Statistical inference I, 2011 Fall, Homework#1

1. Asymptotic analysis

Show that $X_n = o_P(Y_n)$ implies $X_n = O_P(Y_n)$.

2. Exponential family

Problem 13 (p.143)

Problem 14 (144)

3. Location-scale family

Suppose that a probability measure $P_{\boldsymbol{\mu},\boldsymbol{\Sigma},\nu}$ defined on the k-dimensional Borel field has a density (w.r.t. a Lebesgue measure)

$$f_{\boldsymbol{\mu},\boldsymbol{\Sigma},\nu}(\mathbf{x}) = \frac{\det(\Sigma)^{-1/2}\Gamma\{(\nu+k)/2\}}{\Gamma(1/2)^k\ \Gamma(\nu/2)\nu^{k/2}}\left\{1+\frac{(\mathbf{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu}}{\nu}\right\}^{-(\nu+k)/2}.$$

(i) Show that $P_{\boldsymbol{\mu},\boldsymbol{\Sigma},\nu}(B) = P_\nu(\boldsymbol{\Sigma}^{-1/2}(B-\boldsymbol{\mu}\boldsymbol{\Sigma}))$ for some P_ν, and write down P_ν.

(ii) For cases of $k=1,2$, show $P_\nu \to_d P$ using Scheffe's theorem (p.59). Please explain in details how to apply the Scheffe's theorem.