Midterm Exam, Quality control 2017 Spring [+ 20 points] +20

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- **Not only answer but also calculation**
- **Numerical answers up to 4 digits, 0.XXXX**

+5 **Q1[+5]** Consider $\overline{X}$-chart with $\mu = \mu_0$ being the target value and σ being known.

+2 1) [+2] An engineer sets a 3σ-chart with $ARL_1 \le 2$ under $\mu_1 = \mu_0 + \sigma$. What is the required sample size n?

$ARL_1 = \frac{1}{1-\beta} \le 2 \Rightarrow 1 \le 2 - 2\beta \Rightarrow \beta \le 0.5$

$\beta = \Phi(3 - k\sqrt{n}) - \Phi(-3 - k\sqrt{n}),\quad k = \frac{\mu_1 - \mu_0}{\sigma} = 1$

$\approx \Phi(3 - \sqrt{n}) \le 0.5$

$\Rightarrow 3 - \sqrt{n} \le \Phi^{-1}(0.5) \Rightarrow 3 - \sqrt{n} \le 0 \Rightarrow 9 \le n$ #

+3 2) [+3] Draw the OC curve for n obtained from the previous question (show details).

$n \ge 9$

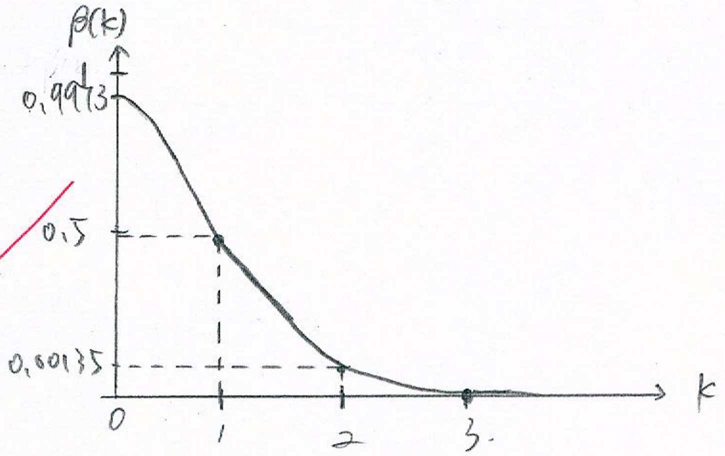
$\beta(k) = \Phi(3 - k\sqrt{9}) - \Phi(-3 - k\sqrt{9})$

$\beta(0) = \Phi(3) - \Phi(-3) = 0.9973$

$\beta(1) = \Phi(0) - \Phi(-6) \approx \Phi(0) = 0.5$

$\beta(2) = \Phi(-3) - \Phi(-9) \approx \Phi(-3) = 0.00135$

$\beta(3) = \Phi(-6) - \Phi(-12) \approx 0.$



+7 **[Q2] [+7]:** Data, $X_{ij}, i=1,\cdots,m, \quad j=1,\cdots,n \overset{iid}{\sim} N(\mu,\sigma^2)$, are collected as follows:

					Mean	Range
i=1	12	16	10	10	12	6
i=2	22	20	22	16	20	6
i=3	15	16	12	11	13.5	5
i=4	15	13	12	12	13	3
i=5	14	14	9	11	12	5

+1 1) [+1] Calculate $\hat{\sigma}$ by the range method. $\bar{\bar{x}}=14.1 \quad \bar{R}=5$

$\boxed{n=4}$ $\hat{\sigma}=\dfrac{\bar{R}}{d_2}=\dfrac{5}{2.059}=2.4284$ #

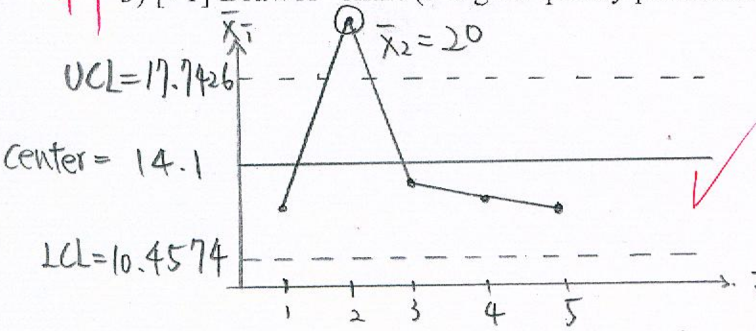
+3 2) [+3] Calculate center, UCL and LCL for $\overline{X}$-chart (3-sigma quality performance)

$\bar{\bar{x}}=\frac{1}{5}\sum_{i=1}^{5}\bar{x}_i=14.1$ ✓ $\nearrow 3.6426$

$UCL=\bar{\bar{x}}+3\dfrac{\hat{\sigma}}{\sqrt{4}}=14.1+\dfrac{3\times 2.4284}{2}=17.7426$ ✓

$LCL=\bar{\bar{x}}-3\dfrac{\hat{\sigma}}{\sqrt{4}}=14.1-\dfrac{3\times 2.4284}{2}=10.4574$ ✓

+1 3) [+1] Draw $\overline{X}$-chart (3-sigma quality performance) with your conclusion



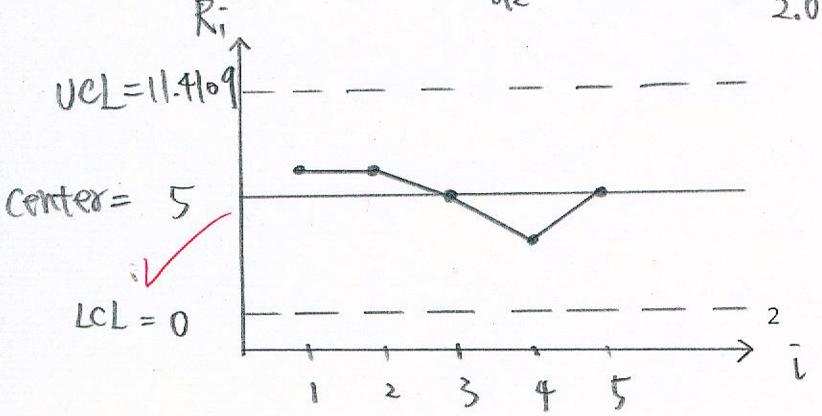
The process is out-of-control since $\bar{x}_2 > UCL=17.7426$ ✓

+2 4) [+2] Draw R-chart (3-sigma quality performance) with your conclusion

$\bar{R}=\frac{1}{5}\sum_{i=1}^{5}R_i=5$ ✓

$\boxed{n=4}$ $UCL=\bar{R}+3\dfrac{d_3}{d_2}\bar{R}=5+3\dfrac{0.88}{2.059}\cdot 5=11.4109$ ✓

$LCL=\bar{R}-3\dfrac{d_3}{d_2}\bar{R}=5-3\dfrac{0.88}{2.059}\cdot 5=-1.4109 \Rightarrow LCL=0$ ✓ since $-1.4109<0$



The process is in-control since $LCL < R_i < UCL$, ✓ for $i=1,2,\cdots,5$

+8 **Q3 [+8]** Consider a p-chart with the target value p_0 and 3-sigma limits. Binomial probabilities can be found below.

```
> pbinom(0:5,size=50,prob=0.04)      cdf  Bin(50,0.04).
[1] 0.1298858 0.4004812 0.6767140 0.8608692 0.9510285 0.9855896
> pbinom(0:5,size=50,prob=0.01)
[1] 0.6050061 0.9105647 0.9861827 0.9984038 0.9998543 0.9999891
```

+1 **1)** [+1] Derive consumer's risk under $p \neq p_0$ in terms of binomial probabilities.

$$\beta = P(LCL \le \hat{p}_i \le UCL \mid p \ne p_0) = P(np_0 - 3\sqrt{np_0(1-p_0)} \le D_i \le np_0 + 3\sqrt{np_0(1-p_0)} \mid p \ne p_0)$$

$$= \sum_{x=0}^{[np_0+3\sqrt{np_0(1-p_0)}]} \binom{n}{x} p^x (1-p)^{n-x} - \sum_{x=0}^{[np_0-3\sqrt{np_0(1-p_0)}]} \binom{n}{x} p^x (1-p)^{n-x}$$ ✓

+2 **2)** [+2] Calculate consumer's risk and ARL under $p_0 = 0.01$, $p = 0.04$ and $n = 50$.

$$\beta = \sum_{x=0}^{[2.6107]} \binom{50}{x}(0.04)^x(1-0.04)^{50-x} - \sum_{x=0}^{[-1.6107]} \binom{50}{x}(0.04)^x(1-0.04)^{50-x} \doteqdot 0.6767140 \ \#$$ ✓

$$ARL_1 = \frac{1}{1-\beta} = \frac{1}{1-0.676714} = 3.0932 \ \#$$ ✓

+2 **3)** [+2] Calculate producer's risk and ARL under $p_0 = 0.01$ and $n = 50$.

$$\alpha = P(\hat{p}_i < LCL \text{ or } \hat{p}_i > UCL \mid p = p_0) = P(D_i < np_0 - 3\sqrt{np_0(1-p_0)} \text{ or } D_i > np_0 + 3\sqrt{np_0(1-p_0)} \mid p = p_0)$$

$$= \sum_{x=0}^{[np_0-3\sqrt{np_0(1-p_0)}]} \binom{50}{x}(0.01)^x(0.99)^{50-x} + \left[1 - \sum_{x=0}^{[np_0+3\sqrt{np_0(1-p_0)}]} \binom{50}{x}(0.01)^x(0.99)^{50-x}\right]$$

$$= 1 - \sum_{x=0}^{[2.6107]} \binom{50}{x}(0.01)^x(0.99)^{50-x} = 1 - 0.9861827 = 0.0138 \ \#$$

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0138} = 72.4638 \ \#$$ ✓

+3 **4)** [+3] An engineer conducts a waterproof testing for 5 electric boards. The number of defective circuits on the board is recorded as follows:

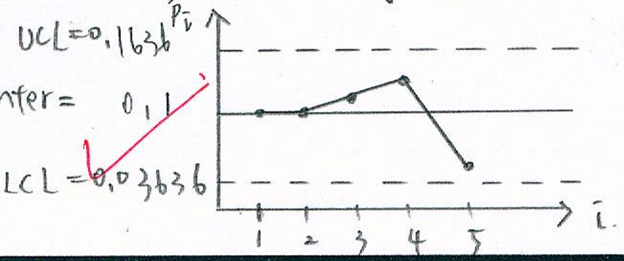
Board ID	1	2	3	4	5
The number of defectives circuits	20	20	23	25	12
The number of circuits	200	200	200	200	200
Defect rates	0.1	0.1	0.115	0.125	0.06

Draw a p-control chart.

$$\bar{p} = \frac{1}{5\times 200}\sum_{i=1}^{5} D_i = (20+20+23+25+12)/1000 = 0.1$$ ✓

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1 + 3\sqrt{\frac{0.1\times 0.9}{200}} = 0.1636$$ ✓

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1 - 3\sqrt{\frac{0.1\times 0.9}{200}} = 0.03636$$ ✓



The process is in-control since $LCL \le \hat{p}_i \le UCL$, $i = 1, 2, \ldots, 5$ #

Tables

Table: the normal distribution $p = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$

z	0	0.5	1	1.5	1.96	2	2.04	2.5	3	3.5	4
P	0.5	0.6915	0.8413	0.9332	0.975	0.9772	0.9793	0.9938	0.99865	0.99977	0.99997

Table: Conversion of range to standard deviation

n	2	3	4	5	6
d_2	1.128	1.683	2.059	2.326	2.534
d_3	0.853	0.888	0.880	0.864	0.848