Statistical Inference Homework#1

$X = (X_1,...,X_n) \sim i.i.d.\, f_\theta(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \theta > 0$

$\Rightarrow \overline{X}$ is CSS for θ.

$H_0 : \theta \le \theta_0$ v.s. $H_1 : \theta > \theta_0$.

Let $T_c(X) = I_{(c,\infty)}(\overline{X})$.

$$R_{T_c}(\theta) = \begin{cases} P_\theta(\overline{X} > c), & \theta \le \theta_0 \quad (Type\ \ I\ \ error) \\ P_\theta(\overline{X} \le c), & \theta > \theta_0 \quad (Type\ \ II\ \ error) \end{cases}, \quad \text{(under 0-1 loss)}$$

$$Y = \sum_{i=1}^{n} X_i \sim f_\theta(y) = \frac{1}{\Gamma(n)\cdot\theta^n} \cdot y^{n-1} \cdot e^{-\frac{y}{\theta}}, \quad (Y \sim \Gamma(n,\theta))$$

$$\therefore R_{T_c}(\theta) = \{\int_{nc}^{\infty} \frac{1}{\Gamma(n)\cdot\theta^n} \cdot y^{n-1} \cdot e^{-\frac{y}{\theta}} dy\} \cdot I_{(0,\theta_0]}(\theta) + \{\int_0^{nc} \frac{1}{\Gamma(n)\cdot\theta^n} \cdot y^{n-1} \cdot e^{-\frac{y}{\theta}} dy\} \cdot I_{(\theta_0,\infty)}(\theta)$$

Q: Daw OC curve for n=1,5,10

$$P_\theta(Y \le nc) = \{\int_0^{nc} \frac{1}{\Gamma(n)\cdot\theta^n} \cdot y^{n-1} \cdot e^{-\frac{y}{\theta}} dy\} \cdot I_{(\theta_0,\infty)}(\theta)$$

Set $\begin{cases} \theta_0 = 1 \\ \alpha = 0.05 \end{cases}$

(i)n=1, $P_\theta(Y \le nc_1) = \{\int_0^{nc_1} \frac{1}{\theta} \cdot e^{-\frac{y}{\theta}} dy\} \cdot I_{(\theta_0,\infty)}(\theta)$.

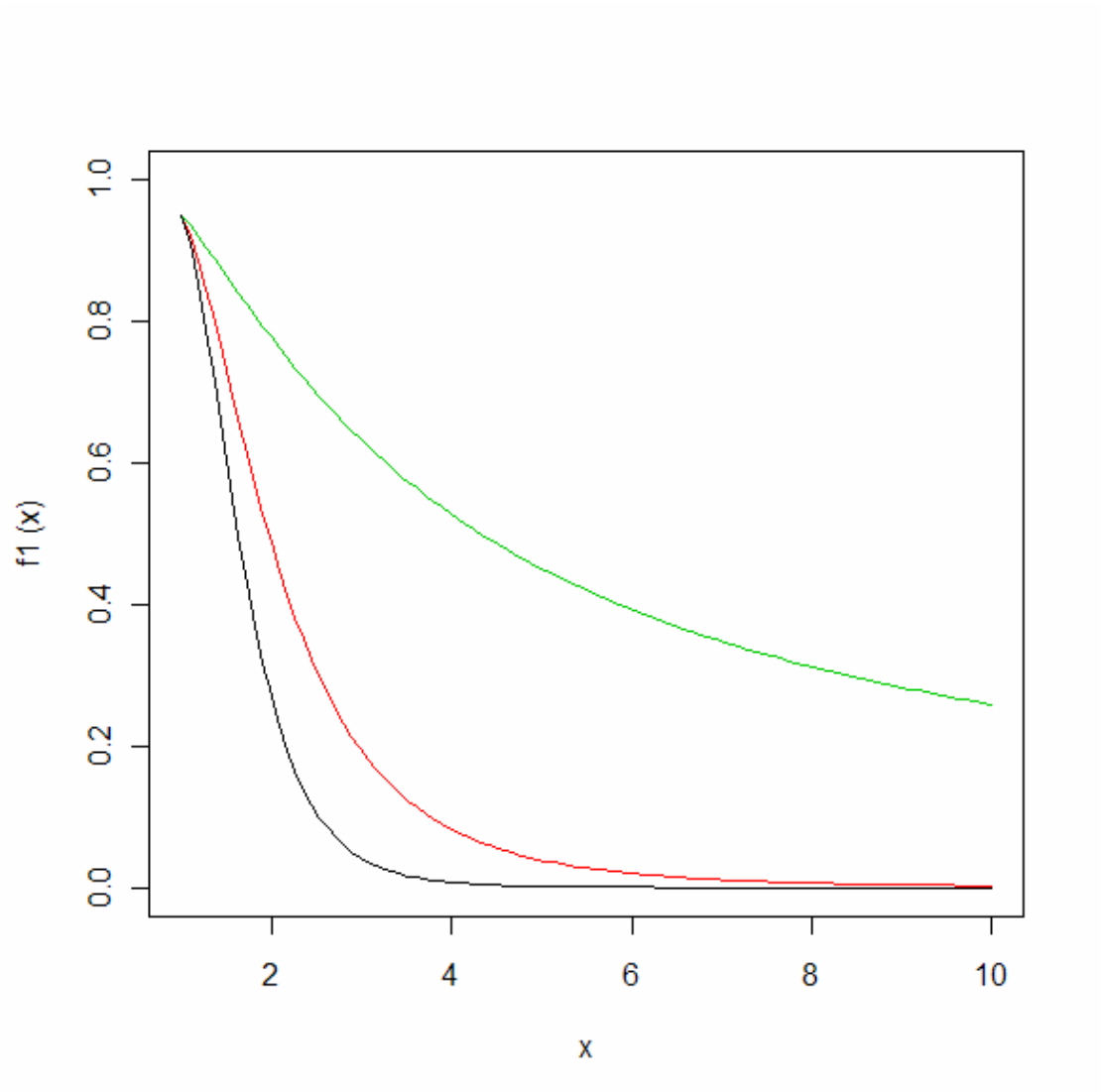
(ii)n=5, $P_\theta(Y \le nc_2) = \{\int_0^{nc_2} \frac{1}{\Gamma(5)\cdot\theta^5} \cdot y^4 \cdot e^{-\frac{y}{\theta}} dy\} \cdot I_{(\theta_0,\infty)}(\theta)$.

(iii)n=10, $P_\theta(Y \le nc_3) = \{\int_0^{nc_3} \frac{1}{\Gamma(10)\cdot\theta^{10}} \cdot y^9 \cdot e^{-\frac{y}{\theta}} dy\} \cdot I_{(\theta_0,\infty)}(\theta)$.

“code”

```
p1=function(x){
pgamma(qgamma(0.95,1,1),1,1/x)
}
p2=function(x){
pgamma(qgamma(0.95,5,1),5,1/x)
}
p3=function(x){
pgamma(qgamma(0.95,10,1),10,1/x)
}
plot(p1, col = 3,ylim =c(0, 1),xlim =c(1, 10))#green
plot(p2, col = 234,xlim =c(1, 10),add=T)#red
plot(p3, xlim =c(1, 10),add=T)#black
```

“OC curve”



翁新傑 Wong HsinChieh