**Exercise 2.1-2.20 (Klein & Moeschberger, p.57-61)**

2.1

$X$ is from exponential distribution with $\lambda$ =0.001

$(a)\ E(\mathrm{X}) = \frac{1}{\lambda} = 1000$

Mean of $X$ is 1000.

$(b)\ S(t_{0.5}) = \frac{1}{2}$, where $S(.)$ is survival function of $X$ .

$$=> S(t_{0.5}) = \exp(-0.001 \times t_{0.5}) = \frac{1}{2}$$

$$=> -0.001 \times t_{0.5} = \log\left(\frac{1}{2}\right)$$

$$=> t_{0.5} = -1000 \log\left(\frac{1}{2}\right) \approx 301.03$$

$\therefore$ median of $X$ is 301.03

$(c)\ \Pr(X > 2000) = S(2000) = \exp(-0.001 \times 2000) \approx 0.13534$

2.2 $X \sim$ Weibull $(\alpha = 2, \lambda = 0.001)$ the pdf is given by

$f(x) = \alpha\lambda x^{\alpha-1} \exp(-\lambda x^{\alpha}),\ \ x \ge 0$

,and the survival function of $X$ is

$$S(t) = \int_t^{\infty} \alpha\lambda x^{\alpha-1} \exp(-\lambda x^{\alpha}) dx = -\exp(-\lambda x^{\alpha})\Big|_t^{\infty} = \exp(-\lambda t^{\alpha})$$

(a) $\Pr(X > 30) = \exp(-0.001 \times 30^2) \approx 0.40657$

$\Pr(X > 45) = \exp(-0.001 \times 45^2) \approx 0.13199$

$\Pr(X > 60) = \exp(-0.001 \times 60^2) \approx 0.02732$

(b) $E(X) = \int_0^{\infty} x\alpha\lambda x^{\alpha-1} \exp(-\lambda x^{\alpha}) dx$

$let\ \ x^{\alpha} = u \quad du = \alpha x^{\alpha-1} dx$

$$= \int_0^{\infty} u^{\frac{1}{\alpha}} \lambda \exp(-\lambda u) du$$

$$= \lambda \frac{\Gamma\left(\frac{1}{\alpha}+1\right)}{\lambda^{\frac{1}{\alpha}+1}} \int_0^{\infty} \frac{\lambda^{\frac{1}{\alpha}+1}}{\Gamma\left(\frac{1}{\alpha}+1\right)} u^{\frac{1}{\alpha}+1-1} \exp(-\lambda u) du$$

$\because \frac{\lambda^{\frac{1}{\alpha}+1}}{\Gamma\left(\frac{1}{\alpha}+1\right)} u^{\frac{1}{\alpha}+1-1} \exp(-\lambda u)$ is pdf of gamma distribution

$$= \lambda \frac{\Gamma\left(\frac{1}{\alpha}+1\right)}{\lambda^{\frac{1}{\alpha}+1}} \times 1 = \frac{\Gamma\left(\frac{1}{\alpha}+1\right)}{\lambda^{\frac{1}{\alpha}}}$$

$$=> \ E(X) = \frac{\Gamma\left(\frac{1}{2}+1\right)}{0.001^{\frac{1}{2}}} = \frac{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{0.001^{\frac{1}{2}}} = \frac{\frac{1}{2}\sqrt{\pi}}{0.001^{\frac{1}{2}}} \approx 28.02496$$

(c) The harzard function is that

$$h(x) = \frac{f(x)}{S(x)} = \frac{\alpha\lambda x^{\alpha-1}\exp(-\lambda x^{\alpha})}{\exp(-\lambda x^{\alpha})} = \alpha\lambda x^{\alpha-1}$$

$$=> \ h(30) = 2 \times 0.001 \times 30^{2-1} \approx 0.06$$

$$h(45) = 2 \times 0.001 \times 45^{2-1} \approx 0.09$$

$$h(60) = 2 \times 0.001 \times 60^{2-1} \approx 0.12$$

(d) $S(t_{0.5}) = \frac{1}{2}$

$$=> \exp(-0.001 \times t_{0.5}{}^{2}) = \frac{1}{2}$$

$$=> -0.001 \times t_{0.5}{}^{2} = \log(0.5)$$

$$=> t_{0.5} = \{-1000\log(0.5)\}^{\frac{1}{2}} \approx 17.35022$$

$\therefore$ median of $X$ is 17.35022

2.3

$X \sim \text{log logistic}(\alpha = 1.5, \lambda = 0.01)$, the pdf is given by

$$f(x) = \frac{\alpha\lambda x^{\alpha-1}}{(1+\lambda x^{\alpha})^2}, x \ge 0$$

(a) $S(t) = \int_t^{\infty} \frac{\alpha\lambda x^{\alpha-1}}{(1+\lambda x^{\alpha})^2} dx = -\frac{1}{1+\lambda x^{\alpha}}\Big|_t^{\infty} = \frac{1}{1+\lambda t^{\alpha}}$

$$=> \Pr(X > 50) = S(50) = \frac{1}{1+0.01 \times 50^{1.5}} \approx 0.22048$$

$$\Pr(X > 100) = S(100) = \frac{1}{1+0.01 \times 100^{1.5}} \approx 0.09091$$

$$\Pr(X > 150) = S(150) = \frac{1}{1+0.01 \times 150^{1.5}} \approx 0.05162$$

(b) $S(t_{0.5}) = \frac{1}{2}$

$$=> \frac{1}{1+0.01\times t_{0.5}^{1.5}} = \frac{1}{2}$$

$$=> 2 = 1+0.01\times t_{0.5}^{1.5}$$

$$=> t_{0.5} = 100^{\frac{1}{1.5}} = 21.54435$$

$\therefore$ median of $X$ is 21.54435

(c) The hazard function is that

$$h(x) = \frac{f(x)}{S(x)} = \frac{\alpha\lambda x^{\alpha-1}}{1+\lambda x^{\alpha}}$$

$$h'(x) = \frac{\alpha(\alpha-1)\lambda x^{\alpha-2}(1+\lambda x^{\alpha}) - \alpha\lambda x^{\alpha-1}(\alpha\lambda x^{\alpha-1})}{(1+\lambda x^{\alpha})^2}$$

$$= \frac{\alpha\lambda x^{\alpha-2}\{(\alpha-1)(1+\lambda x^{\alpha}) - \alpha\lambda x^{\alpha}\}}{(1+\lambda x^{\alpha})^2}$$

$$= \frac{\alpha\lambda x^{\alpha-2}\{\alpha+\alpha\lambda x^{\alpha} - 1 - \lambda x^{\alpha} - \alpha\lambda x^{\alpha}\}}{(1+\lambda x^{\alpha})^2}$$

$$= \frac{\alpha\lambda x^{\alpha-2}\{\alpha - 1 - \lambda x^{\alpha}\}}{(1+\lambda x^{\alpha})^2}$$

let $h'(x) = 0$

$$=> \frac{\alpha\lambda x^{\alpha-2}\{\alpha - 1 - \lambda x^{\alpha}\}}{(1+\lambda x^{\alpha})^2} = 0$$

$$=> \{\alpha - 1 - \lambda x^{\alpha}\} = 0$$

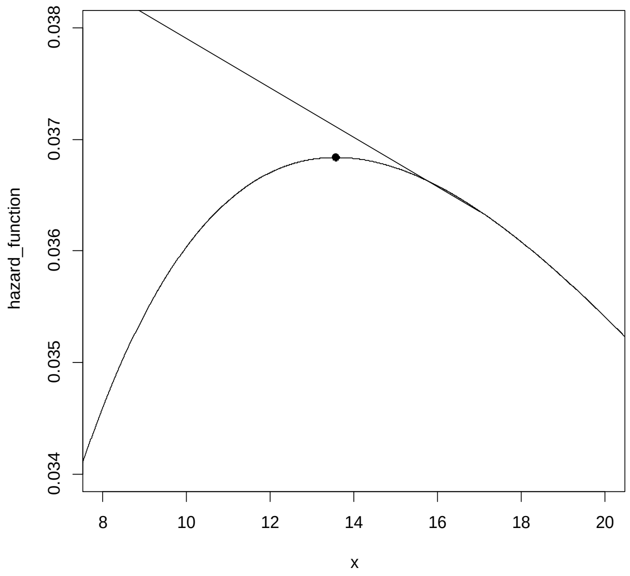
$$=> x = \left(\frac{\alpha-1}{\lambda}\right)^{\frac{1}{\alpha}}$$

If $x > \left(\frac{\alpha-1}{\lambda}\right)^{\frac{1}{\alpha}}$ *then* $h'(x) < 0$

If $x < \left(\frac{\alpha-1}{\lambda}\right)^{\frac{1}{\alpha}}$ *then* $h'(x) > 0$

$h(x)$ is increasing when $0 \le x < \left(\frac{\alpha-1}{\lambda}\right)^{\frac{1}{\alpha}} = \left(\frac{1.5-1}{0.01}\right)^{\frac{1}{1.5}} \approx 13.57209$

$h(x)$ is decreasing when $x > \left(\frac{\alpha-1}{\lambda}\right)^{\frac{1}{\alpha}} \approx 13.57209$




Black point is $x=\left(\frac{\alpha-1}{\lambda}\right)^{\frac{1}{\alpha}}\approx 13.57209$, also the maximum of hazard function which $\lambda=0.01,\alpha=1.5$.

$(d)$

$$E(X)=\int_0^\infty x\frac{\alpha\lambda x^{\alpha-1}}{(1+\lambda x^\alpha)^2}dx$$

$$\text{Let } u=\frac{1}{1+\lambda x^\alpha}\quad\leftrightarrow\quad\left\{\frac{1}{\lambda}(u^{-1}-1)\right\}^{\frac{1}{\alpha}}$$

$$\Rightarrow du=-\frac{\alpha\lambda x^{\alpha-1}}{(1+\lambda x^\alpha)^2}dx$$

$$=\int_1^0-\left\{\frac{1}{\lambda}(u^{-1}-1)\right\}^{\frac{1}{\alpha}}du=\lambda^{-\frac{1}{\alpha}}\int_0^1\left\{u^{-1}(1-u)\right\}^{\frac{1}{\alpha}}du=\lambda^{-\frac{1}{\alpha}}\int_0^1 u^{-\frac{1}{\alpha}+1-1}(1-u)^{\frac{1}{\alpha}+1-1}du$$

$$=\lambda^{-\frac{1}{\alpha}}B\left(-\frac{1}{\alpha}+1,\frac{1}{\alpha}+1\right),\ \text{where } B(.) \text{ is beta function}$$

given that $\alpha=1.5,\lambda=0.01$

$$E(X)=0.01^{-\frac{1}{1.5}}B\left(-\frac{1}{1.5}+1,\frac{1}{1.5}+1\right)$$

$$\approx 52.10283$$

2.4 $X$ is a r.v ,with survival function that

$$S(x)=\exp[1-\exp\{(\lambda x)^{\alpha}\}]$$

$$=> f(x)=-S'(x)=-\exp[1-\exp\{(\lambda x)^{\alpha}\}][-\exp\{(\lambda x)^{\alpha}\}]\alpha(\lambda x)^{\alpha-1}\lambda$$

$$=\exp[1-\exp\{(\lambda x)^{\alpha}\}]\exp\{(\lambda x)^{\alpha}\}\alpha\lambda^{\alpha}x^{\alpha-1}$$

$$=> h(x)=\frac{f(x)}{S(x)}=\exp\{(\lambda x)^{\alpha}\}\alpha\lambda^{\alpha}x^{\alpha-1}$$

(a) The hazard function is that

$$h(x)=\exp\{(\lambda x)^{\alpha}\}\alpha\lambda^{\alpha}x^{\alpha-1}$$

$$h'(x)=\exp\{(\lambda x)^{\alpha}\}\alpha\lambda^{\alpha}x^{\alpha-1}\alpha\lambda^{\alpha}x^{\alpha-1}+\exp\{(\lambda x)^{\alpha}\}\alpha(\alpha-1)\lambda^{\alpha}x^{\alpha-2}$$

$$=\exp\{(\lambda x)^{\alpha}\}\alpha(\alpha-1)\lambda^{\alpha}x^{\alpha-2}(\alpha\lambda^{\alpha}x^{\alpha}+\alpha-1)$$

Case I : given $\alpha=0.5$

Let $h'(x)=0$

$=>\ h'(x)=\exp\{(\lambda x)^{0.5}\}0.5(0.5-1)\lambda^{0.5}x^{0.5-2}(0.5\lambda^{0.5}x^{0.5}+0.5-1)=0$

$=>(0.5\lambda^{0.5}x^{0.5}+0.5-1)=0$

$=> x=\dfrac{1}{\lambda}$

If $x>\dfrac{1}{\lambda}$ *then* $h'(x)>0$ .

If $0\le x<\dfrac{1}{\lambda}$ *then* $h'(x)<0$.

$h(x)$ is increasing when $x>\dfrac{1}{\lambda}$ .

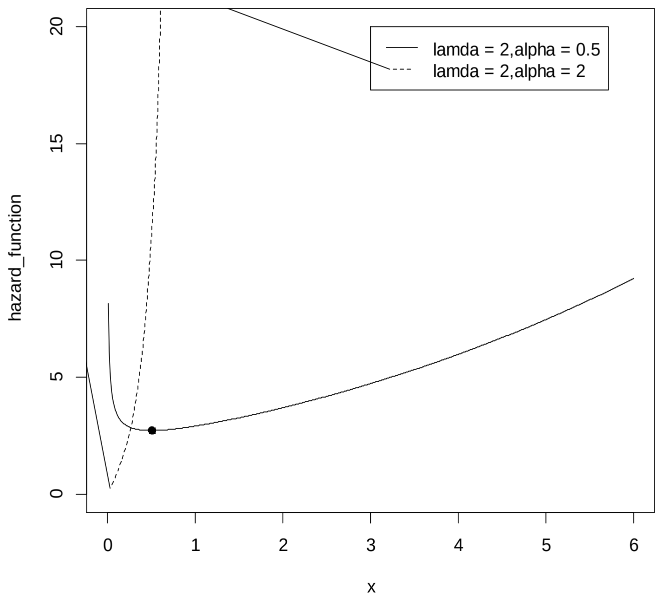
$h(x)$ is decreasing when $0\le x<\dfrac{1}{\lambda}$ .

(b)

Case II : given $\alpha=2$

$h'(x)=\exp\{(\lambda x)^{2}\}2(2-1)\lambda^{2}x^{2-2}(2\lambda^{2}x^{2}+2-1)\ge 0\ \ ,\forall x\ge 0$

$h(x)$ is monotone increasing for all $x\ge 0$




Black point is $x = \frac{1}{\lambda} = \frac{1}{2}$, also the minimum of hazard function which $\lambda = 2, \alpha = 0.5$. When $\lambda = 2, \alpha = 2$ of the hazard function is monotone increasing.

2.5

$X \sim \log\text{normal}(\mu = 3.177, \sigma = 2.084)$

$\Rightarrow X = e^Y$ , where $Y \sim N(\mu = 3.177, \sigma = 2.084)$

(a)

$E(X) = Ee^Y = M_Y(t)\big|_{t=1}$, with $M_Y(t)$ is mgf of normal

$$= \exp\left(\mu + \frac{1}{2}\sigma^2\right)\Bigg|_{\mu=3.177,\sigma=2.084} \approx 210.2985$$

$\Pr(X \le t) = \Pr(\log X \le \log t) = \Pr(Y \le \log t) = \Phi\left(\frac{\log t - \mu}{\sigma}\right)\Bigg|_{\mu=3.177,\sigma=2.084}$, with $\Phi(.)$ is cdf of $N(0,1)$.

$$\Rightarrow F(t_{0.5}) = \Phi\left(\frac{\log t_{0.5} - 3.177}{2.084}\right) = \frac{1}{2}$$

$$\Rightarrow \Phi^{-1}\left(\frac{1}{2}\right) = \frac{\log t_{0.5} - 3.177}{2.084}$$

$$\Rightarrow 3.177 = \log t_{0.5}$$

$$\Rightarrow t_{0.5} = \exp(3.177) \approx 23.97472$$

Mean of $X$ is 210.2985 and median of $X$ is 23.97472

(b)

$$S(100) = 1 - F(100) \approx 0.24658$$

$$S(200) = 1 - F(200) \approx 0.15436$$

$$S(300) = 1 - F(300) \approx 0.11267$$

(c)

$$h(x)=\frac{f(x)}{S(x)}=\frac{\frac{1}{2.084x}\phi\left(\frac{\log x-3.177}{2.084}\right)}{1-\Phi\left(\frac{\log x-3.177}{2.084}\right)}\text{, where } f(x)=F'(x)=\frac{1}{2.084x}\phi\left(\frac{\log x-3.177}{2.084}\right),$$

with $\phi(.)$ is pdf of N(0,1)

$$h'(x)=\frac{f'(x)S(x)-f(x)\{-f(x)\}}{\{S(x)\}^2}=\frac{f'(x)S(x)-f(x)\{-f(x)\}}{\{S(x)\}^2}=\frac{f'(x)}{S(x)}+\frac{\{f(x)\}^2}{\{S(x)\}^2},$$

$$\text{where } f'(x)=\frac{-1}{2.084x^2}\phi\left(\frac{\log x-3.177}{2.084}\right)+\frac{1}{2.084x}\phi\left(\frac{\log x-3.177}{2.084}\right)\left(-\frac{\log x-3.177}{2.084}\right)\frac{1}{2.084x}$$

$$=\left(\frac{1}{2.084x}\right)^2\phi\left(\frac{\log x-3.177}{2.084}\right)\left\{-2.084-\left(\frac{\log x-3.177}{2.084}\right)\right\}$$

$$=\frac{1}{2.084x}\left\{-2.084-\left(\frac{\log x-3.177}{2.084}\right)\right\}f(x)$$

$$h'(x)=\frac{\frac{1}{2.084x}\left\{-2.084-\left(\frac{\log x-3.177}{2.084}\right)\right\}f(x)}{S(x)}+\frac{\{f(x)\}^2}{\{S(x)\}^2}$$

set $h'(x)=0$

$$=>\ \frac{1}{2.084x}\left\{-2.084-\left(\frac{\log x-3.177}{2.084}\right)\right\}+\frac{f(x)}{S(x)}=0$$

$$=>h(x)=\frac{1}{2.084x}\left\{2.084+\left(\frac{\log x-3.177}{2.084}\right)\right\}$$

$$=>x=h^{-1}\left[\frac{1}{2.084x}\left\{2.084+\left(\frac{\log x-3.177}{2.084}\right)\right\}\right] \qquad (1)$$

But $x$ is hard to compute,I just set $x_t$ is the solution of equation (1).

$x_t$ is a critical point.

Set $h'(x)>0$

$$=>\frac{1}{2.084x}\left\{-2.084-\left(\frac{\log x-3.177}{2.084}\right)\right\}+\frac{f(x)}{S(x)}>0$$

$$=>h(x)<\frac{1}{2.084x}\left\{2.084+\left(\frac{\log x-3.177}{2.084}\right)\right\}$$

$$=>x<h^{-1}\left[\frac{1}{2.084x}\left\{2.084+\left(\frac{\log x-3.177}{2.084}\right)\right\}\right]$$

$=>h(x)$ is increasing when $0<x<x_t$

Similarly $h'(x)<0$ $=>$ $x>x_t$

By the previous results,we can know that $h(x)$ first increasing then decreasing change in the time $x_t$.

I use the bisection iteration to find the change point $x_t = 0.3470216$.

The following is the algorithm:

Step 1. Find two initial point $x_L$ and $x_R$ satisfy $h'(x_L)h'(x_R) < 0$

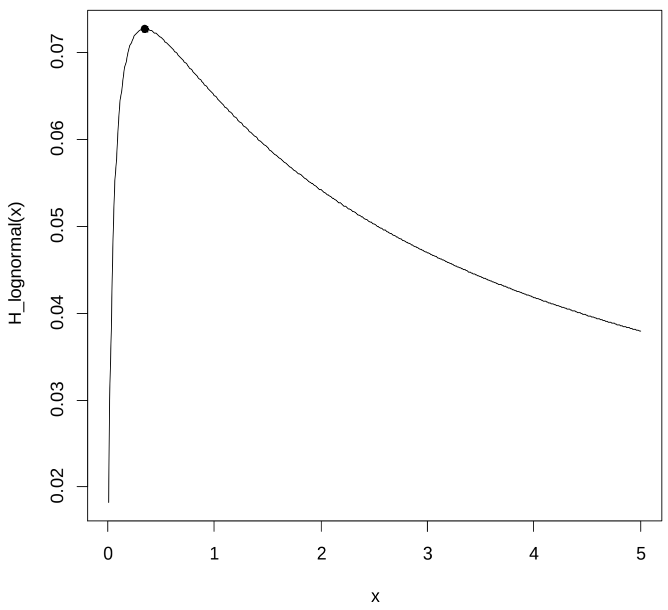
Step 2. Set $x_{New} = \frac{x_L + x_R}{2}$ if $h'(x_{New})h'(x_R) < 0$ then set $x_L = x_{New}$

otherwise set $x_R = x_{New}$

Step3.if $|x_L - x_R| < \varepsilon$, where $\varepsilon$ is very small value then stop

otherwise goes to step1.

Graph1




Black point in the graph 1 is $x_t = 0.3470216$.

2.10

(a)

$X$ is a r.v with hazard function

$$h(x)=\begin{cases}\theta_1 & ,t_0\le x<t_1\\ \theta_2 & ,t_1\le x<t_2\\ \theta_3 & ,t_2\le x<t_3\\ \vdots & \\ \theta_{k-1} & ,t_{k-2}\le x<t_{k-1}\\ \theta_k & ,t_{k-1}\le x\end{cases}$$

$$h(x)=\frac{f(x)}{S(x)}=\frac{d}{dx}\{-\log S(x)\}$$

for $t_{n-1}\le x\le t_n$ , $1\le n\le k$

$$\int_{t_{n-1}}^{t_n}\frac{d}{dx}\{-\log S(x)\}dx=\int_{t_{n-1}}^{t_n}\theta_n dx=\theta_n(t_n-t_{n-1})$$

$$\Rightarrow -\log S(t_n)-\{-\log S(t_{n-1})\}=\theta_n(t_n-t_{n-1})$$

$$\begin{aligned}\Rightarrow -\log S(t_n)&=\theta_n(t_n-t_{n-1})+\{-\log S(t_{n-1})\}\\ &=\theta_n(t_n-t_{n-1})+\theta_{n-1}(t_{n-1}-t_{n-2})+\{-\log S(t_{n-2})\}\\ &=.....=\theta_n(t_n-t_{n-1})+\theta_{n-1}(t_{n-1}-t_{n-2})+....+\{-\log S(t_1)\}\\ &=\theta_n(t_n-t_{n-1})+\theta_{n-1}(t_{n-1}-t_{n-2})+....+\theta_1(t_1-t_0)+\{-\log S(t_0)\}, t_0=0\\ &=\theta_n(t_n-t_{n-1})+\theta_{n-1}(t_{n-1}-t_{n-2})+....+\theta_1 t_1\end{aligned}$$

$$\Rightarrow S(t_n)=\exp\left[-\{\theta_n(t_n-t_{n-1})+\theta_{n-1}(t_{n-1}-t_{n-2})+....+\theta_1 t_1\}\right]$$

$$\Rightarrow S(x)=\begin{cases}\exp\left[-\{\theta_1 x\}\right] & ,t_0\le x<t_1\\ \exp\left[-\{\theta_2(x-t_1)+\theta_1 t_1\}\right] & ,t_1\le x<t_2\\ \exp\left[-\{\theta_3(x-t_2)+\theta_2(t_2-t_1)+\theta_1 t_1\}\right] & ,t_2\le x<t_3\\ \vdots & \\ \exp\left[-\{\theta_k(x-t_{k-1})+\theta_{k-1}(t_{k-1}-t_{k-2})+....+\theta_1 t_1\}\right] & ,t_{k-1}\le x\end{cases}$$

2.11

$$S(x)=\begin{cases}\exp\{-\lambda(x-\phi)^{\alpha}\} & ,x\ge\phi\\ 1 & ,x<\phi\end{cases}$$ is the survival function of $X$ .

(a)

$$f(x)=-S'(x)=\begin{cases}-\lambda\alpha(x-\phi)^{\alpha-1}\exp\{-\lambda(x-\phi)^{\alpha}\} & ,x\ge\phi\\ 0 & ,x<\phi\end{cases}$$

$$h(x)=\frac{f(x)}{S(x)}=\begin{cases}-\lambda\alpha(x-\phi)^{\alpha-1} & ,x\ge\phi\\ 0 & ,x<\phi\end{cases}$$

(b)

$$\Pr(X-\phi \ge \mathrm{t}) = \Pr(X \ge \mathrm{t}+\phi) = \exp(-\lambda t^{\alpha})$$

$$\therefore X-\phi \sim \text{Weibull}(\alpha,\lambda)$$

$$E(X) = E(X-\phi+\phi) = E(X-\phi)+\phi$$

$$= \frac{\Gamma\left(\frac{1}{\alpha}+1\right)}{\lambda^{\frac{1}{\alpha}}}+\phi \quad \text{(by 2.2(b))}$$

Given that $\alpha = 1, \lambda = 0.0075$ and $\phi = 100$

$$E(X) = \frac{\Gamma\left(\frac{1}{1}+1\right)}{0.0075^{\frac{1}{1}}}+100 \quad \approx 233.333$$

by 2.2(d)

$Y \sim \text{Weibull}(\alpha,\lambda)$

median of $Y$ is $\left(-\frac{1}{\lambda}\log\frac{1}{2}\right)^{\frac{1}{\alpha}}$

=> median of $X$ is $\left(-\frac{1}{\lambda}\log\frac{1}{2}\right)^{\frac{1}{\alpha}}+\phi$

given that $\alpha = 1, \lambda = 0.0075$ and $\phi = 100$

median of $X$ is $\left(-\frac{1}{0.0075}\log\frac{1}{2}\right)^{\frac{1}{1}}+100 \approx 192.41962$

2.19

There is a bivariate survival function :

$$S(x,y) = (1-x)(1-y)(1+0.5xy) \quad ,0<x<1 \ ,0<y<1$$

(a) The marginal survival function of $X$ and $Y$ are

$$S(x) = S(x,0) = 1-x$$

$$S(y) = S(0,y) = 1-y$$

2.20

There is a bivariate survival function :

$$S(x,y) = \exp(-x-y-0.5xy) \quad ,0<x \ ,0<y$$

(a) The marginal survival function of $X$ and $Y$ are

$$S(x) = S(x,0) = \exp(-x)$$

$$S(y) = S(0,y) = \exp(-y)$$

- HW for Newton Raphson

$$l(\lambda) = \log L(\lambda) = \sum_{i=1}^{n} \log\{\exp(-\lambda l_i) - \exp(-\lambda r_i)\}$$

So the first order of loglikelihood is that

$$l'(\lambda) = \frac{\partial \log L(\lambda)}{\partial \lambda} = \sum_{i=1}^{n} \frac{-l_i \exp(-\lambda l_i) + r_i \exp(-\lambda r_i)}{\exp(-\lambda l_i) - \exp(-\lambda r_i)}$$

The scecond order of loglikelihood is that

$$l''(\lambda) = \frac{\partial^2 \log L(\lambda)}{\partial \lambda^2}$$

$$= \sum_{i=1}^{n} \frac{\{l_i^2 \exp(-\lambda l_i) - r_i^2 \exp(-\lambda r_i)\}\{\exp(-\lambda l_i) - \exp(-\lambda r_i)\} - \{-l_i \exp(-\lambda l_i) + r_i \exp(-\lambda r_i)\}^2}{\exp(-\lambda l_i) - \exp(-\lambda r_i)}$$

Using Newton-Raphson technique to get MLE of $\lambda$

Newton-Raphson:

First choose a initial point $\lambda_0$ then updated $\lambda$ by the following iteration

$\lambda_{i+1} = \lambda_i - l'(\lambda_i) / l''(\lambda_i)$.

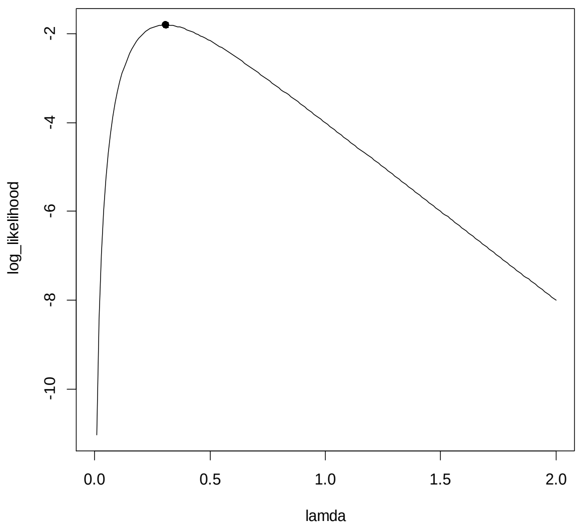
Stop when $|x_{i+1} - x_i| < \varepsilon$, where $\varepsilon$ is very small value.

Given the data [0,7], [0,8], [0,5], [4,11]

So that $l_1 = 0, l_2 = 0, l_3 = 0, l_4 = 4$
$r_1 = 7, r_2 = 8, r_3 = 5, r_4 = 11$

Choose initial value $\lambda_0 = 0.2 \quad \varepsilon = 10^{-5}$

$\hat{\lambda}_{\text{MLE}} = 0.3060561$ by Newton-Raphson technique.




Black point is $\hat{\lambda}_{\text{MLE}} = 0.3060561$.