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+6

Quiz #3, Quality control, 2018 Spring [+ 6 points]

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Let $X_1,..,X_n \overset{iid}{\sim} N(\mu,\sigma^2)$.

+2 **(1)** [+2] Define $d_2 = E[R/\sigma]$, where R is the range. Derive the value of d_2 when n=2 (with mathematical derivation)

$R = \max(X_1, X_2) - \min(X_1, X_2) = |X_1 - X_2|$

$W = \frac{R}{\sigma} = \left|\frac{X_1}{\sigma} - \frac{X_2}{\sigma}\right| = \left|\frac{X_1-\mu}{\sigma} - \frac{X_2-\mu}{\sigma}\right| = |Z_1 - Z_2|$

$E(W) = E|Z_1 - Z_2|$

$Z_1 - Z_2 \sim N(0,2)$ $\rightarrow$ $Z = \frac{Z_1 - Z_2}{2} \sim N(0,1)$ $\rightarrow$ continued behind the paper

+1 **(2)** [+1] We obtained sizes of chips (43, 44, 47, 46, 45). Find an estimate $\hat{\sigma}$ by the range method. [up to 2 digits: 0.00] $n=5$

$R = \max_{i=1..5} X_i - \min_{i=1..5} X_i = 47 - 43 = 4$

$\hat{\sigma} = \frac{R}{d_2} = \frac{4}{2.326} = 1.71969 \simeq 1.72$

Table: Conversion of range to standard deviation

n	2	3	4	5	6
d_2	1.128	1.683	2.059	2.326	2.534

+2 **(3)** [+2] Consider testing $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$ with level $\alpha = 0.0027$.

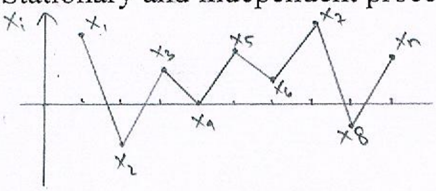
Let $\delta = \mu_1 - \mu_0$ be the shift of mean. Under $H_1 : \mu_1 = \mu_0 + \delta$, derive the consumer's risk (Type II error) in terms of δ, σ, and Φ.

$\beta = P(\text{Accept } H_0 \mid H_0 \text{ is false})$

$= P(|Z_0| \leq Z_{\alpha/2} \mid \mu_1)$ $\rightarrow$ continued behind the paper

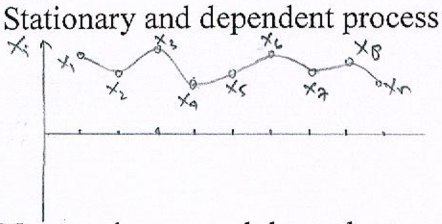
(4) [+1] Write figures to explain three processes for $X_1, X_2, \ldots, X_n$.

+1 (a) Stationary and independent process



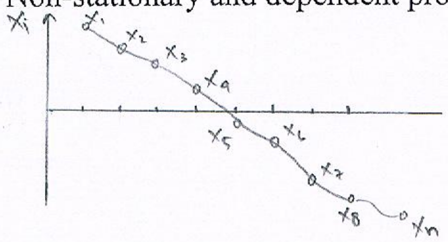
$\rightarrow$ Random walk without drift

(b) Stationary and dependent process



$\rightarrow \mu_1 = \mu_2 = \ldots$

(c) Non-stationary and dependent process



$\rightarrow \mu_1 > \mu_2 > \ldots$

1. $E(W) = \sqrt{2} \cdot E|Z|$

$$E|Z| = \int |z| \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}} dz = 2\int_0^{\infty} z \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}} dz$$

$$= -\frac{2}{\sqrt{2\pi}} \left(e^{-\frac{z^2}{2}} \Big|_0^{\infty} \right) = -\sqrt{\frac{2}{\pi}} (0-1) = \sqrt{\frac{2}{\pi}}$$

so, $E(W) = d_2 = \sqrt{2} \cdot \sqrt{\frac{2}{\pi}} = \frac{2}{\sqrt{\pi}} \simeq 1{,}128$

3. $$\beta = P\left(\left| \frac{\bar{X} - \mu_1}{\sigma/\sqrt{n}} \right| \leqslant z_{0.000135} \;\Big|\; \mu_1 \right)$$

$$= P\left(-z_{0.000135} \leqslant \frac{\bar{X} - \mu_1}{\sigma/\sqrt{n}} \leqslant z_{0.000135} \right)$$

$$= P\left(-z_{0.000135} - \frac{\mu_1 - \mu_0}{\sigma/\sqrt{n}} \leqslant \frac{\bar{X} - \mu_0 + \mu_0 - \mu_1}{\sigma/\sqrt{n}} \leqslant z_{0.000135} - \frac{\mu_1 - \mu_0}{\sigma/\sqrt{n}} \right)$$ where $\delta = \mu_1 - \mu_0$

$$= P\left(-z_{0.000135} - \frac{\delta\sqrt{n}}{\sigma} \leqslant Z \leqslant z_{0.000135} - \frac{\delta\sqrt{n}}{\sigma} \right)$$

$$= \Phi\left(z_{0.000135} - \frac{\delta\sqrt{n}}{\sigma} \right) - \Phi\left(-z_{0.000135} - \frac{\delta\sqrt{n}}{\sigma} \right)$$

$$= \Phi\left(3 - \frac{\delta\sqrt{n}}{\sigma} \right) - \Phi\left(-3 - \frac{\delta\sqrt{n}}{\sigma} \right)$$