

**Homework #3**

**#7.9**

$$\sum_{i=1}^{20} D_i = 117$$

$$\bar{p} = \frac{\sum_{i=1}^{20} D_i}{(100)(20)} = \frac{117}{2000} = 0.0585$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 + 3\sqrt{\frac{0.0585(0.9415)}{100}} = 0.0585 + 3(0.0235) = 0.1289$$

$$\text{Since } \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 - 3\sqrt{\frac{0.0585(0.9415)}{100}} = 0.0585 - 3(0.0235) = -0.0119,$$

$LCL = 0$ . Moreover, the following fraction nonconforming control chart shows that the Sample 12 is out of control, we need to exclude the Sample 12 and re-determine the center line and control limits.

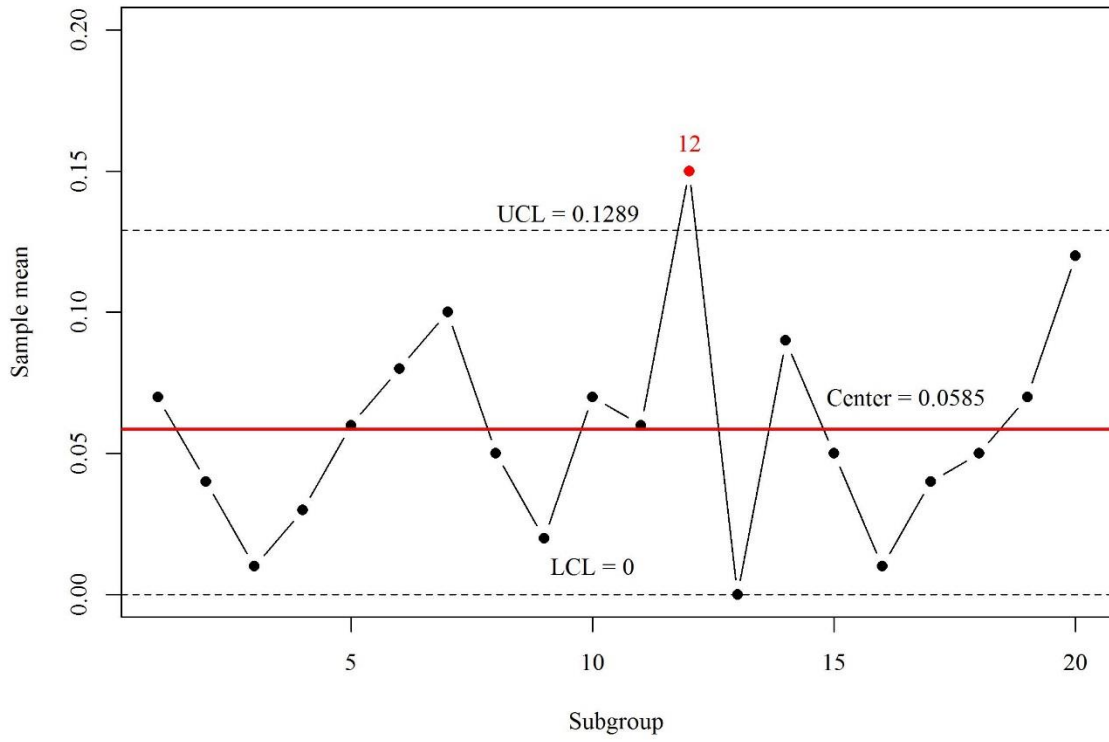
$$\bar{p} = \frac{\sum_{i=1}^{20} D_i - D_{12}}{(100)(19)} = \frac{117 - 15}{1900} = 0.05368$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.05368 + 3\sqrt{\frac{0.05368(0.94632)}{100}} = 0.1213$$

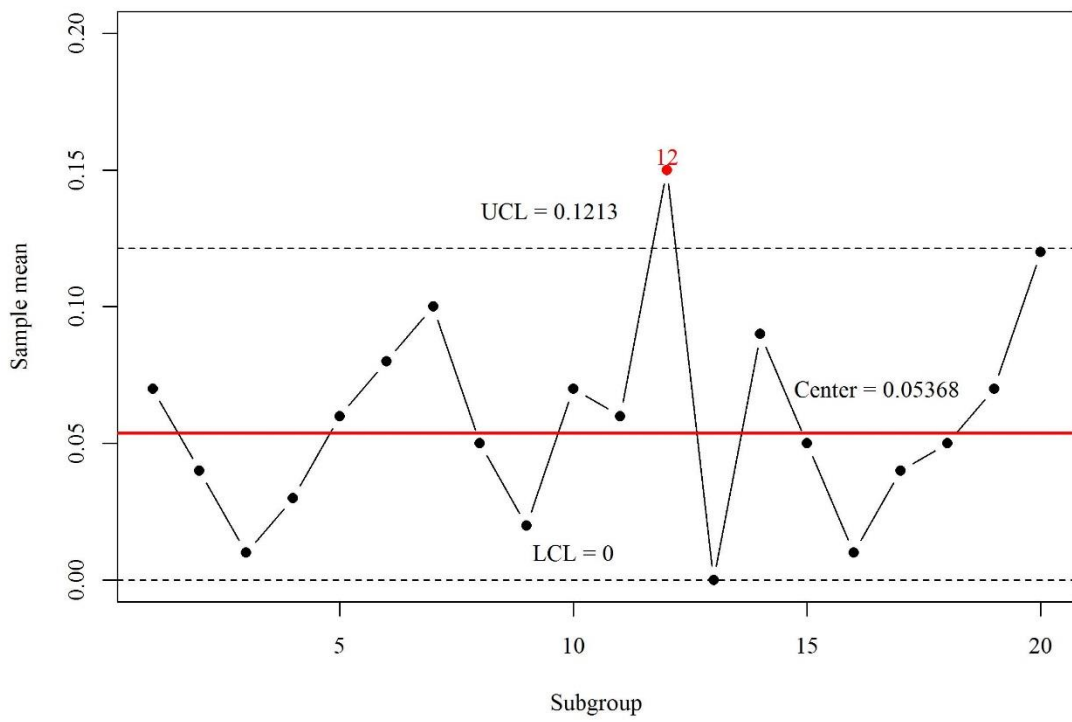
$$\text{Since } \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.05368 - 3\sqrt{\frac{0.05368(0.94632)}{100}} = -0.0139,$$

$LCL = 0$ . After revising the control limits, there is no other sample points out of control. Therefore, the process is in control and the revised control limits for fraction nonconforming control chart are 0.1213 and 0 with the center line 0.05368.

Fraction Nonconforming Control Chart



Fraction Nonconforming Control Chart



**#7.10**

$$\sum_{i=1}^{20} D_i = 69$$

$$\bar{p} = \frac{\sum_{i=1}^{20} D_i}{(150)(20)} = \frac{69}{3000} = 0.023$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.023 + 3\sqrt{\frac{0.023(0.977)}{150}} = 0.023 + 3(0.01224) = 0.0597$$

$$\text{Since } \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.023 - 3\sqrt{\frac{0.023(0.977)}{150}} = 0.023 - 3(0.01224) = -0.0137,$$

$LCL = 0$ . Moreover, the following fraction nonconforming control chart shows that the Sample 9 and 17 are out of control, we need to exclude the Sample 9 and 17 and then re-determine the center line and control limits.

$$\bar{p} = \frac{\sum_{i=1}^{20} D_i - D_9 - D_{17}}{(150)(18)} = \frac{69 - 10 - 15}{2700} = 0.0163$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0163 + 3\sqrt{\frac{0.0163(0.9837)}{150}} = 0.0473$$

$$\text{Since } \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0163 - 3\sqrt{\frac{0.0163(0.9837)}{150}} = -0.0147,$$

$LCL = 0$ . After revising the control limits, Sample 1 is out of control. Then, we need to exclude the Sample 1, 9, and 17, and re-calculate the center line and control limits.

$$\bar{p} = \frac{\sum_{i=1}^{20} D_i - D_1 - D_9 - D_{17}}{(150)(17)} = \frac{69 - 8 - 10 - 15}{2500} = 0.0141$$

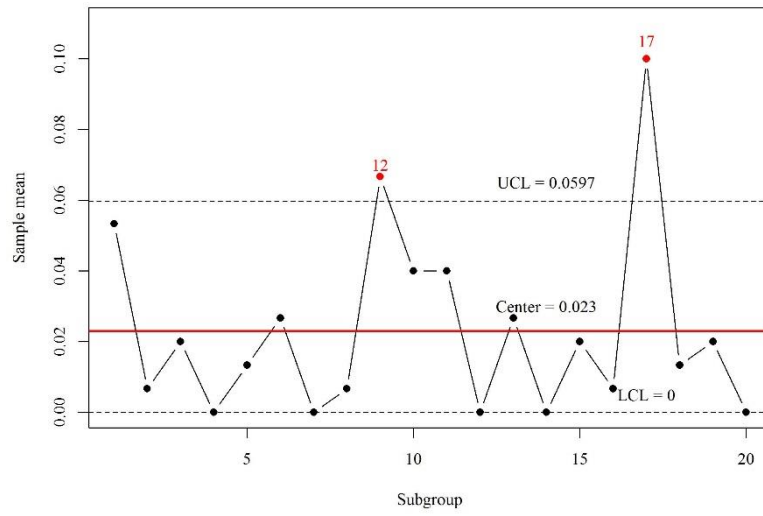
$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0141 + 3\sqrt{\frac{0.0141(0.9859)}{150}} = 0.0430$$

$$\text{Since } \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0141 - 3\sqrt{\frac{0.0141(0.9859)}{150}} = -0.0148,$$

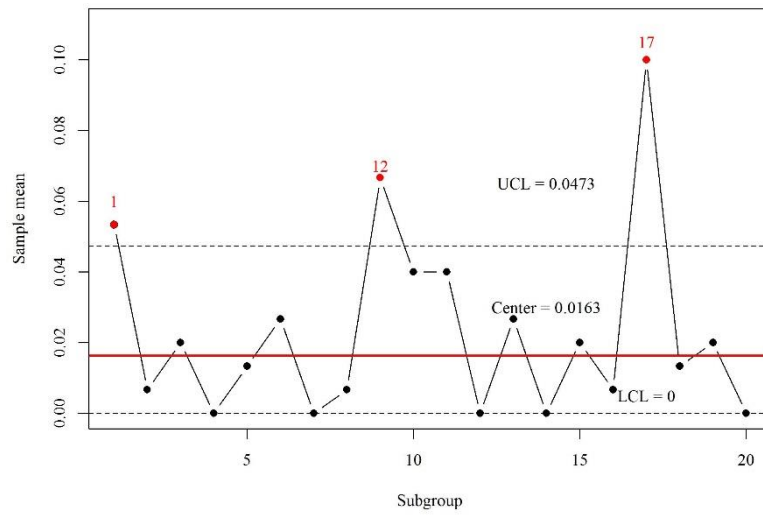
$LCL = 0$ . After revising the control limits, there is no other sample points out of control.

Therefore, the process is in control and the revised control limits for fraction nonconforming control chart are 0.0430 and 0 with the center line 0.0141.

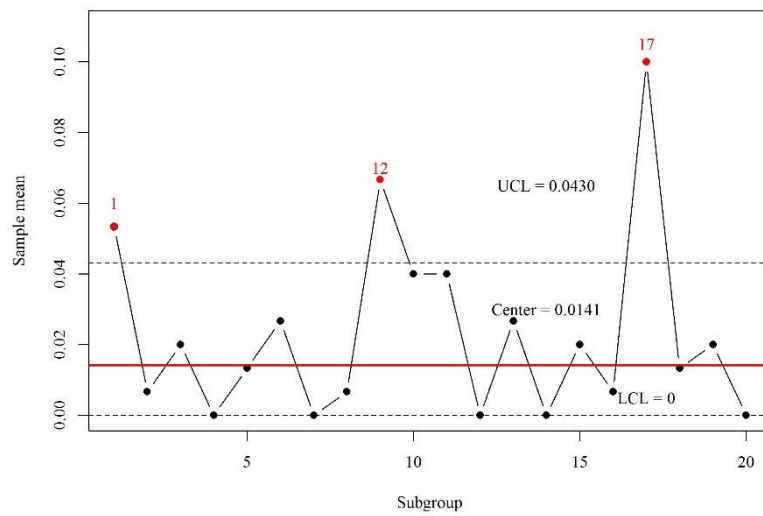
Fraction Nonconforming Control Chart



Fraction Nonconforming Control Chart



Fraction Nonconforming Control Chart



#7.12

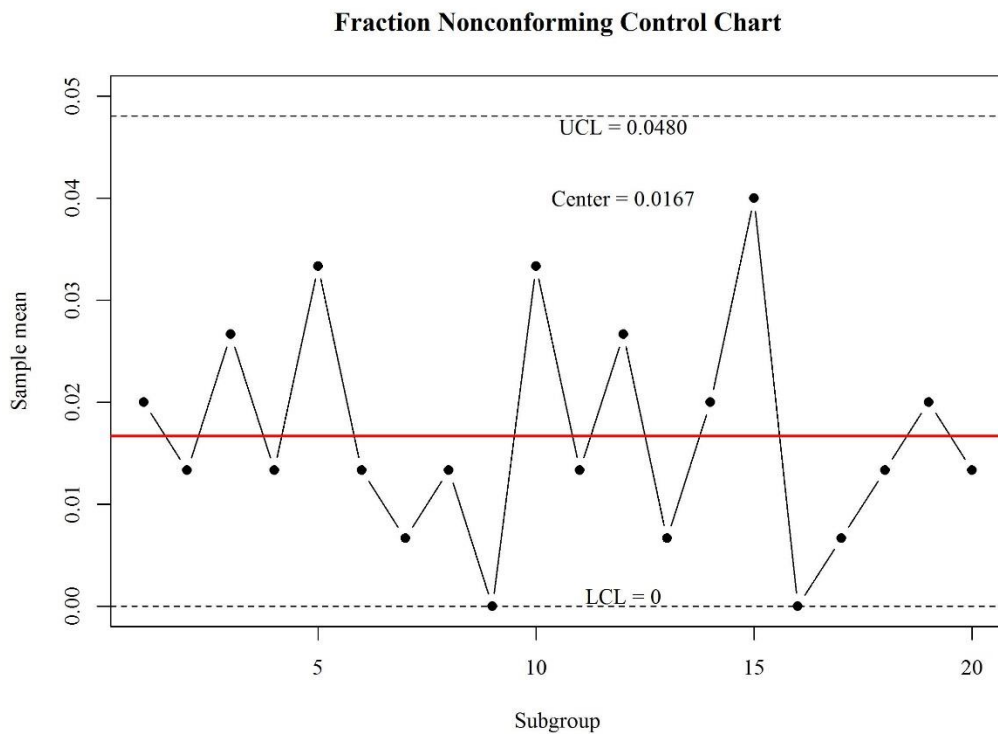
(a)  $\sum_{i=1}^{20} D_i = 50$

$$\bar{p} = \frac{\sum_{i=1}^{20} D_i}{(150)(20)} = \frac{50}{3000} = 0.0167$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0167 + 3\sqrt{\frac{0.0167(0.9833)}{150}} = 0.0480$$

Since  $\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0167 - 3\sqrt{\frac{0.0167(0.9833)}{150}} = -0.0147,$

$LCL = 0$ . There is no sample points out of control. Therefore, the process is in control and the control limits for fraction nonconforming control chart are 0.0480 and 0 with the center line 0.0167.



(b) Since we wish to have

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} > 0$$

This implies that

$$n > \frac{(3^2)(1-\bar{p})}{\bar{p}} = \frac{(3^2)(1-0.0167)}{0.0167} = 529.92.$$

Therefore, the smallest sample size that could be used for this process and still give a

positive lower control limit on the chart is 530.

《CODE》

```
##### 7.9 #####
rm(list=ls(all=TRUE))
data = read.csv("table7e-3.csv")
n = 100
X = rep(NA,20)
for(j in 1:20){
  X[j] = data[j,2]
}
p = X/n
pbar = mean(p)
UCL = pbar + 3*sqrt((pbar*(1-pbar))/n)
LCL = max(0,pbar - 3*sqrt((pbar*(1-pbar))/n))
par(family="serif")
plot(p,type="b",pch=16,xlab="Subgroup",ylab="Sample
mean",ylim=c(0,0.2),col=ifelse(p==p[12], "red", "black"))
title(main="Fraction Nonconforming Control Chart")
abline(a = pbar,b = 0,lty=1,col="red",lwd=2)
abline(a = UCL,b = 0,lty=2)
abline(a = LCL,b = 0,lty=2)
text(16.5,0.07,"Center = 0.0585")
text(9.5,0.135,"UCL = 0.1289")
text(10,0.01,"LCL = 0")
text(12,0.155,"12",col="red")
##### Exclude 12 #####
rm(list=ls(all=TRUE))
data = read.csv("table7e-3.csv")
n = 100
X = rep(NA,20)
for(j in 1:20){
  X[j] = data[j,2]
}
p = X/n
pbar = mean(p[-12])
UCL = pbar + 3*sqrt((pbar*(1-pbar))/n)
LCL = max(0,pbar - 3*sqrt((pbar*(1-pbar))/n))
par(family="serif")
plot(p,type="b",pch=16,xlab="Subgroup",ylab="Sample
mean",ylim=c(0,0.2),col=ifelse(p==p[12], "red", "black"))
title(main="Fraction Nonconforming Control Chart")
abline(a = pbar,b = 0,lty=1,col="red",lwd=2)
abline(a = UCL,b = 0,lty=2)
abline(a = LCL,b = 0,lty=2)
text(16.5,0.07,"Center = 0.05368")
text(9.5,0.135,"UCL = 0.1213")
text(10,0.01,"LCL = 0")
text(12,0.155,"12",col="red")
##### 7.10 #####
rm(list=ls(all=TRUE))
```

```

data = read.csv("table7e-4.csv")
n = 150
X = rep(NA,20)
for(j in 1:20){
  X[j] = data[j,2]
}
p = X/n
pbar = mean(p)
UCL = pbar + 3*sqrt((pbar*(1-pbar))/n)
LCL = max(0,pbar - 3*sqrt((pbar*(1-pbar))/n))
par(family="serif")
cols = rep("black",20)
cols[c(9,17)] = "red"
plot(p,type="b",pch=16,xlab="Subgroup",ylab="Sample mean",ylim=c(0,0.11),col=cols)
title(main="Fraction Nonconforming Control Chart")
abline(a = pbar,b = 0,lty=1,col="red",lwd=2)
abline(a = UCL,b = 0,lty=2)
abline(a = LCL,b = 0,lty=2)
text(14,0.03,"Center = 0.023")
text(14,0.065,"UCL = 0.0597")
text(17,0.005,"LCL = 0")
text(9,0.07,"12",col="red")
text(17,0.105,"17",col="red")
##### Exclude 9&16 #####
rm(list=ls(all=TRUE))
data = read.csv("table7e-4.csv")
n = 150
X = rep(NA,20)
for(j in 1:20){
  X[j] = data[j,2]
}
p = X/n
pp = p[-9]
pp = pp[-16]
pbar = mean(pp)
UCL = pbar + 3*sqrt((pbar*(1-pbar))/n)
LCL = max(0,pbar - 3*sqrt((pbar*(1-pbar))/n))
par(family="serif")
cols = rep("black",20)
cols[c(9,17)] = "red"
ch = c(1,rep(16,19))
plot(p,type="b",xlab="Subgroup",ylab="Sample mean",ylim=c(0,0.11),col=cols,pch=ch)
points(1,p[1],col="red",pch=16)
title(main="Fraction Nonconforming Control Chart")
abline(a = pbar,b = 0,lty=1,col="red",lwd=2)
abline(a = UCL,b = 0,lty=2)
abline(a = LCL,b = 0,lty=2)
text(14,0.03,"Center = 0.0163")
text(14,0.065,"UCL = 0.0473")
text(17,0.005,"LCL = 0")
text(1,0.06,"1",col="red")
text(9,0.07,"12",col="red")

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```

text(17,0.105,"17",col="red")
##### Exclude 1&9&16 #####
rm(list=ls(all=TRUE))
data = read.csv("table7e-4.csv")
n = 150
X = rep(NA,20)
for(j in 1:20){
  X[j] = data[j,2]
}
p = X/n
pp = p[-9]
pp = pp[-16]
pp = pp[-1]
pbar = mean(pp)
UCL = pbar + 3*sqrt((pbar*(1-pbar))/n)
LCL = max(0,pbar - 3*sqrt((pbar*(1-pbar))/n))
par(family="serif")
cols = rep("black",20)
cols[c(9,17)] = "red"
ch = c(1,rep(16,19))
plot(p,type="b",xlab="Subgroup",ylab="Sample mean",ylim=c(0,0.11),col=cols,pch=ch)
points(1,p[1],col="red",pch=16)
title(main="Fraction Nonconforming Control Chart")
abline(a = pbar,b = 0,lty=1,col="red",lwd=2)
abline(a = UCL,b = 0,lty=2)
abline(a = LCL,b = 0,lty=2)
text(14,0.03,"Center = 0.0141")
text(14,0.065,"UCL = 0.0430")
text(17,0.005,"LCL = 0")
text(1,0.06,"1",col="red")
text(9,0.07,"12",col="red")
text(17,0.105,"17",col="red")
##### 7.12 #####
##### (a) #####
rm(list=ls(all=TRUE))
data = read.csv("table7e-6.csv")
n = 150
X = rep(NA,20)
for(j in 1:20){
  X[j] = data[j,2]
}
p = X/n
pbar = mean(p)
UCL = pbar + 3*sqrt((pbar*(1-pbar))/n)
LCL = max(0,pbar - 3*sqrt((pbar*(1-pbar))/n))
par(family="serif")
plot(p,type="b",pch=16,xlab="Subgroup",ylab="Sample mean",ylim=c(0,0.05))
title(main="Fraction Nonconforming Control Chart")
abline(a = pbar,b = 0,lty=1,col="red",lwd=2)
abline(a = UCL,b = 0,lty=2)
abline(a = LCL,b = 0,lty=2)

```



```
text(12,0.04,"Center = 0.0167")
text(12,0.047,"UCL = 0.0480")
text(12,0.001,"LCL = 0")
##### (b) #####
(9*(1-0.0167))/(0.0167)
```