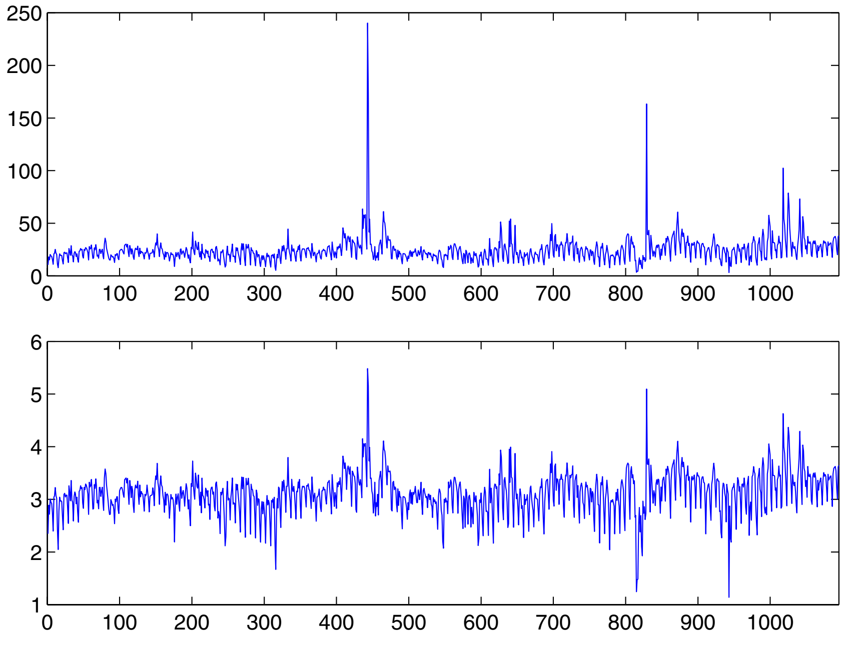

Fig. 1. *Top panel*: Spot prices of the Phelix base (day) index from October 1, 2000 to September 30, 2003. *Bottom panel*: Spot log-prices for the same period.

4.2. Deseasonalizing the data

As previously mentioned we assume that the observed log-spot price $\log S_t$ is a sum of a deterministic component f_t and a stochastic component Y_t (see Eq. (1)). For the deterministic seasonal component f_t sinusoidal (Pilipovic, 1997; Weron et al., 2004a), constant piece-wise functions (Pindyck, 1999; Knittel and Roberts, 2001; Huisman and De Jong, 2003; Haldrup and Nielsen, 2004), a combination of both methods (Lucia and Schwartz, 2002; De Jong, 2005; Kosater and Mosler, 2006) have been suggested in the literature. Alternative approaches by Simonsen (2003) and Weron et al. (2004b) approximate the underlying periodical structure using a wavelet decomposition. We pursue a hybrid approach of constant piece-wise functions and a sinusoidal cycle to capture long-term seasonal effects. Hence, we specify dummy variables for daily and monthly effects, a trend component and an additional sinusoidal component with one-year cycle:

$$f(t) = \alpha + \beta \cdot t + d \cdot D_{\text{day}} + m \cdot D_{\text{mon}} + \gamma \cdot \sin\left((t+\tau)\frac{2\pi}{365}\right), \tag{13}$$

where α, β, γ and τ are all constant parameters. Note that initially for each day (day $= 1,\ldots,7$) and month (mon $= 1,\ldots,12$) a dummy variable D_{day}, D_{mon} was used. Hereby d and m denote the corresponding parameter vector. In a first step, the function $f(t)$ was calibrated via numerical optimization using non-linear least squares regression in Matlab. The initial results roughly validated general assumptions about intra-week and intra-year price patterns for power markets (e.g. Pindyck, 1999): Prices are higher at the beginning of the week reaching their peak on Tuesday and then, from the middle

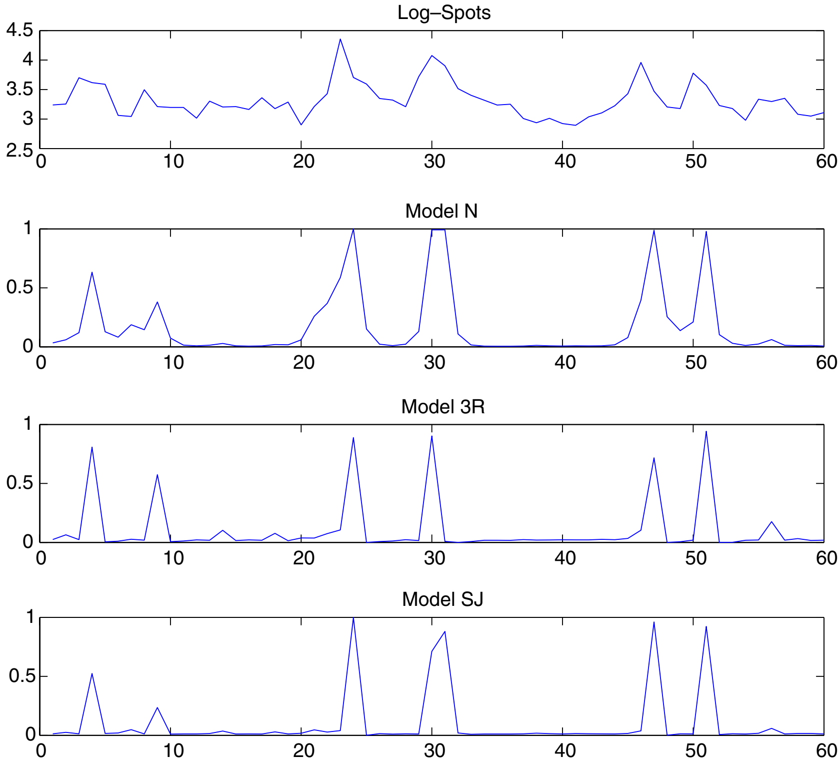

Fig. 2. *Top panel*: Deseasonalized log-spot price from July 3, 2003 to August 31, 2003. *Upper middle panel*: Probability of being in the spike regime of the estimated two-regimes model with normal distribution for the spike regime for the same time period. *Lower middle panel*: Probability of being in the spike regime for the three-regime model with normally distributed jumps. *Bottom panel*: Probability of being in the spike regime for the two-regimes model with single jumps for the same period.

Table 1
Summary statistics for the daily system price data, the log-prices, the price changes and the log-price changes and other related time series prior to removing any deterministic or seasonal components

Series	N	Mean	Median	Min	Max	Std Dev	Skew	Kurt
Phelix base								
P_t	1095	24.496	22.950	3.120	240.26	12.83	7.538	102.58
$\ln P_t$	1095	3.118	3.133	1.138	5.48	0.39	−0.005	4.08
$P_t - P_{t-1}$	1094	0.025	−1.125	−131.36	214.75	12.762	4.065	108.40
$\ln P_t - \ln P_{t-1}$	1094	0.001	−0.05	−1.96	2.369	0.370	0.881	4.41

of the week, they constantly decline to reach their lowest level over the weekend. A similar cycle can be observed during the year: Prices usually tend to be higher during the cold winter months and are somewhat lower during the rest of the year. However, while in Eq. (13) parameters for the constant, trend and sinusoidal cycle are significantly different from zero, for daily and monthly effects only a fraction of the parameters were significant.

Furthermore, the residuals show asymmetry and excess kurtosis that may be due to a number of extreme outliers and autocorrelation in the data.

We further investigated the influence of outliers or large price jumps on the estimation of the seasonal component. For this purpose, following Clewlow and Strickland (2000) we use a recursive filter to identify price jumps in the sample distribution of daily log-returns. The filter consists of an iterative procedure that is repeated until no more jumps can be identified: In the first step we calculate the sample standard deviation $\hat{s}$ of the log-returns before we identify returns beyond a certain range – measured in multiples of $\hat{s}$ – and identify these as extreme returns. Clewlow and Strickland (2000) suggest three standard deviation as the limit. Returns within that limit are treated as 'normal' price returns, while the other returns are identified as outliers. We replace the outliers in the log-spot prices by the median of all prices having the same weekday and month as the outlier. Then the next iteration is performed. The procedure stops after five iterations. Fitting the model (13) to the series which has been adjusted for outliers yields better results. The residuals are nearly symmetric and show substantially less kurtosis than before replacement of the outliers. Starting with a model including all possible independent variables, in a backwards stepwise regression non-significant variables were excluded from the model, remaining with a model including only significant variables (see Table 2): the constant, trend and sinusoidal cycle, dummy variables for Tuesday, Wednesday, Thursday, Saturday and Sunday as well as for the months February, April, October and November.

Note that with respect to the mean-reverting nature of the electricity prices, a joint estimation of the deterministic component and parameters of the stochastic model may be favorable. However, following the literature on spot price modeling of electricity prices (e.g. Ethier and Mount, 1998; Huisman and Mahieu, 2003; Haldrup and Nielsen, 2004; Weron et al., 2004a) there seems to be a preference for adjusting the data for seasonal effects before the stochastic component is estimated. Furthermore, since our main focus is on comparison of the performance of different stochastic models, it appears to be more adequate to use the same deseasonalized time series as input for all models. The deseasonalized log-price (the stochastic component) Y_t was obtained by subtracting f_t from the original log-spot price series. All stochastic processes estimated in the next subsection are considered as models for Y_t.

4.3. Models and estimation methodology

Among the models we consider for the stochastic component of the deseaonalized log-prices Y_t, are the classical mean-reverting diffusion (see Eq. (2)), the non-linear OU model of Barlow (2002) as specified in Eq. (3), the jump-diffusion model as specified in Eq. (5) and the special jump-diffusion of Kluge (2004) (see Eq. (6)). Furthermore, we consider

Table 2
Parameter estimates for the non-linear least-square regression after detection and adjustment for outliers

Parameter estimate													
α	β	γ	τ	d_{tue}	d_{wed}	d_{thu}	d_{sat}	d_{sun}	d_{feb}	d_{apr}	d_{jun}	d_{oct}	d_{nov}
Deseasonalization													
3.0493	0.0003	−0.1463	−140.39	0.0897	0.1027	0.0912	−0.2448	−0.5270	0.0802	0.0900	0.1079	−0.1921	−0.1172
(0.02)	**	(0.01)	(5.00)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)

four variants of the two-regime model as specified in Eqs. (9) and (10),[3] and in addition the three-regime model as given by Eq. (12). As discussed in the introduction, all these models have been propagated in the literature as appropriate for modeling electricity spot prices and it will be interesting to see the results of a comparative study. In summary, we have

(1) Model MR: Mean-reverting diffusion.
(2) Model B: Non-linear Ornstein–Uhlenbeck (Barlow).
(3) Model JD: Merton's jump-diffusion with a normal jump distribution with mean v and variance τ^2.
(4) Model K: Kluge's electricity price model with a normal jump distribution with mean v and variance τ^2.
(5) Model N: F is the distribution function of a normal distribution with mean v and variance τ^2.
(6) Model LN: F is the distribution function of a lognormal distribution where the underlying normal distribution has mean v and variance τ^2.
(7) Model E: $F = 1 - e^{-\frac{x}{v}}$ equals the distribution function of an exponential distribution with mean v.
(8) Model SJ: Single jump version of Model N with $p_{21} = 1$.
(9) Model 3R: 3 regime model.

For estimation purposes, all models will be considered in discretized form. The mean-reversion process (Model MR) is then equivalent to a Gaussian AR(1) process:

$$Y_t = c + \phi Y_{t-1} + \epsilon_t, \quad t = 2, 3, \ldots, \ \epsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2), \tag{14}$$

where the relation between the AR(1) parameters and the original parameters is given by the equations $c = \alpha \cdot \mu$ and $\phi = 1 - \alpha$. The non-linear OU model (Model B) can be estimated by a two step procedure. Given a value for the non-linearity parameter β, the transformed prices follow a classical mean-reverting OU process and using Eq. (14) likelihood values can be obtained as a function of β. These likelihood values now have to be maximized with respect to β.

The estimation of Kluge's electricity price model is not straightforward. Due to the unobservability of the two processes X and Z and the fact that jumps have a lasting effect on subsequent prices due to the autoregressive structure of the jump process, Kluge suggests an iterative filtering procedure. Starting with an initial guess for the model parameters it is possible to identify jumps with the help of a 3-σ rule. Once a jump is identified the subsequent log-prices can be cleaned from the jump effect and one can search for the next jump. After all jumps have been eliminated, new process parameters can be estimated from the cleaned series and the jump size information. The classical jump-diffusion (Model JD) will be considered as a restricted version of Model K where both speeds of mean reversion take the same value: $\alpha = \beta$. The estimation can now be performed using the algorithm described for Model K.

The first three variants of the two-regime model differ only by the distributional assumption F for the spike regime as specified in Eq. (10) and all regime-switching models

[3] The models E and LN lead to infinite mean price processes and are listed for pure comparison purposes. They are inappropriate for pricing derivatives.

use a mean-reverting process for the base regime as given in Eq. (14). Given a series $y_1, \ldots, y_T$ of realizations from the observable stochastic process $(Y_t)_{t\in\mathbb{N}}$ with $Y_t = S_{t,R_t}$, the parameter estimation can be performed using the EM algorithm introduced by Dempster et al. (1977).

4.4. Estimation results

The following paragraphs discuss the estimation results for the various different models, we have discussed above. Standard errors are given for all estimates in parentheses below and are calculated via estimation of the White robust covariance matrix by means of finite difference approximations for the second derivative of the log-likelihood function.

The estimation results for the mean-reversion and jump models are reported in Tables 3 and 4.

Comparing the results we find that the speed of mean reversion $\alpha = 1 - \phi$ and the innovation variance σ^2 are remarkably lower for the jump-diffusion models (Models JD, K) than for the mean-reverting models (Models MR, B) (see Table 4). The reason is simply that the jump models separate extreme prices from the normal prices while the mean-reverting models do not. We proceed with reporting the estimation results for the regime-switching models. The model and distribution parameter estimates for the five different variants are reported in Table 5 Panel (a)–(c) and Table 6 Panel (a) and (b).

We start our discussion of the estimation results with Models N, LN and E, initially suggested by Huisman and Mahieu (2003). All three variants of the basic model with two independent regimes yield reasonable parameter estimates with comparably small standard errors. The parameter estimates for the AR(1) process in the base regime are consistent throughout the three different variants. The estimated probability of being in the base regime equals approximately 90% for model N and LN and approximately 97% for model E. At the same time, model E estimates a significantly higher variance in the spike regime (7.7) than models N and LN (0.76 and 0.43) which can obviously be explained by the fact that the exponential distribution has only one degree of freedom to fit the mean and the variance of the spike regime whereas the normal and lognormal distribution offer the flexibility of two separate parameters. For all three variants, the estimated variance in the spike regime is significantly higher than the one in the base regime – the factors range from 12 for model N to 150 for model E. As a summary, we can state

Table 3
Estimation results for Barlow's model

Model	Parameter			
	β	c	ϕ	σ
Model MR	0	2.05 (0.02)	0.373 (0.05)	0.238 (0.02)
Model B	0.088 (0.01)	3.500 (0.08)	0.375 (0.03)	0.310 (0.02)

Model MR reports the estimation results, when the non-linearity parameter β is restricted to zero, in which case Model B reduces to a standard mean-reverting diffusion for the log-price process. Model B contains the estimation results when β is estimated.
The speed of mean reversion α equals $\alpha = 1 - \phi$ and the long-term mean equals $\mu = c/\alpha$.

Table 4
Estimation results for Kluge's model

Model	Parameter						
	c	ϕ	σ	ψ	λ	v	τ
Model JD	1.525	0.501	0.224		0.020	−0.051	0.574
	(0.10)	(0.03)	(0.01)		(0.01)	(0.12)	(0.12)
Model K	1.369	0.551	0.210	0.257	0.024	−0.075	0.579
	(0.09)	(0.03)	(0.01)	(0.06)	(0.01)	(0.11)	(0.10)

Model JD reports the estimation results for the restricted Model K which equals a classical jump-diffusion (Model JD). The row "Model K" contains the estimation for Kluge's electricity price model.
The speed of mean reversion α equals $\alpha = 1 - \phi$ and the long-term mean equals $\mu = c/\alpha$. λ is the intensity of the Poisson process and the jump distribution is normal with mean v and variance τ^2. $\beta = 1 - \psi$ equals the speed of mean reversion for the mean-reverting jump process in model K.

Table 5
Parameter estimates and basic statistics for the basic model with two independent regimes

Regime	Parameter estimates						Statistics		
	c	ϕ	σ	v	τ	p_{ii}	$P(R=i)$	$E(Y_{t,i})$	$V(Y_{t,i})$
Panel (a): Model N									
Base	1.105	0.639	0.145	–	–	0.953	0.891	3.065	0.036
$(i=1)$	(0.17)	(0.06)	(0.01)			(0.01)			
Spike	–	–	–	2.916	0.658	0.618	0.109	2.916	0.433
$(i=2)$				(0.10)	(0.10)	(0.06)			
Panel (b): Model LN									
Base	1.081	0.647	0.150	–	–	0.960	0.913	3.065	0.038
$(i=1)$	(0.24)	(0.08)	(0.01)			(0.01)			
Spike	–	–	–	1.016	0.296	0.587	0.087	2.886	0.763
$(i=2)$				(0.04)	(0.05)	(0.10)			

	Parameter estimates					Statistics		
	c	ϕ	σ	v	p_{ii}	$P(R=i)$	$E(Y_{t,i})$	$V(Y_{t,i})$
Panel (c): Model E								
Base	1.142	0.627	0.173	–	0.982	0.966	3.058	0.049
$(i=1)$	(0.68)	(0.22)	(0.02)		(0.01)			
Spike	–	–	–	2.783	0.481	0.034	2.783	7.745
$(i=2)$				(0.14)	(0.07)			

Standard errors are given in parenthesis.
Panel (a): Estimation results for a two-state regime-switching model with Gaussian distribution in the spike regime (Model N).
Panel (b): Estimation results for the two-state regime-switching model with a lognormal distribution in the spike regime (Model LN).
Panel (c): Estimation results of the two-state regime-switching model with an exponential distribution in the spike regime (Model E).

that the basic model with two independent regimes seems to be a promising candidate for capturing the specific features in the dynamic of electricity prices. From this preliminary analysis, it seems advisable to use at least a two parameter distribution family to model the spike regime. The estimation results for the two-state model with single jumps (Model SJ) are different. Since the process has to switch back to the base regime after a spike has

Table 6
Parameter estimates and basic statistics for model SJ and 3R

Regime	Parameter estimates						Statistics		
	c	ϕ	σ	ν	τ	p_{ii}	$P(R=i)$	$E(Y_{t,i})$	$V(Y_{t,i})$
Panel (a): Model SJ									
Base	1.360	0.555	0.169	–	–	0.943	0.946	3.057	0.041
$(i=1)$	(0.10)	(0.03)	(0.01)			(0.02)			
Spike	–	–	–	2.898	0.814		0.054	2.898	0.663
$(i=2)$				(0.11)	(0.12)				
	Parameter estimates						Statistics		
	c	ϕ	σ	ν_i	τ_i	p_{ii}	$P(R=i)$	$E(Y_{t,i})$	
Panel (b): Model 3R									
Base	1.251	0.590	0.155	–	–	0.939	0.891	3.053	
$(i=1)$	(0.19)	(0.06)	(0.02)			(0.02)			
Initial jump	–	–	–	0.249	0.404		0.055	3.302	
$(i=2)$				(0.11)	(0.06)				
Reverse jump	–	–	–	−0.249	0.404		0.055	3.053	
$(i=3)$				(0.11)	(0.06)				

Standard errors are given in parenthesis.
Panel (a): Estimation results of the two-state regime-switching model with single jumps (Model SJ). The spike regime uses a normal distribution.
Panel (b): Estimation results of a three-state regime-switching model with Gaussian jumps (Model 3R).

occurred the probability of being in the spike regime is halved from 10.9% to 5.4%. As a compensation, the variance of the spike regime increases by about 50% to 0.66.

Finally, we examine the estimation results of the three-state regime switching model (Model 3R) proposed by Huisman and Mahieu (2003). We see that the estimates for the AR(1)-process describing the base regime do not significantly differ from the two-state models with normal spikes or single jumps. The probability of being in one of the two jump regimes is 5.5% and the mean jump size is 0.249. The table does not contain the variance for the base and spike regime as the model is not stationary due to the random walk hypothesis for the spike regime. The conceptual difference between the different specifications for the spike regime are visualized in Fig. 2. The graph shows the series of deseasonalized log-prices and the probability of being in the spike regime for the Models N, 3R, and SJ.

Comparing the parameter estimates of the regime-switching models with those for the mean-reverting and jump-diffusion models, we clearly see some of the differences between these modeling approaches. Regime switching models have the power of two independent processes to model the price dynamics separately, jump-diffusion models distinguish between jumps and 'normal' electricity prices as well. Consequently, estimates of $\alpha = 1 - \phi$ are normally lower for the regime switching (base regime) and the jump-diffusion model, that means the speed of adjustment should be highest for the pure mean-reverting model. This is because the simple mean-reverting model has to pull prices back to the normal level after extreme values have occurred whereas in the regime-switching models these spikes are modeled by the spike regime and thus do not influence the parameters in the mean-reverting process of the base regime.

4.5. Model performance measures

We now want to investigate which of the estimated models provides the best fit to the data. Model SJ is nested into Model N, Model JD is nested into model K, and Model MR is nested into model B. In all three cases the null-hypothesis of the restricted model can be rejected at the 1% level using a likelihood ratio test. For model evaluation, we report log-likelihood values and information criteria such as the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) for the remaining models (Table 7).

From this preliminary analysis it seems that the regime-switching models perform better than their competitors. In detail, we find that according to the chosen parsimony model criteria the regime-switching model with two independent regimes and a normal distribution for spike regime outperforms the other models – a result which we expected from our analysis of the parameter estimates. In order to assess the model performance between these remaining non-nested models, we perform pairwise likelihood-ratio tests, for non-nested models as suggested in Vuong (1989). The tests show that model B is worse than all other models under consideration at the 1% level. Similarly, model K is worse than all regime-switching models at the 1% level and model E is worse than the remaining regime-switching models at the 1% level. Finally, the test is unable to separate between models N, LN and 3R at the 5% level. This shows, that the regime-switching models with two independent states and a flexible distribution in the spike regime (at least two parameters) and the three state regime-switching model outperform all its competitors based on the likelihood ratio test for non-nested models.

4.6. Forecasting performance

After examining the goodness-of-fit of the estimated models, it may also be of particular interest to investigate their out-of-sample or forecasting performance. Therefore, this section provides forecasting results for a one-year out-of-sample period. Based on the estimated model parameters from the previous section, we provide one-day-ahead interval and density forecasts for EEX spot prices. Interval forecasts may be especially relevant for risk management purposes where one is rather interested in predicting intervals for future price movements than simply point estimates.

The estimation window that is used to determine the model parameters is the same as in the in-sample analysis from October 1, 2000 to September 30, 2003 totaling 1095

Table 7
Number of parameters k, log-likelihood, Akaike information criterion (AIC), and Bayesian information criterion (BIC) for the estimated models

Model	k	$L(\theta)$	AIC	BIC
Barlow (B)	4	25.1	−42.1	−22.1
Kluge (K)	7	184.0	−354.0	−319.0
Regime switching				
Normal spikes (N)	7	407.9	−801.8	−766.8
Lognormal spikes (LN)	7	392.0	−769.9	−734.9
Exponential spikes (E)	6	301.2	−590.4	−560.4
Three regimes (3R)	6	386.1	−760.2	−730.2

Log-likelihood values for the regime-switching models are calculated with probabilities from the smoothed inferences.

observations. The verification window for evaluation of the out-of-sample forecasts comprises daily data from October 1, 2003 to September 30, 2004 including 366 observations. Due to the poor in-sample results in comparison to their competitors, we excluded the mean-reversion type model Barlow (B) as well as the regime switching model with exponential spikes (E) from the analysis. Recall that model JD is nested into model Kluge (K) and was rejected at the 1% level by the likelihood ratio test in the previous section. Hence, the analysis was restricted to the model K, and the three regime switching models N, LN and 3R.

Given the estimated model parameters and an observation y_t from the verification sample, we are able to calculate a model dependent confidence interval for the next observation y_{t+1}. Following Christoffersen (1998) and Christoffersen and Diebold (2000), we evaluated the quality of the interval forecasts by comparing the nominal coverage of the models to the true coverage. Since comparing the nominal and true coverage may be sensitive to the choice of the confidence level α, we decided to investigate the coverage for four different values of α. Thus, for each of the models we calculated confidence intervals (CI) $[c_{l,t,\alpha}, c_{u,t,\alpha}]$ and determined the actual percentage of exceedances of the 50%, 90%, 95% and 99% two sided day-ahead CI. If the model implied interval forecasts were accurate then the percentage of exceedances should be approximately 50%, 10%, 5% and 1%, respectively.

With a total number of *366* days the expected number of observations beyond the 50% confidence intervals would be approximately 183, respectively 37 for the 90%, 18 for the 95% and 4 for the 99% confidence interval. Table 8 reports the actual number of exceedances. For the 50% CI we find that the number of exceedances is below the expected fraction for all models. However, while for the model 'Kluge' less than 30% of exceedances could be observed, the regime-switching models provide results between 43.17% and 46.17% being much closer to the expected fraction of 50% exceedances. Obviously, for the 50% confidence level, forecasted intervals provided by the model 'Kluge' are too wide. Similar results can be found for other confidence levels: only 2.19% exceedances are observed for the 90%, respectively 1.37% for the 95% and 0.27% for the 99% confidence intervals. We conclude that the one-day-ahead interval forecasts of the model K are overly conservative for any of the considered confidence levels.

Table 8
Number of observations and fraction of exceedances ($x_t \notin [c_{l,t}, c_{u,t}]$) for 50%, 90%, 95% and 99% confidence levels

Model	50% CI		90% CI	
	# Exc.	Fraction (%)	# Exc.	Fraction (%)
Kluge (K)	108	29.51	8	2.19
RS normal spikes (N)	158	43.17	26	7.12
RS lognormal spikes (LN)	169	46.17	22	6.83
RS three regimes (3R)	160	43.72	25	6.85
	95% CI		99% CI	
	# Exc.	Fraction (%)	# Exc.	Fraction (%)
Kluge (K)	5	1.37	1	0.27
RS normal spikes (N)	11	3.01	4	1.09
RS lognormal spikes (LN)	7	1.91	2	0.55
RS three regimes (3R)	10	2.73	5	1.37

Clearly better results are obtained for the regime-switching models where the nominal coverage for different confidence levels is much closer to the theoretical coverage. The best performance – in terms of being closest to the theoretical number of exceedances – is given by the model with a Gaussian distribution for the spike regime: we observe a fraction of 7.12% exceedances for the 90% CI, respectively 3.01% for the 95% and 1.09% for the 99% confidence level. Results for the other two-regime switching models LN and 3R are only slightly worse. In total the regime-switching models seem to give intervals somewhat too wide for the 50%, 90% and 95% confidence level while for the 99% CI the models N and 3R only fail to predict the extreme price jumps. For the more heavy-tailed lognormal distribution intervals at the 99% level are more conservative and only two exceedances are observed for the out-of-sample period.

Overall, the results confirm the superior fit of the regime-switching models to the data in comparison to a stochastic model with jump like the model 'Kluge'. Further, the best results are obtained once more for the two-regime model N with a Gaussian distribution in the spike regime. Despite the good performance of the regime-switching models, confidence intervals are slightly too wide. This may be due to the fact that in comparison to the estimation period during the out-of-sample period from October 1, 2003 to September 30, 2004 spot prices generally exhibited lower volatility and less price spikes could be observed.

4.7. A distributional test

Note that due to the special behavior of electricity spot prices, stylized facts suggest that the spikes are rather unpredictable. Furthermore, tests being based on confidence intervals may be unstable in the sense that they are sensitive to the choice of the confidence level α. Therefore, one should additionally apply tests investigating the complete density forecast instead of a number of quantiles only. To test for the appropriateness of the predicted distribution, we perform a distribution test proposed by Crnkovic and Drachman (1996). The test utilizes the information on the entire distribution and is based on the following methodology.

Assume that we are interested in the distribution of the spot price y_{t+1}, $t > 0$, which is being forecasted at time t. Let further the probability density of y_{t+1} be $f(y_{t+1})$ and the associated distribution function be $F(y_{t+1}) = \int_{-\infty}^{y_{t+1}} f(x)\mathrm{d}x$. To conduct the test, we determine $\widehat{F}(y_{t+1})$ using the parameter estimates from the in-sample period and the observations y_s, $s = 0, \ldots, t$. Rosenblatt (1952) shows that if $\widehat{F}$ is the correct loss distribution, the transformation $u_{t+1} = \int_{-\infty}^{y_{t+1}} \hat{f}(x)\mathrm{d}x = \widehat{F}(y_{t+1})$ is independent and identically distributed uniformly on $[0, 1]$. The method can be applied to test for violations of either independence or uniformity. To test for uniformity, Crnkovic and Drachman (1996) suggest using the Kuiper statistic that is based on the distance between the empirical and the theoretical cumulative distribution function of the uniform distribution. Table 9 reports test results based on the modified Kuiper test statistic in Stephens (1970) for the considered models and the out-of-sample forecasting period from October 1, 2003 to September 30, 2004. Fig. 3 shows a plots of the probability integral transforms of the one-day-ahead forecasts for the model 'Kluge' and the regime-switching model N.

We obtain similar results as in Section 4.6 with respect to interval forecasts: again the model K gives the worst results, indicating that probability integral transforms of the one-day-ahead forecasts are non-uniformly distributed. The test rejects the hypothesis even at

Table 9
Test results for violations of uniformity: Kuiper distance T, modified Kuiper test statistic T^* and p-value (Stephens, 1970), *, **indicate rejection of uniformity at the 5% and 1% significance level, respectively

Model	T	T^*	p-Value
Kluge (K)	0.2331	4.5140**	0.0000
RS normal spikes (N)	0.0831	1.6020	0.1093
RS lognormal spikes (LN)	0.1281	2.4690**	0.0002
RS three regimes (3R)	0.0920	1.7728*	0.0431

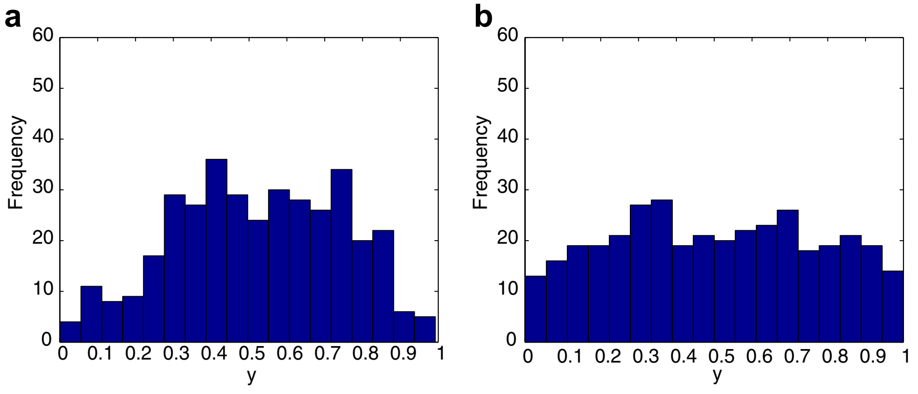

Fig. 3. Bar plot of the probability integral transforms of the one-day ahead forecasts for EEX logreturns for the period October 1, 2003–September 30, 2004. Results for the model 'Kluge' (*Left panel*) and the regime-switching model with Gaussian spikes (N) (*Right panel*).

the 1% level. The results obtained for the regime-switching models are superior in terms of the modified Kuiper test statistic. Empirical observed probability integral transforms of the one-day-ahead forecasts are closer to the uniform distribution. However, for the models LN and 3R the test still rejects the null-hypothesis of uniformity at the 1%, respectively 5% level. Again the best forecast results are obtained by the model N where the hypothesis of an uniform distribution is not rejected even at the 10% level. The superior forecast results of the regime-switching approach are also indicated by Fig. 3 that shows bar plots of the probability integral transforms for the models K and N. Despite the better fit of the model N we find that for the out-of-sample period still a higher fraction of observations is in the area $[0.25, 0.75]$ confirming the results on interval forecasts of the previous section where the number of extreme values is also slightly underestimated. Again, we argue that this behavior may be explained by the lower volatility and less spiky behavior of the spot prices in the out-of-sample period.

Overall, we conclude that the best in-sample and out-of-sample results are obtained by the regime-switching model N with a mean-reversion process for the base regime and a Gaussian distribution with substantially higher variance in the spike regime.

4.8. Futures pricing

For the valuation of futures contracts, the ability to forecast the expected future spot price at maturity is essential and will decide on the reliability of the derived futures prices.

Therefore, we conduct a backtesting analysis, where the two-regime switching models N and 3R and the most appropriate of the benchmark models, namely the model 'Kluge' are compared with respect to their predicting power. Approaches for the valuation of forward and futures contracts can be conceptually divided into two groups: On the one hand, there is the class of no-arbitrage or cost-of-carry models which is based on considerations on a hedging strategy that consists of holding the underlying asset of the forward contract until maturity. The long position in the underlying is funded by a short position in the money market account. Risk drivers determining the forward price in this case include the cost-of-storage for forwards on commodities, cost-of-delivery and interest rate risk. As electricity is a non-storable good, this approach fails in the present context. Eydeland and Geman (1998) circumvent this problem when valuing electricity options by using the forward contracts as hedging instrument. Lucia and Schwartz (2002) solve the problem by making an ad-hoc assumption about the market price of risk whose governing the change from the objective to the pricing measure.

On the other hand, the value of a forward contract can be retrieved from equilibrium considerations. In the context of electricity forwards, one possible approach has been suggested by Bessembinder and Lemmon (2002). Generally, the forward premium, i.e. the difference between the forward price and the expected future spot price represents the equilibrium compensation for facing the risk of uncertainty about futures prices: In the case of risk-averse buyers/consumers, there exists a high demand for forward contracts which lets forward prices rise. If the risk aversion on the producer side is higher, the forward prices will decrease. In the classical terminology, the case where the expected future spot price is higher than the forward price is called normal backwardation, whereas the opposite relation is denoted as contango. Both situations resemble a typical insurance contract: In the normal backwardation case the producers are buying insurance against falling prices whereas in the contango case, consumers buy insurance against raising prices. Longstaff and Wang (2004) examine forward risk premiums by comparing the hourly spot prices with the hourly day-ahead forward prices in the intraday market in a non-parametric way. This approach is not feasible in our setting as the EEX does not offer a true spot market. Our analysis, however, can be based on a set of appropriate stochastic models describing the evolution of the underlying under the objective probability measure. With the help of these models, the expected future stock price E_tP_T under the objective probability measure can be retrieved – analytically or by simulation. Additionally, we can collect at time t forward prices $F_{t,T}$ with expiration date T. Under the two assumptions that (1) the stochastic model for the underlying is appropriate and (2) the energy market is sufficiently efficient, the difference between these two quantities provides a consistent estimate for the unknown forward premium:

$$f_{t,T} := F_{t,T} - E_tP_T. \tag{15}$$

In order to make the results comparable along different maturities, one can transform the obtained expression for the futures premium and express it in form of an annualized excess yield λ:

$$F_{t,T} = \mathrm{e}^{-\lambda\tau} \cdot E_tP_t, \tag{16}$$

where the time to maturity is expressed as a year fraction.

In what follows, we ignore the interest rate risk and do not separate between futures and forward contracts. For our analysis we use all futures quotes available on September

Table 10
EEX futures quotes on September 30, 2003, settlement prices, expected future spot prices, futures premiums and annualized excess yields for different models

Market data			Expected future spot prices								
Contract	Quote	Settlement	Model K			Model N			Model 3R		
			E_tP_t	$f_{t,T}$	λ (%)	E_tP_t	$f_{t,T}$	λ (%)	E_tP_t	$f_{t,T}$	λ (%)
Monthly											
October 2003	33.38	**34.00**	27.48	5.90	−229.01	27.70	5.68	−219.62	28.00	5.38	−206.93
November 2003	34.54	**30.99**	29.30	5.24	−98.45	29.53	5.01	−93.77	30.02	4.52	−83.92
December 2003	32.98	**28.47**	33.81	−0.83	9.86	34.04	−1.06	12.55	34.58	−1.60	18.80
January 2004	34.30	**27.84**	32.05	2.25	−20.13	32.20	2.10	−18.75	33.05	1.25	−11.02
February 2004	34.35	**26.54**	32.16	2.19	−15.82	32.57	1.78	−12.78	32.98	1.37	−9.77
March 2004	29.15	**31.05**	28.60	0.55	−3.80	28.74	0.41	−2.83	29.21	−0.06	0.41
April 2004	27.93	**25.57**	29.75	−1.82	10.82	29.81	−1.88	11.16	30.29	−2.36	13.90
Quarterly											
Q4 2003	33.85	**31.15**	30.20	3.65	−45.27	30.43	3.42	−42.26	30.87	2.98	−36.56
Q1 2004	32.55	**28.48**	30.91	1.64	−10.31	31.14	1.41	−8.83	31.72	0.83	−5.15
Q2 2004	26.25	**26.48**	28.83	−2.58	12.49	29.03	−2.78	13.41	29.47	−3.22	15.41
Q3 2004	27.22	**29.40**	31.56	−4.34	14.75	31.76	−4.54	15.38	32.27	−5.05	16.97
Q4 2004	31.20	**29.69**	33.60	−2.40	5.91	33.87	−2.67	6.54	34.33	−3.13	7.62

30, 2003. These include seven futures with a delivery period of one month and maturities between October 2003 and April 2004 and five futures with a delivery period of three months and maturities between the fourth quarter of 2003 and the fourth quarter 2004. We calculate the expected future spot price – which is an average over the respective maturity period – and estimates for the forward/futures premiums as well as the annualized excess yield according to Eqs. (15) and (16) for the three different models that were also considered in the sections on forecasting and distributional tests: the model with stochastic jumps by Kluge (Model K) and the two-regime switching models N and 3R.[4] The results are presented in Table 10. We can state the following facts: (1) The estimates for the expected future mean are quite stable among the different models, (2) the futures prices are greater than the expected future spot most of the times for the first six months and less than the expected spot for the second, third and fourth quarter in 2004 and (3) our results underpin what is commonly reported about electricity futures: The futures premium is positive and the implied excess yield negative – most of the time.

These results are in accordance with the findings of Longstaff and Wang (2004) and Botterud et al. (2002). Nevertheless, the analysis could be performed in a superior way by using not only the futures quotes of a single trading day but a series of price quotes for every particular contract. First, this would allow to assess the intertemporal dynamic of the forward risk premium and second, statements about the significance of the result

[4] Model LN is excluded in this section as it predicts an infinite expected spot price.

can be derived. Furthermore, the reader may notice that identifying a market consistent annualized access yield is an important step toward market consistent derivative pricing. Choosing a time dependent or possibly stochastic excess yield $(\lambda_t)_{t\geqslant 0}$ combined with a probabilistic model for the spot price process $(S_t)_{t\geqslant 0}$ identifies a risk-neutral dynamic of the spot price process. This risk-neutral dynamic reproduces the observed futures prices and can therefore be used to retrieve market consistent prices of other derivatives.

5. Conclusion

We conducted an extensive empirical analysis of spot price modeling in the German EEX Power market. We examined the explanatory power and the goodness-of-fit of various regime-switching model specifications and benchmarked them against a set of mean-reversion and jump-diffusion models. We find that the regime-switching models clearly outperform the benchmark models in terms of Log-likelihood, Akaike and Bayesian information criterion. The two-regime model with a Gaussian distribution in the spike regime outperforms its competitors in almost all respects. Another result is that while all models provide similar results for predictions of the mean, the jump-diffusion and especially the regime-switching models are able to additionally capture price jumps and spikes. Using confidence interval predictions and distributional tests, the superiority of the regime-switching models is confirmed in out-of-sample tests.

Regarding electricity futures, we find that the considered models predict similar values for future means. Additionally, we denote that for the first six months electricity futures quotes are consistently greater than the expected future spot, a situation which is denoted as contango. This observation is consistent with the observed right-skewness in electricity spot prices.

We confirm the adequacy of regime-switching models for modeling electricity spot prices and recommend them for further investigation of electricity spot and futures markets. However, we point out that the scope of our investigation is limited to the EEX and caution should be exercised in generalizing the results also to other markets. Longstaff and Wang (2004) point out the individual structure of power markets while Mugele et al. (2005) obtain quite diverse results considering electricity spot prices from different European power markets.

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