Homework #1

#1

In this case, we have $a_0 = 1$, $a_1 = 2$, $a_3 = 3$, and $\Delta = 1$. Then we could obtain $z_0(t) = t - 1$, $z_1(t) = t - 2$, and $z_2(t) = t - 3$. Therefore,

$$B_4(t) = \begin{cases} \dfrac{(t-1)^3}{2}, & 1 \le t < 2 \\ \dfrac{-7(t-2)^3 + 3(t-2)^2 + 3(t-2) + 1}{2}, & 2 \le t < 3 \end{cases}$$

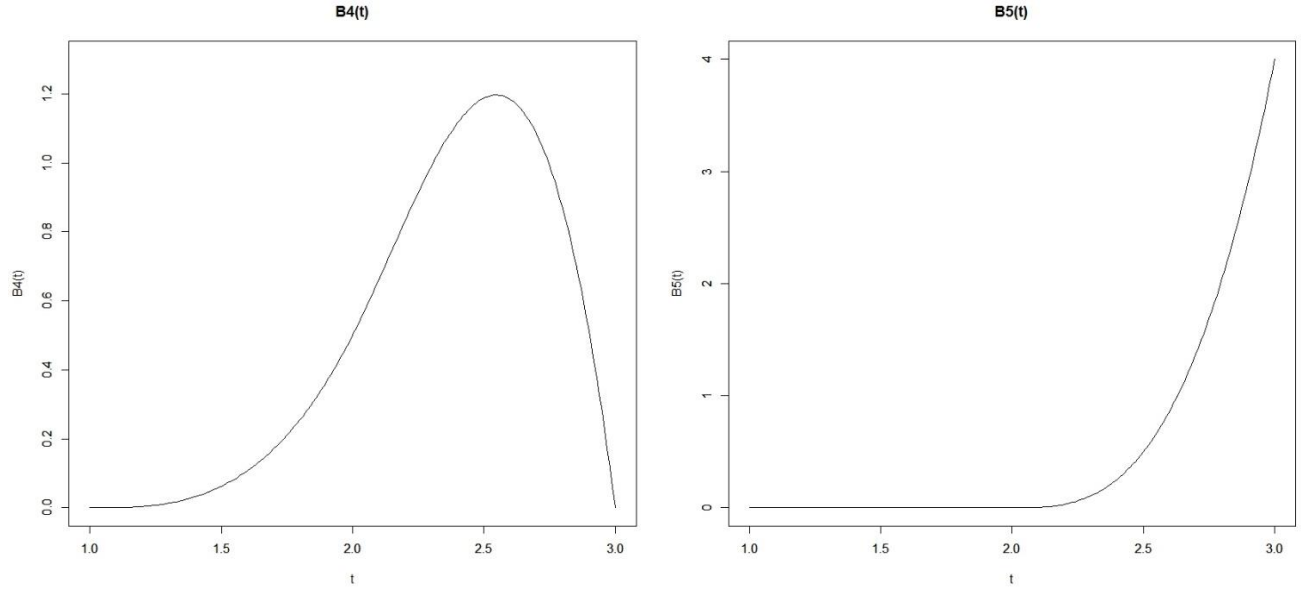
$$B_4'(t) = \begin{cases} \dfrac{3(t-1)^2}{2}, & 1 \le t < 2 \\ \dfrac{-21(t-2)^2 + 6(t-2) + 3}{2}, & 2 \le t < 3 \end{cases}$$

$$B_4''(t) = \begin{cases} 3(t-1), & 1 \le t < 2 \\ -21(t-2) + 3(t-2), & 2 \le t < 3 \end{cases}$$

Also, $B_4(1) = 0$, $B_4(2-) = \frac{1}{2}$, $B_4(2) = \frac{1}{2}$, $B_4(3-) = 0$. Furthermore, we could obtain $B_4'(1) = 0$, $B_4'(\frac{15+4\sqrt{2}}{7}) = 0$, $B_4''(1) = 0$, and $B_4''(\frac{15+4\sqrt{2}}{7}) < 0$. Hence $t = \frac{15+4\sqrt{2}}{7} \cong 2.95$ is a maximum value in the interval of $[2,3)$. Then we could obtain the plots. Similarly, we could work on $B_5(t)$.

$$B_5(t) = \begin{cases} 0 & , 1 \le t < 2 \\ 4(t-2)^3 & , 2 \le t < 3 \end{cases}, \quad B_5'(t) = \begin{cases} 0 & , 1 \le t < 2 \\ 12(t-2)^2 & , 2 \le t < 3 \end{cases}, \text{ and } \quad B_5''(t) = \begin{cases} 0 & , 1 \le t < 2 \\ 24(t-2) & , 2 \le t < 3 \end{cases}$$

Then $B_5(1) = 0$, $B_5(2-) = 0$, $B_5(2) = 0$, $B_5(3-) = 4$. Furthermore, we could obtain $B_5'(1) = 0$, $B_5'(2) = 0$, $B_5''(1) = 0$, and $B_5''(2) = 0$. Thus, we could get the following plots.

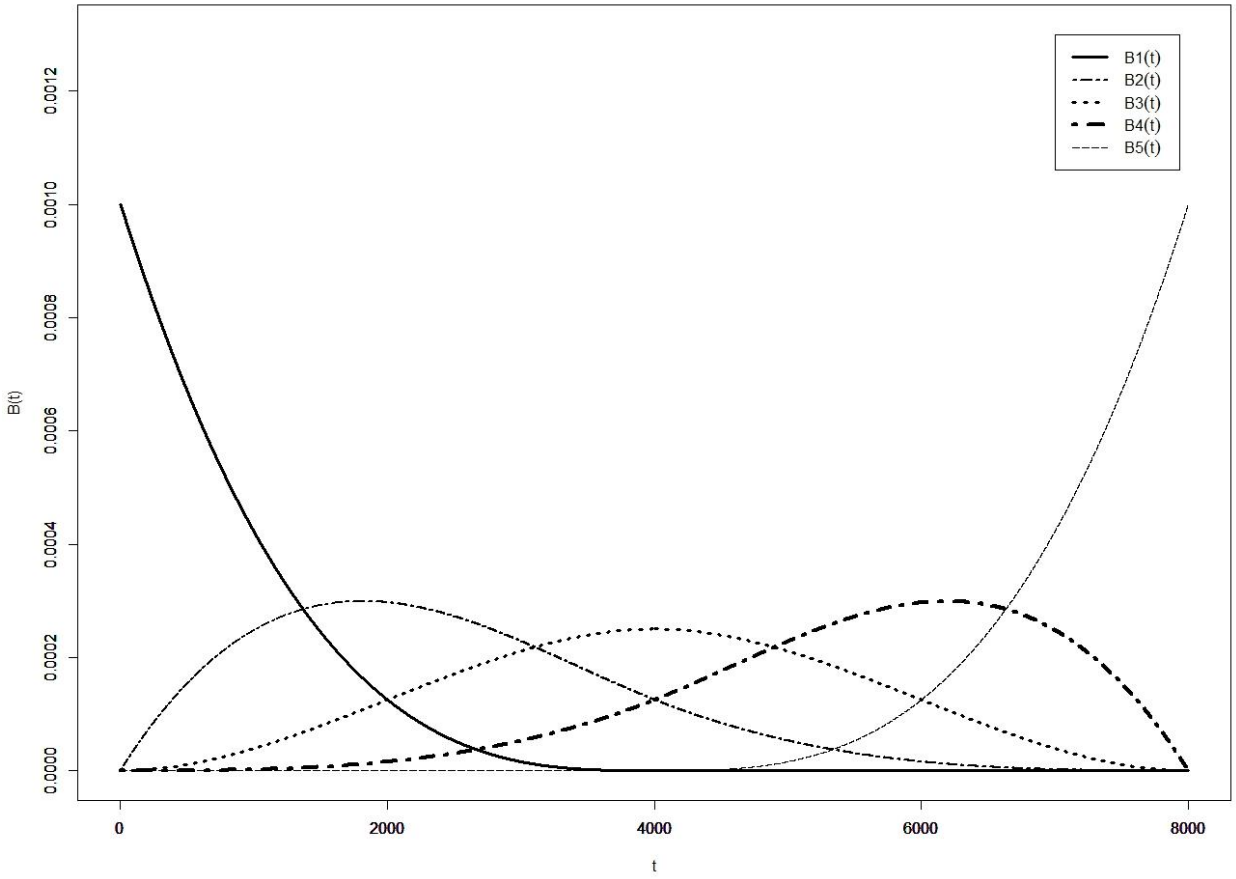


《CODE》

```
a0 = 1
a1 = 2
a2 = 3
seg1 = seq(1,2,by=0.01)
seg2 = seq(2,3,by=0.01)
xr = c(1,3)
delta = 1
z0 = function(t){
   (t-a0)/delta
}
z1 = function(t){
   (t-a1)/delta
}
z2 = function(t){
   (t-a2)/delta
}

########## B4 ##############
B4 = function(t){
   if(t>=a0 && t<a1){
      (z0(t)^3)/(2*delta)
   }else if(t>=a1 && t<a2){
      (-7*z1(t)^3+3*z1(t)^2+3*z1(t)+1)/(2*delta)
   }else{
      0
   }
}

########## B5 ##############
B5 = function(t){
   if(t>=a0 && t<a1){
      0
   }else if(t>=a1 && t<a2){
      (4*z1(t)^3)/delta
   }else{
      0
   }
}

layout(matrix(1:2,1,2))

########## B4 ##############
plot(seg1,B4(seg1),type="l",xlim=xr,ylim=c(0,1.3),xlab = "",ylab="")
par(new=TRUE)
plot(seg2,B4(seg2),type="l",xlim=xr,ylim=c(0,1.3),xlab="t",ylab="B4(t)",main="B4(t)")

########## B5 ##############
plot(seg1,0*c(1:101),type="l",xlim=xr,ylim=c(0,4),xlab = "",ylab="")
par(new=TRUE)
plot(seg2,B5(seg2),type="l",xlim=xr,ylim=c(0,4),xlab="t",ylab="B5(t)",main="B5(t)")
```

#2

(1) Using the following code, we could obtain the plots of $B_1(t)$, $B_2(t)$, $B_3(t)$, $B_4(t)$, and $B_5(t)$.



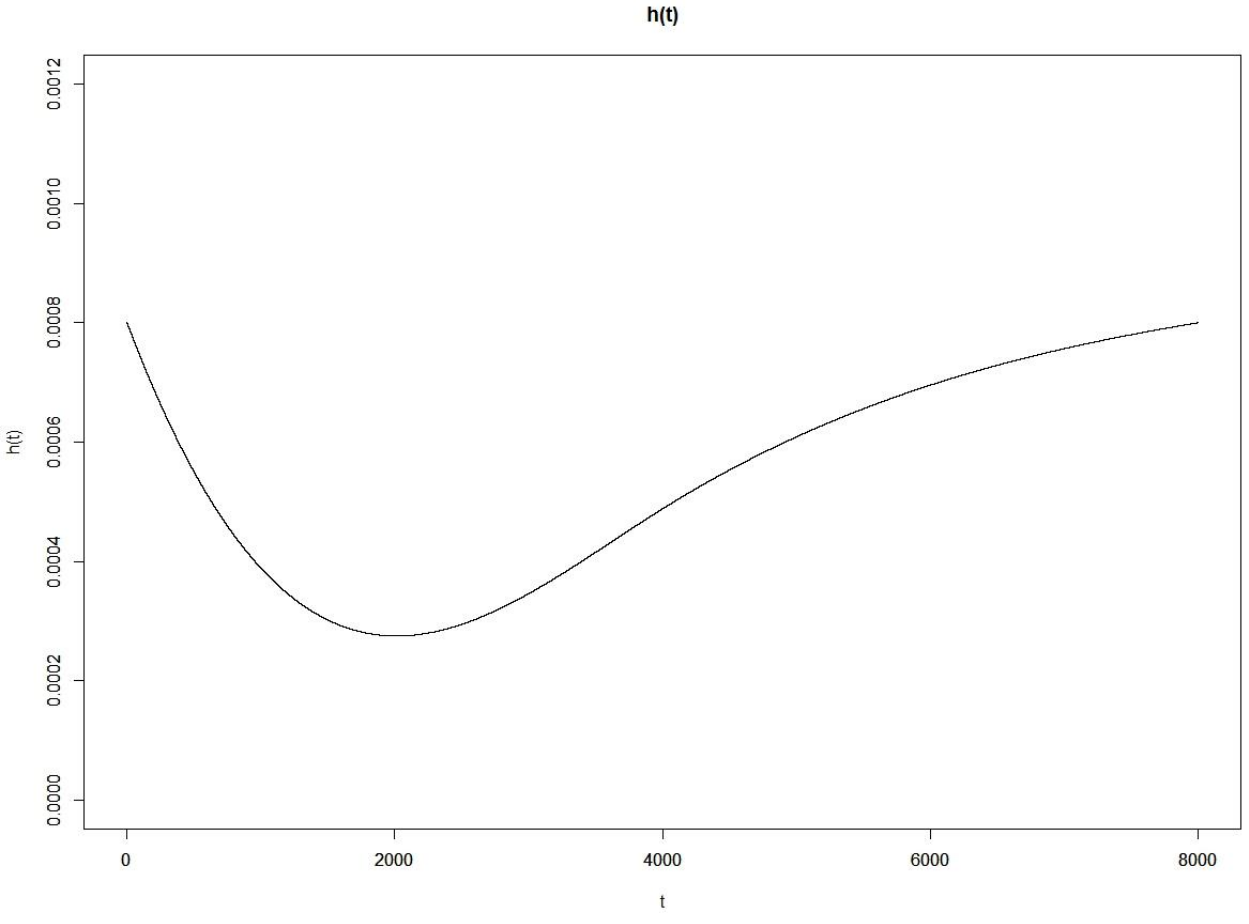
《CODE》

```
a0 = 0
a1 = 4000
a2 = 8000
delta = 4000
seg1 = seq(0,4000,by=0.01)
seg2 = seq(4000,8000,by=0.01)
xr = c(0,8000)

z0 = function(t){
   (t-a0)/delta
}
z1 = function(t){
   (t-a1)/delta
}
z2 = function(t){
   (t-a2)/delta
}

B1 = function(t){
   if(t>=a0 && t<a1){
      -(4*(z1(t)^3))/delta
   }else if(t>=a1 && t<a2){
```

```
      0
   }else{
      0
   }
}
B2 = function(t){
   if(t>=a0 && t<a1){
      (7*z0(t)^3-18*z0(t)^2+12*z0(t))/(2*delta)
   }else if(t>=a1 && t<a2){
      -z2(t)^3/(2*delta)
   }else{
      0
   }
}


B3 = function(t){
   if(t>=a0 && t<a1){
      (-2*z0(t)^3+3*z0(t)^2)/delta
   }else if(t>=a1 && t<a2){
      (2*(z1(t)^3)-3*(z1(t)^2)+1)/delta
   }else{
      0
   }
}

B4 = function(t){
   if(t>=a0 && t<a1){
      (z0(t)^3)/(2*delta)
   }else if(t>=a1 && t<a2){
      (-7*z1(t)^3+3*z1(t)^2+3*z1(t)+1)/(2*delta)
   }else{
      0
   }
}

B5 = function(t){
   if(t>=a0 && t<a1){
      0
   }else if(t>=a1 && t<a2){
      (4*z1(t)^3)/delta
   }else{
      0
   }
}

########## B1 ##############
plot(seg1,B1(seg1),type="l",xlim=xr,ylim=c(0,0.0013),lty=1,lwd=3,xlab = "",ylab="")
par(new=TRUE)
plot(seg2,0*c(1:400001),type="l",xlim=xr,ylim=c(0,0.0013),lty=1,lwd=3,xlab= "",ylab="")

par(new=TRUE)
########## B2 ##############
```

```
plot(seg1,B2(seg1),type="l",xlim=xr,ylim=c(0,0.0013),lty=6,lwd=2,xlab = "",ylab="")
par(new=TRUE)
plot(seg2,B2(seg2),type="l",xlim=xr,ylim=c(0,0.0013),lty=6,lwd=2,xlab= "",ylab="")
par(new=TRUE)

########## B3 ##############
plot(seg1,B3(seg1),type="l",xlim=xr,ylim=c(0,0.0013),lty=3,lwd=3,xlab = "",ylab="")
par(new=TRUE)
plot(seg2,B3(seg2),type="l",xlim=xr,ylim=c(0,0.0013),lty=3,lwd=3,xlab ="",ylab="")
par(new=TRUE)

########## B4 ##############
plot(seg1,B4(seg1),type="l",xlim=xr,ylim=c(0,0.0013),lty=4,lwd=4,xlab = "",ylab="")
par(new=TRUE)
plot(seg2,B4(seg2),type="l",xlim=xr,ylim=c(0,0.0013),lty=4,lwd=4,xlab ="",ylab="")

par(new=TRUE)
########## B5 ##############
plot(seg1,0*c(1:400001),type="l",xlim=xr,ylim=c(0,0.0013),lty=5,lwd=1,xlab = "",ylab="")
par(new=TRUE)
plot(seg2,B5(seg2),type="l",xlim=xr,ylim=c(0,0.0013),lty=5,lwd=1,xlab= "t",ylab="B(t)")
k = expression(B1(t),B2(t),B3(t),B4(t),B5(t))
legend(7000,0.0013,legend=k,lty=c(1,6,3,4,5),lwd=c(3,2,3,4,1))
```

(2) Choose $\theta_1 = 0.8$, $\theta_2 = 0.005$, $\theta_3 = 1.2$, $\theta_4 = 1.5$, and $\theta_5 = 0.8$, then using the following code, we could obtain the plot of $h(t) = \sum_{i=1}^{5} \theta_i B_i(t)$.



《CODE》

```
a0 = 0
a1 = 4000
a2 = 8000
delta = 4000
seg1 = seq(0,4000,by=0.01)
seg2 = seq(4000,8000,by=0.01)
xr = c(0,8000)

z0 = function(t){
   (t-a0)/delta
}
z1 = function(t){
   (t-a1)/delta
}
z2 = function(t){
   (t-a2)/delta
}

B1 = function(t){
   if(t>=a0 && t<a1){
      -(4*(z1(t)^3))/delta
   }else if(t>=a1 && t<a2){
      0
   }else{
      0
   }
}
B2 = function(t){
   if(t>=a0 && t<a1){
      (7*z0(t)^3-18*z0(t)^2+12*z0(t))/(2*delta)
   }else if(t>=a1 && t<a2){
      -z2(t)^3/(2*delta)
   }else{
      0
   }
}


B3 = function(t){
   if(t>=a0 && t<a1){
      (-2*z0(t)^3+3*z0(t)^2)/delta
   }else if(t>=a1 && t<a2){
      (2*(z1(t)^3)-3*(z1(t)^2)+1)/delta
   }else{
      0
   }
}

B4 = function(t){
   if(t>=a0 && t<a1){
      (z0(t)^3)/(2*delta)
```

```
   }else if(t>=a1 && t<a2){
      (-7*z1(t)^3+3*z1(t)^2+3*z1(t)+1)/(2*delta)
   }else{
      0
   }
}

B5 = function(t){
   if(t>=a0 && t<a1){
      0
   }else if(t>=a1 && t<a2){
      (4*z1(t)^3)/delta
   }else{
      0
   }
}

############## Choose theta ##############
theta1 = 0.8
theta2 = 0.005
theta3 = 1.2
theta4 = 1.5
theta5 = 0.8

############## Define h(t) ##############
hf = function(t){
   theta1*B1(t)+theta2*B2(t)+theta3*B3(t)+theta4*B4(t)+theta5*B5(t)
}

l = min(range(hf(seg1)),range(hf(seg2)))
u = max(range(hf(seg1)),range(hf(seg2)))

############## Plot ##############
plot(seg1,hf(seg1),type="l",xlim=xr,ylim=c(l-0.1,u+0.1),xlab = "",ylab="")
par(new=TRUE)
plot(seg2,hf(seg2),type="l",xlim=xr,ylim=c(l-0.1,u+0.1),xlab="t",ylab="h(t)", main="h(t)")
```