HW#2, Survival data analysis I, Due on 3/29 (2016 Spring) **Name: Lo, Yen-Wen**

Q. Work on details of Example 3.5.1. At least including Tables for $t_j^*, d_j, n_j, \hat{S}(t_j^*+)$, Figure 3.14, R code.

Sol: Example 2.4.2 presented a set of data on the lifetimes of the brake pads on 98 automobiles. In Table 2.1 on page 69, we obtain the values of $\hat{S}(t+;u_{\min}) = \hat{S}(t_j^*+) = \prod_{j:t_j^*\le t}\left(1-\frac{d_j}{n_j}\right)$, $t_1^* < t_2^* < \ldots < t_k^*$ are the distinct observed failures times, $d_j = \sum_{i=1}^{98} I(t_i = t_j^*, \delta_i = 1)$, $n_j = \sum_{i=1}^{98} I(u_i \le t_j^* \le t_i)$, with $j = 1,\ldots,88$, and t_i, u_i are given in Table 2.1, with $i = 1,\ldots,98$.

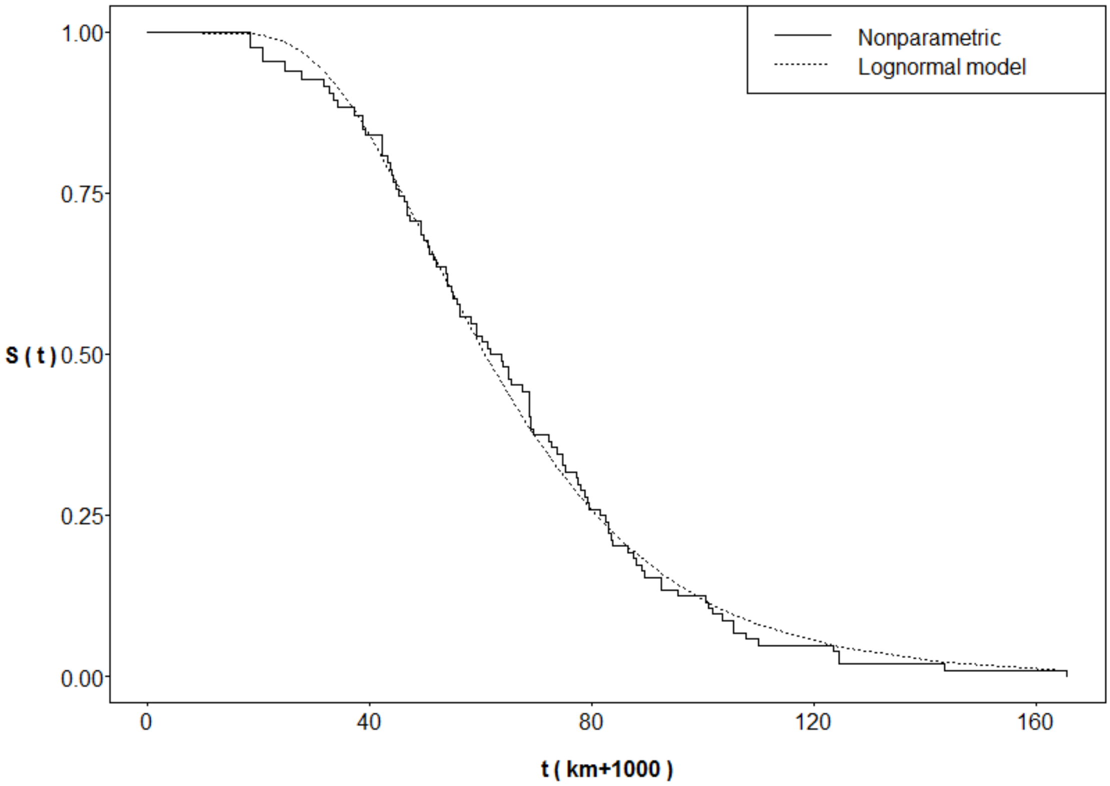
t_j^*	d_j	n_j	$1-\frac{d_j}{n_j}$	$\hat{S}(t_j^*+)$	t_j^*	d_j	n_j	$1-\frac{d_j}{n_j}$	$\hat{S}(t_j^*+)$	t_j^*	d_j	n_j	$1-\frac{d_j}{n_j}$	$\hat{S}(t_j^*+)$
18.6	1	40	$\frac{39}{40}$	0.9750	43.4	1	78	$\frac{77}{78}$	0.7977	50.8	1	66	$\frac{65}{66}$	0.6553
20.8	1	48	$\frac{47}{48}$	0.9547	43.8	1	77	$\frac{76}{77}$	0.7873	51.5	1	65	$\frac{64}{65}$	0.6452
24.8	1	65	$\frac{64}{65}$	0.9400	44.1	1	76	$\frac{75}{76}$	0.7770	52.0	1	64	$\frac{63}{64}$	0.6351
27.8	1	73	$\frac{72}{73}$	0.9271	44.2	1	75	$\frac{74}{75}$	0.7666	53.9	1	65	$\frac{64}{65}$	0.6254
31.8	1	82	$\frac{81}{82}$	0.9158	44.8	1	74	$\frac{73}{74}$	0.7563	54.0	2	64	$\frac{62}{64}$	0.6058
32.9	1	82	$\frac{81}{82}$	0.9046	45.2	1	74	$\frac{73}{74}$	0.7460	54.9	1	62	$\frac{61}{62}$	0.5961
33.6	1	81	$\frac{80}{81}$	0.8935	46.3	1	74	$\frac{73}{74}$	0.7360	55.0	1	61	$\frac{60}{61}$	0.5863
34.3	1	80	$\frac{79}{80}$	0.8823	46.7	1	73	$\frac{72}{73}$	0.7259	55.9	1	60	$\frac{59}{60}$	0.5765
37.2	1	80	$\frac{79}{80}$	0.8713	46.8	1	72	$\frac{71}{72}$	0.7158	56.2	2	59	$\frac{57}{59}$	0.5570
38.7	1	81	$\frac{80}{81}$	0.8605	47.4	1	71	$\frac{70}{71}$	0.7057	58.4	1	58	$\frac{57}{58}$	0.5474
38.8	1	80	$\frac{79}{80}$	0.8498	49.2	2	70	$\frac{68}{70}$	0.6856	59.3	1	57	$\frac{56}{57}$	0.5378
39.3	1	80	$\frac{79}{80}$	0.8391	49.8	1	68	$\frac{67}{68}$	0.6755	59.4	1	56	$\frac{55}{56}$	0.5282
42.4	3	81	$\frac{78}{81}$	0.8081	50.5	1	67	$\frac{66}{67}$	0.6654	60.3	1	55	$\frac{54}{55}$	0.5186

t_j^*	d_j	n_j	$1-\frac{d_j}{n_j}$	$\hat{S}(t_j^*+)$	t_j^*	d_j	n_j	$1-\frac{d_j}{n_j}$	$\hat{S}(t_j^*+)$	t_j^*	d_j	n_j	$1-\frac{d_j}{n_j}$	$\hat{S}(t_j^*+)$
61.4	1	54	$\frac{53}{54}$	0.5090	75.2	1	34	$\frac{33}{34}$	0.3169	92.5	1	16	$\frac{15}{16}$	0.1440
61.9	1	53	$\frac{52}{53}$	0.4994	77.2	1	33	$\frac{32}{33}$	0.3073	92.6	1	15	$\frac{14}{15}$	0.1344
63.7	1	52	$\frac{51}{52}$	0.4898	77.6	1	32	$\frac{31}{32}$	0.2977	95.7	1	14	$\frac{13}{14}$	0.1248
64.0	1	51	$\frac{50}{51}$	0.4802	78.1	1	31	$\frac{30}{31}$	0.2881	100.6	1	13	$\frac{12}{13}$	0.1152
65.0	1	50	$\frac{49}{50}$	0.4705	78.7	1	30	$\frac{29}{30}$	0.2785	101.2	1	12	$\frac{11}{12}$	0.1056
65.1	1	49	$\frac{48}{49}$	0.4609	79.4	1	29	$\frac{28}{29}$	0.2689	101.9	1	11	$\frac{10}{11}$	0.0960
65.5	1	48	$\frac{47}{48}$	0.4513	79.5	1	28	$\frac{27}{28}$	0.2593	103.6	1	10	$\frac{9}{10}$	0.0864
67.6	1	47	$\frac{46}{47}$	0.4417	81.6	1	27	$\frac{26}{27}$	0.2497	105.6	2	9	$\frac{7}{9}$	0.0672
68.8	2	46	$\frac{44}{46}$	0.4225	82.6	1	26	$\frac{25}{26}$	0.2401	107.8	1	7	$\frac{6}{7}$	0.0576
68.9	2	44	$\frac{42}{44}$	0.4033	83.0	2	25	$\frac{23}{25}$	0.2209	110.0	1	6	$\frac{5}{6}$	0.0480
69.0	2	42	$\frac{40}{42}$	0.3841	83.6	1	23	$\frac{22}{23}$	0.2113	123.5	1	5	$\frac{4}{5}$	0.0384
69.6	1	40	$\frac{39}{40}$	0.3745	83.8	1	22	$\frac{21}{22}$	0.2017	124.5	1	4	$\frac{3}{4}$	0.0288
72.2	1	39	$\frac{38}{39}$	0.3649	86.7	1	21	$\frac{20}{21}$	0.1921	124.6	1	3	$\frac{2}{3}$	0.0192
72.8	1	38	$\frac{37}{38}$	0.3553	87.6	1	20	$\frac{19}{20}$	0.1825	143.6	1	2	$\frac{1}{2}$	0.0096
73.8	1	37	$\frac{36}{37}$	0.3457	88.0	1	19	$\frac{18}{19}$	0.1729	165.5	1	1	0	0.0000
74.7	1	36	$\frac{35}{36}$	0.3361	89.1	1	18	$\frac{17}{18}$	0.1633					
74.8	1	35	$\frac{34}{35}$	0.3265	89.5	1	17	$\frac{16}{17}$	0.1536					

Note: 1. Units of t_j^* are 1000 km.

2. The digit of $\hat{S}(t_j^*+)$ in the Table just show to .0000. The details are in R code.

By the front Table, we use R to draw the figure of nonparametric and log-normal estimates of $S(t)$ for brake pad life. The figure shows the $\hat{S}(t_j^*+)$ along with a parametric estimate based on a log-normal distribution for the lifetimes. The maximum likelihood estimations for log-normal are $\hat{\mu}=4.109$, $\hat{\sigma}=0.421$. Use the formula on page 22 (1.3.11), the estimated log-normal survivor function is written as $\hat{S}(t_j^*)=1-\Phi(\frac{\log t_j^*-\hat{\mu}}{\hat{\sigma}})$.



R code:

```
##Example 3.5.1(p.118)##
#Key in the data of Table 2.1(p.69)
brake_pad=read.table("Table2.1.txt",header=FALSE)
u=brake_pad$V1;u
t=brake_pad$V2;t
##Sorting the 98 lifetimes from little to large##
s1=order(brake_pad$V1)
brake_pad=brake_pad[s1,]
N=length(unique(brake_pad$V2));N
#[1] 88
tj=sort(unique(brake_pad$V2));tj
# [1]   18.6  20.8  24.8  27.8  31.8  32.9  33.6  34.3  37.2  38.7  38.8  39.3
#[13]   42.4  43.4  43.8  44.1  44.2  44.8  45.2  46.3  46.7  46.8  47.4  49.2
#[25]   49.8  50.5  50.8  51.5  52.0  53.9  54.0  54.9  55.0  55.9  56.2  58.4
#[37]   59.3  59.4  60.3  61.4  61.9  63.7  64.0  65.0  65.1  65.5  67.6  68.8
#[49]   68.9  69.0  69.6  72.2  72.8  73.8  74.7  74.8  75.2  77.2  77.6  78.1
#[61]   78.7  79.4  79.5  81.6  82.6  83.0  83.6  83.8  86.7  87.6  88.0  89.1
#[73]   89.5  92.5  92.6  95.7 100.6 101.2 101.9 103.6 105.6 107.8 110.0 123.5
#[85] 124.5 124.6 143.6 165.5

#Count the number of dj
dj=table(sort(brake_pad$V2));dj
# 18.6  20.8  24.8  27.8  31.8  32.9  33.6  34.3  37.2  38.7  38.8  39.3  42.4
#    1     1     1     1     1     1     1     1     1     1     1     1     3
# 43.4  43.8  44.1  44.2  44.8  45.2  46.3  46.7  46.8  47.4  49.2  49.8  50.5
#    1     1     1     1     1     1     1     1     1     1     2     1     1
# 50.8  51.5    52  53.9    54  54.9    55  55.9  56.2  58.4  59.3  59.4  60.3
#    1     1     1     1     2     1     1     1     2     1     1     1     1
# 61.4  61.9  63.7    64    65  65.1  65.5  67.6  68.8  68.9    69  69.6  72.2
#    1     1     1     1     1     1     1     1     2     2     2     1     1
# 72.8  73.8  74.7  74.8  75.2  77.2  77.6  78.1  78.7  79.4  79.5  81.6  82.6
#    1     1     1     1     1     1     1     1     1     1     1     1     1
#   83  83.6  83.8  86.7  87.6    88  89.1  89.5  92.5  92.6  95.7 100.6 101.2
#    2     1     1     1     1     1     1     1     1     1     1     1     1
#101.9 103.6 105.6 107.8   110 123.5 124.5 124.6 143.6 165.5
#    1     1     2     1     1     1     1     1     1     1
dj=as.vector(table(sort(brake_pad$V2)));dj
# [1] 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1
#[31] 2 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 1 1 1 1 1 1 1 1 1 1
#[61] 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1
```

#Count the number of nj

```
nj=rep(0,N)
for(j in 1:N){
        for(i in 1:98){
                if(t[i]>=tj[j] && u[i]<=tj[j])
        nj[j]=nj[j]+1
        }
};nj
# [1] 40 48 65 73 82 82 81 80 80 81 80 80 81 78 77 76 75 74 74 74 73 72 71 70 68
#[26] 67 66 65 64 65 64 62 61 60 59 58 57 56 55 54 53 52 51 50 49 48 47 46 44 42
#[51] 40 39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 23 22 21 20 19 18 17 16 15
#[76] 14 13 12 11 10  9  7  6  5  4  3  2  1
```

#Estimate S(tj*)_hat

```
Stj=cumprod(1-(dj/nj));Stj
#        18.6        20.8        24.8        27.8        31.8        32.9
#0.975000000 0.954687500 0.940000000 0.927123288 0.915816906 0.904648407
#        33.6        34.3        37.2        38.7        38.8        39.3
#0.893479908 0.882311410 0.871282517 0.860525943 0.849769368 0.839147251
#        42.4        43.4        43.8        44.1        44.2        44.8
#0.808067723 0.797707881 0.787348038 0.776988196 0.766628353 0.756268510
#        45.2        46.3        46.7        46.8        47.4        49.2
#0.746048666 0.735966927 0.725885188 0.715803449 0.705721711 0.685558233
#        49.8        50.5        50.8        51.5          52        53.9
#0.675476495 0.665394756 0.655313017 0.645231278 0.635149540 0.625378008
#          54        54.9          55        55.9        56.2        58.4
#0.605834946 0.596063414 0.586291883 0.576520351 0.556977289 0.547374232
#        59.3        59.4        60.3        61.4        61.9        63.7
#0.537771175 0.528168119 0.518565062 0.508962005 0.499358948 0.489755892
#          64          65        65.1        65.5        67.6        68.8
#0.480152835 0.470549778 0.460946722 0.451343665 0.441740608 0.422534495
#        68.9          69        69.6        72.2        72.8        73.8
#0.403328381 0.384122268 0.374519211 0.364916155 0.355313098 0.345710041
#        74.7        74.8        75.2        77.2        77.6        78.1
#0.336106985 0.326503928 0.316900871 0.307297814 0.297694758 0.288091701
#        78.7        79.4        79.5        81.6        82.6          83
#0.278488644 0.268885588 0.259282531 0.249679474 0.240076418 0.220870304
#        83.6        83.8        86.7        87.6          88        89.1
#0.211267247 0.201664191 0.192061134 0.182458077 0.172855021 0.163251964
#        89.5        92.5        92.6        95.7       100.6       101.2
#0.153648907 0.144045851 0.134442794 0.124839737 0.115236680 0.105633624
```

101.9 103.6 105.6 107.8 110 123.5
#0.096030567 0.086427510 0.067221397 0.057618340 0.048015284 0.038412227
124.5 124.6 143.6 165.5
#0.028809170 0.019206113 0.009603057 0.000000000

#Log-normal distribution

mu=4.109
sigma=0.421

#The log-normal survivor function

St=1-pnorm((log(tj)-mu)/sigma);St
# [1] 0.997574147 0.994632065 0.983553545 0.968708829 0.938564643 0.928137642
# [7] 0.921032689 0.913570589 0.879057451 0.859123675 0.857749010 0.850794447
#[13] 0.804969040 0.789340304 0.782993335 0.778200161 0.776596415 0.766914060
#[19] 0.760405598 0.742313462 0.735672783 0.734008077 0.723985192 0.693638685
#[25] 0.683461018 0.671566856 0.666465801 0.654562505 0.646064901 0.613891543
#[31] 0.612206093 0.597085932 0.595411880 0.580407067 0.575432210 0.539435684
#[37] 0.524991621 0.523397918 0.509160555 0.492031317 0.484349389 0.457264881
#[43] 0.452840844 0.438286819 0.436847959 0.431123013 0.401883448 0.385804776
#[49] 0.384485882 0.383170235 0.375344663 0.342795462 0.335599343 0.323868191
#[55] 0.313589612 0.312463828 0.307993151 0.286413070 0.282250255 0.277117742
#[61] 0.271062182 0.264138802 0.263162096 0.243353326 0.234382926 0.230876474
#[67] 0.225703176 0.224001588 0.200578151 0.193769808 0.190811630 0.182887304
#[73] 0.180081085 0.160264911 0.159640572 0.141377118 0.116481551 0.113738139
#[79] 0.110615988 0.103373859 0.095440533 0.087397904 0.080019339 0.046487634
#[85] 0.044653614 0.044474258 0.020770969 0.008769045

#Draw the Figure 3.14

```
plot(c(0,tj),c(1,Stj),type="s",xlim=c(0,166),ylim=c(0,1),xlab="",ylab="",axes=FALSE)
lines(c(0,tj),c(1,St),type="l",lty=3)
axis(1,at=seq(0,160,40),mgp=c(3,0.2,0),tcl=-0.15)
axis(2,las=1,at=seq(0,1,0.25),mgp=c(3,0.2,0),tcl=-0.15)
box()
mtext(side=1,line=2,"t ( km+1000 )",font=2)
mtext(side=2,las=1,line=2,"S ( t )",font=2)
legend("topright",c("Nonparametric","Lognormal model"),lty=c(1,3))
```