Statistical inference I, 2011 Fall, Homework#2

<u>Exercise 122(2nd ed., J. Shao, p.159):</u>

$X_1,..,X_1 \stackrel{iid}{\sim} Bin(n=1,p)$. Let a and b be positive constants. Find the asymptotic relative efficiency of $(a+n\overline{X})/(a+b+n)$ w.r.t. $\overline{X}$.

<u>Ans</u>

By CLT, $\sqrt{n}(\overline{X}-p) \to_d Y = N(0,p(1-p))$.

Let $T(X) = (a+n\overline{X})/(a+b+n)$. Then,

$\sqrt{n}(T(X)-p)$

$= a\sqrt{n}/(a+b+n) + \sqrt{n}\{n\overline{X}/(a+b+n)-p\}$

$= o(1) - \overline{X}\{\sqrt{n}(a+b)/(a+b+n)\} + \sqrt{n}(\overline{X}-p)$

$= o_P(1) + \sqrt{n}(\overline{X}-p)$

$\to_d Y$

$amse_T(p) = amse_{\overline{X}}(p) = p(1-p)/n$.

Hence, ARE is 1.□