Homework#2, Statistical Inference II, 2013 Spring

1. Let $Y \sim Bin(n=5, p)$.

1) Derive a UMP test for $H_0 : p = 1/3$ vs. $H_1 : p = 2/3$ of level $\alpha = 0.05$.

2) Derive a UMP test for $H_0 : p = 1/3$ vs. $H_1 : p > 1/3$.

2. I mention in class

3. Data $(X_1,..., X_n)$ follows independently and identically a scale exponential distribution with the p.d.f. $f_a(x) = \exp\{-(x-a)\} I_{(a,\infty)}(x)$. Derive the UMP test for $H_0 : a = a_0$ vs. $H_1 : a > a_0$ using the properties of MLR. Here, prove the MLR by drawing the graph.

<u>1.Answer</u>

$P_0(Y=5) = 1/243 = 0.004$

$P_0(Y=4) = 10/243 = 0.041$

$P_0(Y=3) = 40/243 = 0.165$

Solving $P_0(Y=5) + P_0(Y=4) + \gamma P_0(Y=3) = 0.05$,

$$\gamma = \frac{0.05 - 0.004 - 0.041}{0.165} = 0.030.$$

$$T(Y) = \begin{cases} 1 & Y > 3 \\ 0.03 & Y = 3 \\ 0 & Y < 3 \end{cases}$$

Test $H_0: \sigma = 1$ v.s. $H_1: \sigma = 2$ based on $X_1, \ldots, X_n \sim i.i.d. N(0,1)$, where μ is known.

(i) $T_1(X) = \sqrt{n}(\overline{X} - \mu) \sim N(0,1), (under \quad H_0)$

Reject H_0, if $|T_1(X)| > z_{1-\frac{\alpha}{2}}$.

(ii) $T_2(X) = (n-1)S^2 \sim \chi^2_{n-1}, (under \quad H_0)$

Reject H_0, if $T_2(X) > \chi^2_{n-1,1-\alpha}$.

(iii) $T_3(X) = \sum_{i=1}^{n} (X_i - \mu)^2 \sim \chi^2_n, (under \quad H_0)$

Reject H_0, if $T_3(X) > \chi^2_{n,1-\alpha}$.

Q2. Obtain the powers of T_1, T_2 and T_3.
Compare them by numerical calculation for n=2,3,5,10.

<u>*Ans.*</u>

1° The powers of T_1, T_2 and T_3.

$$\beta_{T_1} = P_{\sigma=2}(|T_1(X)| > z_{1-\frac{\alpha}{2}}) = P_{\sigma=2}(|\frac{T_1(X)}{2}| > \frac{z_{1-\frac{\alpha}{2}}}{2}),$$

where $\frac{T_1(X)}{2} \sim N(0,1) \quad (under \quad H_1).$

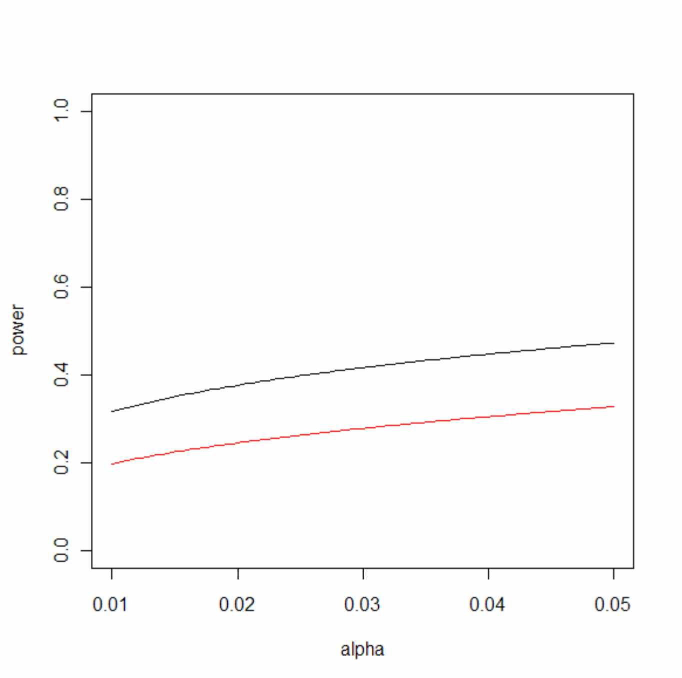
$$\beta_{T_2} = P_{\sigma=2}(T_2(X) > \chi^2_{n-1,1-\alpha}) = P_{\sigma=2}(\frac{T_2(X)}{4} > \frac{\chi^2_{n-1,1-\alpha}}{4}),$$

where $\frac{T_2(X)}{4} \sim \chi^2_{n-1} \quad (under \quad H_1).$

$$\beta_3 = P_{\sigma=2}(T_3(X) > \chi^2_{n,1-\alpha}) = P_{\sigma=2}(\frac{T_3(X)}{4} > \frac{\chi^2_{n,1-\alpha}}{4}),$$

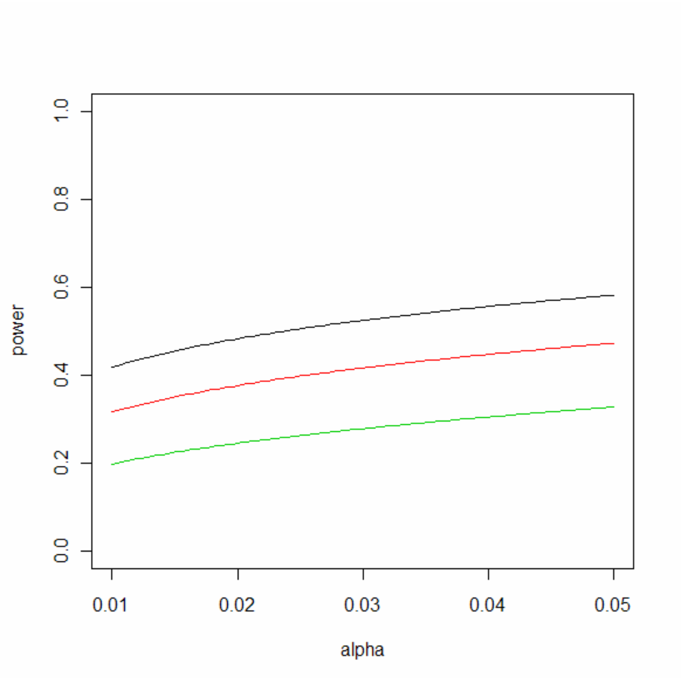
where $\frac{T_3(X)}{4} \sim \chi^2_n \quad (under \quad H_1).$

2° Compare for n=2



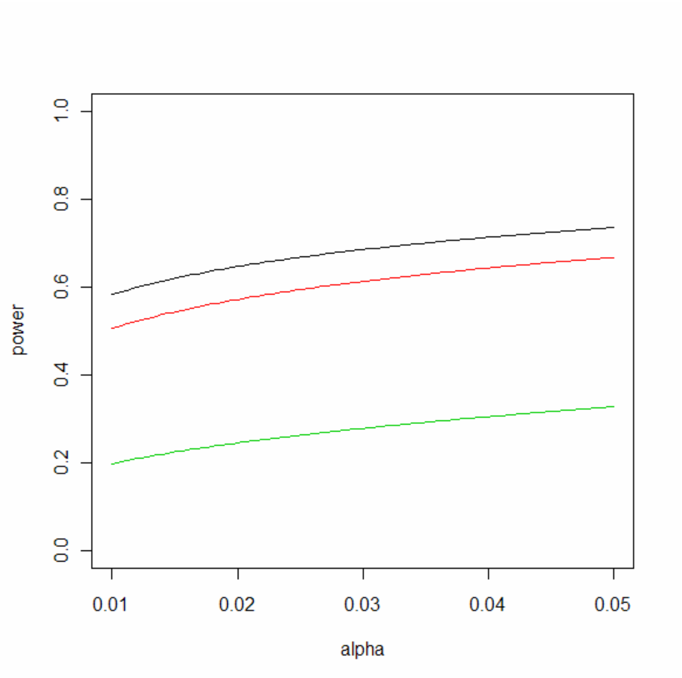
$\beta_{T_1} = \beta_{T_2} < \beta_{T_3}$,for n=2

3° Compare for n=3



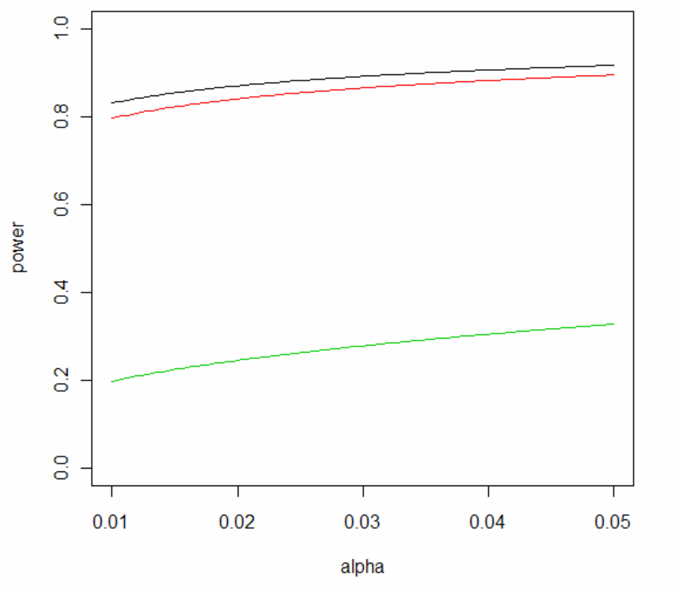
$\beta_{T_1} < \beta_{T_2} < \beta_{T_3}$,for n=3

4° Compare for n=5



$\beta_{T_1} < \beta_{T_2} < \beta_{T_3}$,for n=5

5° Compare for n=10



$\beta_{T_1} < \beta_{T_2} < \beta_{T_3}$,for n=10

“code”

```
p1=function(x){
1-pnorm(qnorm(1-x/2,0,1)/2,0,1)+pnorm(-qnorm(1-x/2,0,1)/2,0,1)
}
p2=function(x){
1-pchisq(qchisq(1-x,n-1)/4,n-1)
}
p3=function(x){
1-pchisq(qchisq(1-x,n)/4,n)
}
plot(p1, col = 3,ylim =c(0, 1),xlim =c(0.01, 0.05),xlab=”alpha”,ylab=”power”)#green
plot(p2, col = 234,xlim =c(0.01, 0.05),add=T)#red
plot(p3, xlim =c(0.01, 0.05),add=T)#black
```