HW#5 Survival analysis II

Name: Shih Jia-Han

The distribution of the clayton family is

$$\overline{G}(t_1, t_2) = \{\overline{G_1}(t_1)^{-\nu} + \overline{G_2}(t_2)^{-\nu} - 1\}^{\frac{-1}{\nu}},$$

where $G_1(t_1) \sim U(0,1)$ and $G_2(t_2) \sim U(0,1)$.

Since $G_1(t_1) \sim U(0,1)$ and $G_2(t_2) \sim U(0,1)$, we have

$$\overline{G_1}(t_1) = 1 - G_1(t_1) \sim 1 - U(0,1) = U(0,1)$$

similarly,

$$\overline{G_2}(t_2) \sim U(0,1)$$

Here we can use Monte Carlo simulation by the method of Inverse transform to generate the data of clayton family. First, we have

$$T_1 = \overline{G_1}(t_1) \sim U(0,1)$$

so we can define

$$T_1 \equiv U_1 \sim U(0,1)$$

Then since T_1 and T_2 are dependent, we need to generate T_2 by $\overline{G_2}(t_2 \mid T_1 = t_1)$.

By the method of Inverse transform we can obtain

$$\overline{G_2}(T_2 \mid T_1 = t_1) \sim U(0,1)$$

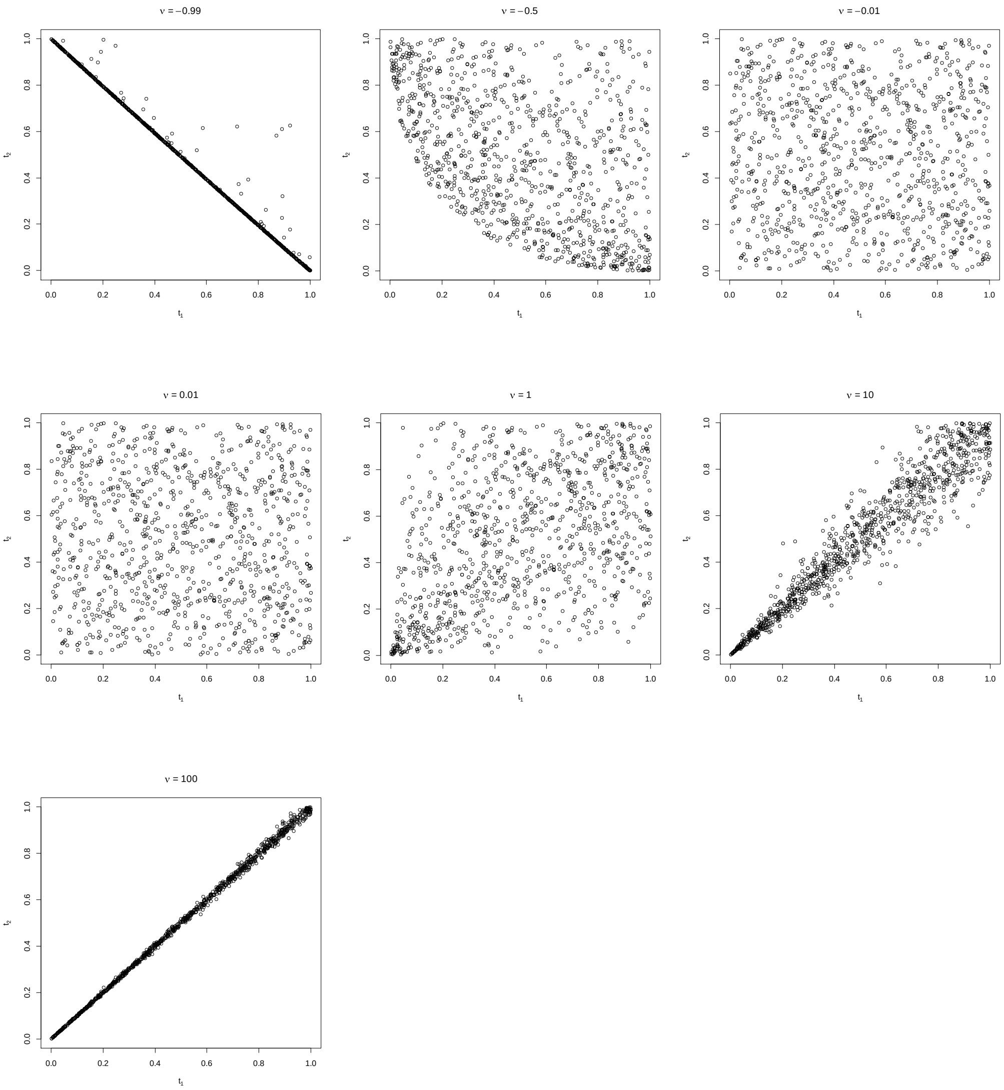
therefore,

$$T_2 \equiv \{(U_2 t_1^{\nu+1})^{\frac{-\nu}{\nu+1}} + 1 - t_1^{-\nu}\}^{\frac{-1}{\nu}},$$

where $U_2 \sim U(0,1)$.

Hence we can generate data T_1 and T_2 in this way.

Now, we generate 1000 data and plot the scatter plot with different ν to observe the dependence between T_1 and T_2.



From the above figure, we can observe the following result:

if $-1 < \nu < 0$, T_1 and T_2 seem to have negative dependence. And the dependence is stronger as $\nu \to -1$.

if ν is very close to 0, T_1 and T_2 seem to be nearly independent.

(since ν cannot be 0, so T_1 and T_2 cannot be independent.)

if $\nu > 0$, T_1 and T_2 seem to have positive dependence. And the dependence is stronger as ν goes larger.

The result is the same as we inferred during class. So it is correct.

R code

```
cf_func=function(n,nu) {
	set.seed(10)
	u1=runif(n)
	u2=runif(n)
	t1=u1
	t2=((u2*t1^(nu+1))^(-nu/(nu+1))+1-t1^(-nu))^(-1/nu)
	data=cbind(t1,t2)
}

plot(cf_func(1000,-0.99),xlab=expression(t[1]),ylab=expression(t[2]),
	main=expression(paste(nu," = ",-0.99)))
plot(cf_func(1000,-0.5),xlab=expression(t[1]),ylab=expression(t[2]),
	main=expression(paste(nu," = ",-0.5)))
plot(cf_func(1000,-0.01),xlab=expression(t[1]),ylab=expression(t[2]),
	main=expression(paste(nu," = ",-0.01)))
plot(cf_func(1000,0.01),xlab=expression(t[1]),ylab=expression(t[2]),
	main=expression(paste(nu," = ",0.01)))
plot(cf_func(1000,1),xlab=expression(t[1]),ylab=expression(t[2]),
	main=expression(paste(nu," = ",1)))
plot(cf_func(1000,10),xlab=expression(t[1]),ylab=expression(t[2]),
	main=expression(paste(nu," = ",10)))
plot(cf_func(1000,100),xlab=expression(t[1]),ylab=expression(t[2]),
	main=expression(paste(nu," = ",100)))
```