Supplement Report of Survival Analysis

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Question:

Express $\Pr(U < T \mid x)$ under the Clayton copula model (3.6). Hint: expression may be in an integral from on $[\,0,1\,]$ (Emura and Pan 2017).

Solution:

First we define:

$C_\theta(u, v) = \Pr(V < u, W < v)$ is the Copula function.

$C_\theta^{[\,0,1\,]} = \dfrac{\partial}{\partial v} C_\theta(u, v) = \Pr(V < u \mid W = v)$

V and W are random variables of $\text{Uniform}(0,1)$.

The survival function of U given x is $S_U(t \mid x)$.

The survival function of T given x is $S_T(t \mid x)$.

$$\begin{aligned}
\Pr(U < T \mid x) &= \Pr\{ S_U(U \mid x) > S_U(T \mid x) \} \\
&= \Pr[V > S_U\{ S_T^{-1}(W \mid x) \}] \\
&= \mathrm{E}(\Pr[V > S_U\{ S_T^{-1}(W \mid x) \} \mid W]) \\
&= \int_0^1 \Pr[V > S_U\{ S_T^{-1}(W \mid x) \} \mid W = w] \cdot 1 \, dw \\
&= \int_0^1 C_\theta^{[\,0,1\,]}(S_U\{ S_T^{-1}(w \mid x) \}, w) \, dw \\
&= \int_0^1 (S_U\{ S_T^{-1}(w \mid x) \}^{-\theta} + w^{-\theta} - 1)^{-1/\theta - 1} w^{-\theta - 1} \, dw
\end{aligned}$$