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# Quality Control HW#2

**(6.34) A TiW layer is deposited on a substrate using a sputtering tool. Table 6E.14 contains layer thickness measurements (in angstroms) on 20 subgroups of four substrates.**

We know:

$$\sum_{i=1}^{20} \bar{X}_i = 8973.75 \ , \ \sum_{i=1}^{20} R_i = 333 \ , \ \mathrm{A}_2 = 0.729 \ , \ n = 4 \ , \ m = 30$$

$$D_3 = 0 \ , \ D_4 = 2.282 \ , \ d_2 = 2.059$$

**(a). Set up $\bar{x}$ and $R$ control charts on this process. Is the process in control? Revise the control limits as necessary.**

$\bar{x}$-chart:

$$center = \bar{\bar{x}} = \frac{13456}{30} = 448.6875$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 448.6875 + 0.729 \cdot 16.65 = 460.8254$$

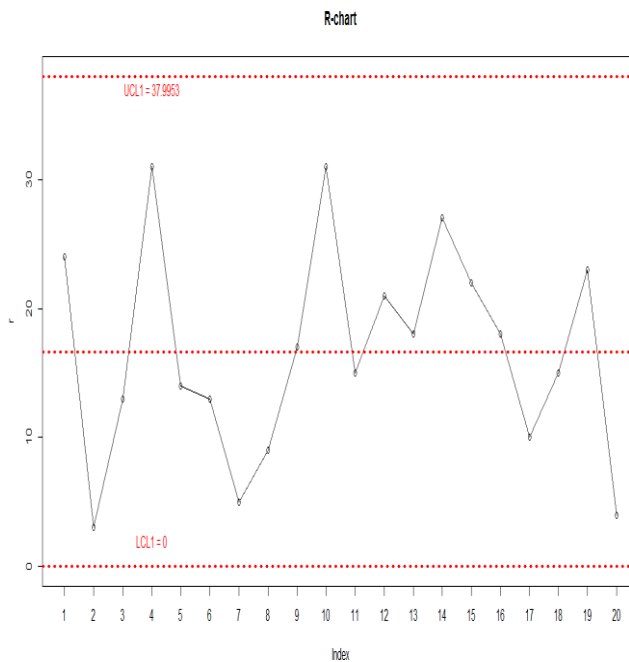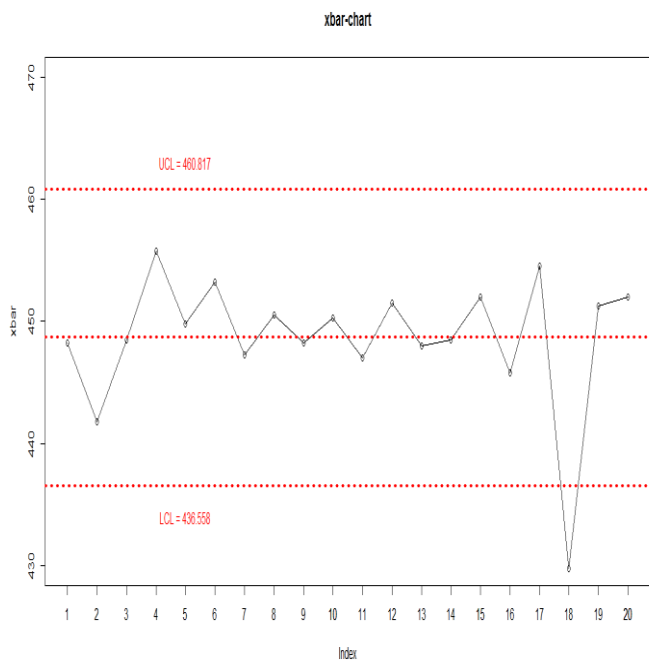$$LCL = \bar{\bar{x}} - A_2\bar{R} = 448.6875 - 0.729 \cdot 16.65 = 436.5496$$

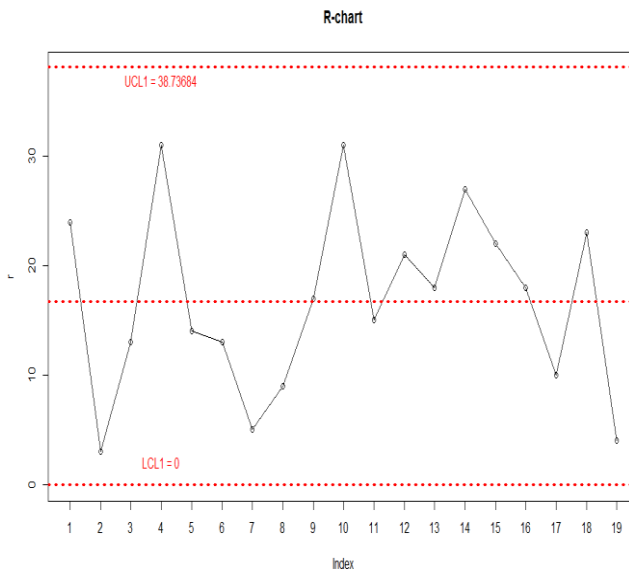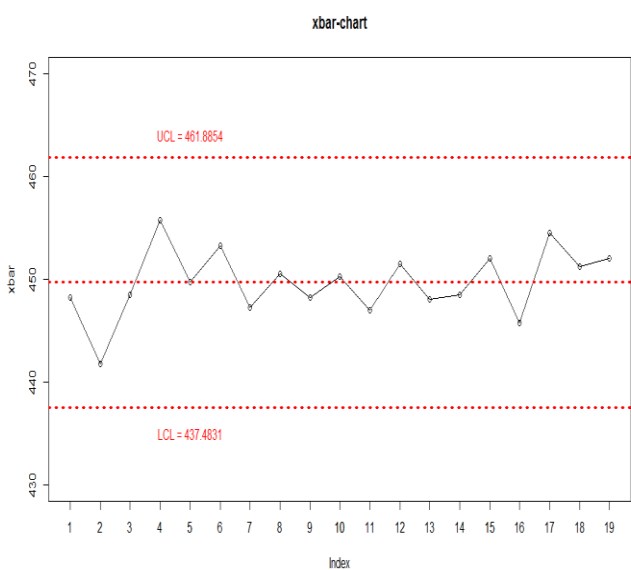$R$-chart:

$$center = \bar{R} = \frac{333}{20} = 16.65$$

$$UCL = D_4\bar{R} = 2.282 \cdot 16.65 = 37.9953$$

$$LCL = D_3\bar{R} = 0$$

The process is not in control, we can find there exist some values larger than UCL or lower than LCL, such likes the $\bar{x}$-chart and $R$-chart below:



Then we need to revised. Take the out-control-point, number 18, then reproduce again:



We can find that all the points are in between the UCL and LCL, the process is in-control.

**(b). Estimate the mean and standard deviation of the process.**

By the plot and the value in front, we know the mean is $\bar{\bar{x}} = 448.6875$.
And the standard deviation is:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{16.73684}{2.059} = 8.1286$$

**(c). Is the layer thickness normally distributed?**

By R, we can conclude that the layer thickness is normally distributed, because the p-value is smaller than 0.05, and , the qq-plot looks ok.



Normal Q-Q Plot
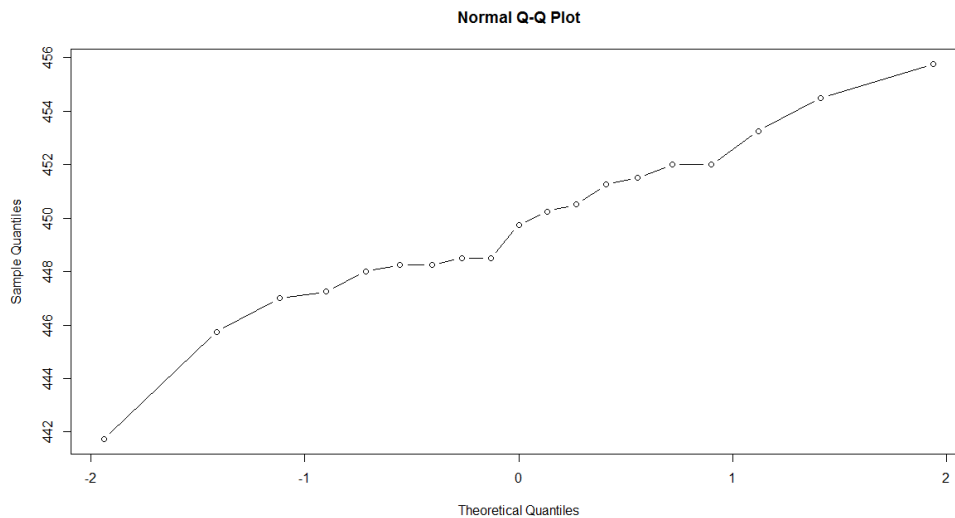
```
> shapiro.test(xbar)

      Shapiro-Wilk normality test

data:  xbar
W = 0.78944, p-value = 0.0006037
```

**(d). If the specifications are at $450 \pm 30$, estimate the process capability.**

The $C_p = \dfrac{UCL - LCL}{6\sigma} = \dfrac{510 - 450}{6 \cdot 8.1286} = 1.236637$ is:

```
[1] 1.236637
```

This means that the process uses less tolerance band, so we can conclude that the few nonconforming will be produced by this process.

```
                                    Code
> # (6.34)
> rm(list = ls())
> x1 =
c(459,443,457,469,443,444,445,446,444,432,445,456,459,441,460,453,451,42
2,444,450)
> x2 =
c(449,440,444,463,457,456,449,455,452,463,452,457,445,465,453,444,460,43
1,446,450)
> x3 =
c(435,442,449,453,445,456,450,449,457,463,453,436,441,438,457,451,450,43
7,448,454)
> x4 =
c(450,442,444,438,454,457,445,452,440,443,438,457,447,450,438,435,457,42
9,467,454)
> data = rbind(x1, x2, x3, x4)
> # find R and x
> r = rep(0,20)
> for(i in 1:20){
+   r[i] = (max(data[,i]) - min(data[,i]))
+ }
> r
 [1] 24  3 13 31 14 13  5  9 17 31 15 21 18 27 22 18 10 15 23  4
> sum(r)
[1] 333
> rbar = mean(r)
> D4 = 2.282 ; D3 = 0
> rbar
[1] 16.65
> xbar = rep(0, 20)
> for(i in 1:20){
+   xbar[i] = mean(data[,i])
+ }
> xbar
 [1] 448.25 441.75 448.50 455.75 449.75 453.25 447.25 450.50 448.25 450.25
[11] 447.00 451.50 448.00 448.50 452.00 445.75 454.50 429.75 451.25 452.00
> xbarsum = sum(xbar)
> A2 = 0.729
> xbarsum
[1] 8973.75
> # xbar-chart
> center = xbarsum / 20
```

```
> UCL = center + A2 * rbar
> LCL = center - A2 * rbar
> center ; UCL ; LCL
[1] 448.6875
[1] 460.8254
[1] 436.5496
> plot(xbar, axat = "n", ylim = c(430, 470), main = "xbar-chart")
> axis(1, at = seq(0, 20, 1), labels = seq(0, 20, 1))
> lines(xbar)
> abline(a = center, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = UCL, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = LCL, b = 0, col = "red", lwd = 3, lty = 3)
> text(5, 463, "UCL = 460.817", col = "red")
> text(5, 434, "LCL = 436.558", col = "red")
> #  R-chart
> center1 = sum(r) / 20
> UCL1 = D4 * center1
> LCL1 = D3 * center1
> center1 ; UCL1 ; LCL1
[1] 16.65
[1] 37.9953
[1] 0
> plot(r, xaxt = "n", ylim = c(0, 38), main = "R-chart")
> axis(1, at = seq(0, 20, 1), labels = seq(0, 20, 1))
> abline(a = center1, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = UCL1, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = LCL1, b = 0, col = "red", lwd = 3, lty = 3)
> text(4, 37, "UCL1 = 37.9953", col = "red")
> text(4, 2, "LCL1 = 0", col = "red")
> > # revise
> rm(list = ls())
> x1 =
c(459,443,457,469,443,444,445,446,444,432,445,456,459,441,460,453,451,44
4,450)
> x2 =
c(449,440,444,463,457,456,449,455,452,463,452,457,445,465,453,444,460,44
6,450)
> x3 =
c(435,442,449,453,445,456,450,449,457,463,453,436,441,438,457,451,450,44
8,454)
> x4 =
c(450,442,444,438,454,457,445,452,440,443,438,457,447,450,438,435,457,46
```

```
7,454)
> data = rbind(x1, x2, x3, x4)
> # find R and X
> r = rep(0,19)
> for(i in 1:19){
+   r[i] = (max(data[,i]) - min(data[,i]))
+ }
> r
 [1] 24  3 13 31 14 13  5  9 17 31 15 21 18 27 22 18 10 23  4
> sum(r)
[1] 318
> rbar = mean(r)
> D4 = 2.282 ; D3 = 0
> rbar
[1] 16.73684
> xbar = rep(0, 19)
> for(i in 1:19){
+   xbar[i] = mean(data[,i])
+ }
> xbar
 [1] 448.25 441.75 448.50 455.75 449.75 453.25 447.25 450.50
 [9] 448.25 450.25 447.00 451.50 448.00 448.50 452.00 445.75
[17] 454.50 451.25 452.00
> xbarsum = sum(xbar)
> A2 = 0.729
> xbarsum
[1] 8544
> # xbar-chart
> center = xbarsum / 19
> UCL = center + A2 * rbar
> LCL = center - A2 * rbar
> center ; UCL ; LCL
[1] 449.6842
[1] 461.8854
[1] 437.4831
> plot(xbar, axat = "n", ylim = c(430, 470), main = "xbar-chart")
> axis(1, at = seq(0, 19, 1), labels = seq(0, 19, 1))
> lines(xbar)
> abline(a = center, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = UCL, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = LCL, b = 0, col = "red", lwd = 3, lty = 3)
> text(5, 464, "UCL = 461.8854", col = "red")
```

```
> text(5, 435, "LCL = 437.4831", col = "red")
> #  R-chart
> center1 = sum(r) / 19
> UCL1 = D4 * center1
> LCL1 = D3 * center1
> center1 ; UCL1 ; LCL1
[1] 16.73684
[1] 38.19347
[1] 0
> plot(r, xaxt = "n", ylim = c(0, 38), main = "R-chart")
> axis(1, at = seq(0, 19, 1), labels = seq(0, 19, 1))
> lines(r)
> abline(a = center1, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = UCL1, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = LCL1, b = 0, col = "red", lwd = 3, lty = 3)
> text(4, 37, "UCL1 = 38.73684", col = "red")
> text(4, 2, "LCL1 = 0", col = "red")
> # (c)
> table = as.data.frame(data)
> xbar = rep(0, 20)
> for(i in 1:20){
+   xbar[i] = mean(data[,i])
+ }
> xbar
 [1] 448.25 441.75 448.50 455.75 449.75 453.25 447.25 450.50 448.25 450.25
[11] 447.00 451.50 448.00 448.50 452.00 445.75 454.50 429.75 451.25 452.00
> qqnorm(sort(xbar), type = "b")
> shapiro.test(xbar)


    Shapiro-Wilk normality test


data:  xbar
W = 0.78944, p-value = 0.0006037> # (d)
> USL = 480 + 30
> LSL = 480 - 30
> Cp = (USL - LSL) / (6 * 8.08645)
> Cp
[1] 1.236637
```

**(6.35) Continuation of Exercise 6.34. Table 6E.15 contains 10 news subgroups of thickness data. Plot this data on the control charts constructed in Exercise 6.34(a). Is the process in statistical control?**

No matter the center, UCL and LCL are all the same as the first revised 19 data:

| $\bar{x}$ -chart: |
| :--- |
| $$center = \bar{\bar{x}} = 449.68421$$ $$UCL = 461.8854$$ $$LCL = 437.4831$$ |
| $R$ -chart: |
| $$center = \bar{R} = 16.73684$$ $$UCL = 38.19347$$ $$LCL = 0$$ |

We want to draw this data on the original plot, and the center, UCL, LCL are the same as original, the plots of control charts are below:



new10data-data-xbar-chart



new10data-R-chart
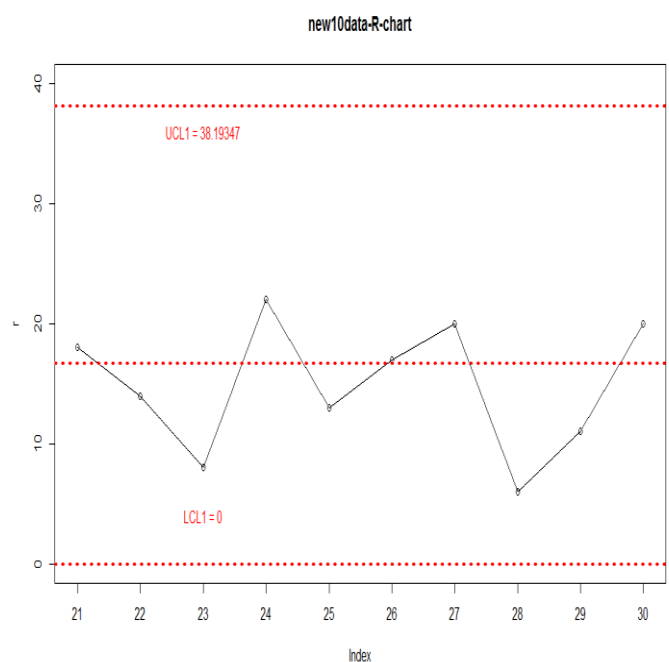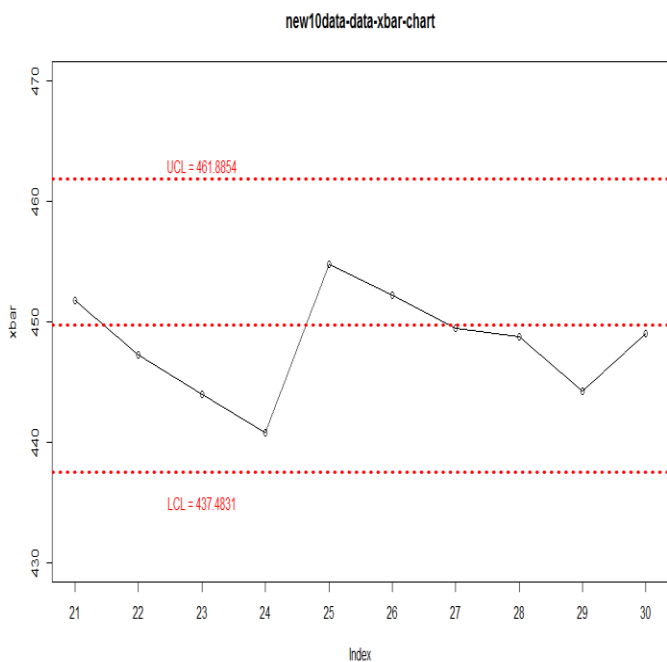
By the control charts of the new 10 thickness data, there are no points exceed the UCL and LCL, so we can know the process is in statistical control.

```
                              Code
> # (6.35)
> rm(list = ls())
> x1 = c(454,449,442,443,446,454,458,450,443,457)
> x2 = c(449,441,442,452,459,448,449,449,440,450)
> x3 = c(443,444,442,438,457,445,453,445,443,452)
> x4 = c(461,455,450,430,457,462,438,451,451,437)
> data = rbind(x1, x2, x3, x4)
> # find X and R
> xbar = rep(0, 10)
> for(i in 1:10){
+   xbar[i] = mean(data[,i])
+ }
> xbar
 [1] 451.75 447.25 444.00 440.75 454.75 452.25 449.50 448.75 444.25 449.00
> A2 = 0.729
> xbarsum = sum(xbar)
> xbarsum
[1] 4482.25
> r = rep(0,10)
> for(i in 1:10){
+   r[i] = (max(data[,i]) - min(data[,i]))
+ }
> r
 [1] 18 14  8 22 13 17 20  6 11 20
> D4 = 2.282 ; D3 = 0
> sum(r)
[1] 149
> # xbar-chart
> center = 449.68421
> UCL = 461.8854
> LCL = 437.4831
> plot(xbar, xaxt = "n", ylim = c(430, 470), main =
"new10data-data-xbar-chart")
> axis(1, at = seq(1, 10, 1), labels = seq(21, 30, 1))
> lines(xbar)
> abline(a = center, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = UCL, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = LCL, b = 0, col = "red", lwd = 3, lty = 3)
text(3, 463, "UCL = 461.8854", col = "red")
text(3, 435, "LCL = 437.4831", col = "red")
> #  R-chart
```

```
> center1 = 16.73684
> UCL1 = 38.19347
> LCL1 = 0
> plot(r, xaxt = "n", ylim = c(2, 32), main = "new10data-R-chart")
> axis(1, at = seq(1, 10, 1), labels = seq(21, 30, 1))
> lines(r)
> abline(a = center1, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = UCL1, b = 0, col = "red", lwd = 3, lty = 3)
> abline(a = LCL1, b = 0, col = "red", lwd = 3, lty = 3)
> text(3, 36, "UCL1 = 38.19347", col = "red")
> text(3, 4, "LCL1 = 0", col = "red")
```

**(6.36) Continuation of Exercise 6.34. Suppose that following the construction of the and control charts in Exercise 6.34, the process engineers decided to change the subgroup size to n=2. Table 6E.16 contains 10 new subgroups pf thickness data. Plot the data on the control charts from Exercise 6.34(a) based on the new subgroup size. Is the process in statistic control?**

We know the revised data:

$$\sum_{i=1}^{20} \bar{X}_i = 8570.5 \ , \ \sum_{i=1}^{20} R_i = 183 \ , \ A_2 = 1.88 \ , \ n = 2$$
$$D_3 = 0 \ , \ D_4 = 3.267$$

---

$\bar{x}$-chart:

$$center = \frac{\sum_{i=1}^{20} \bar{X}_i}{19} = 451.0789$$
$$UCL = 449.85 + 1.88 \cdot 9.631579 = 469.1863$$
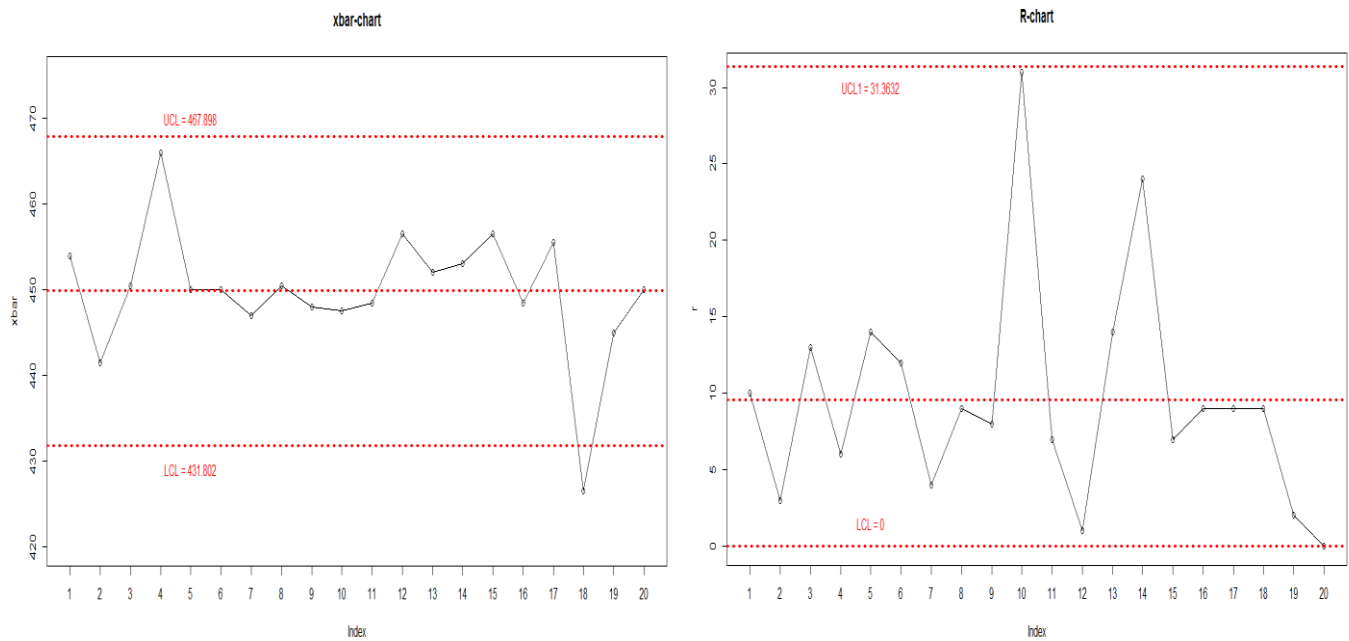$$LCL = 449.85 - 1.88 \cdot 9.631579 = 432.9716$$

---

$R$-chart:

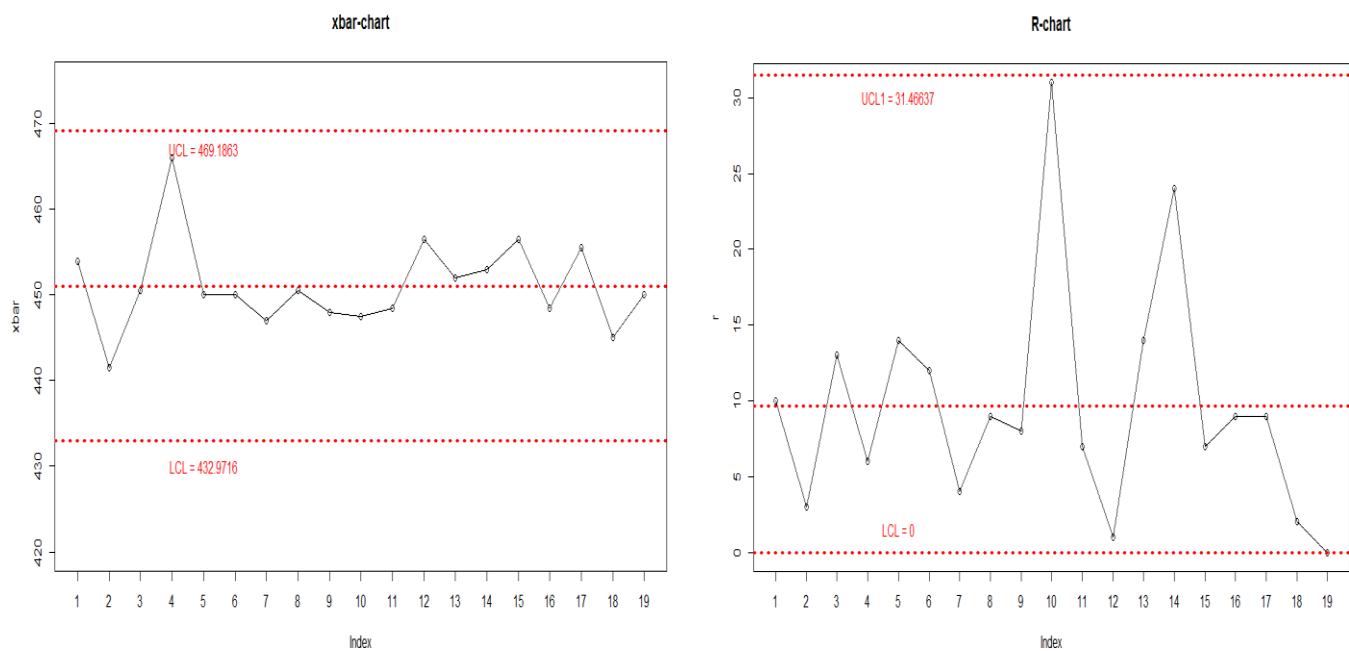$$center = \frac{\sum_{i=1}^{20} R_i}{19} = \frac{183}{19} = 9.631579$$
$$UCL = 3.267 \cdot 9.631579 = 31.46637$$
$$LCL = 0$$

By the control chart, we can find the point on $x=18$ is exceed the LCL, so we know the process is out of control.
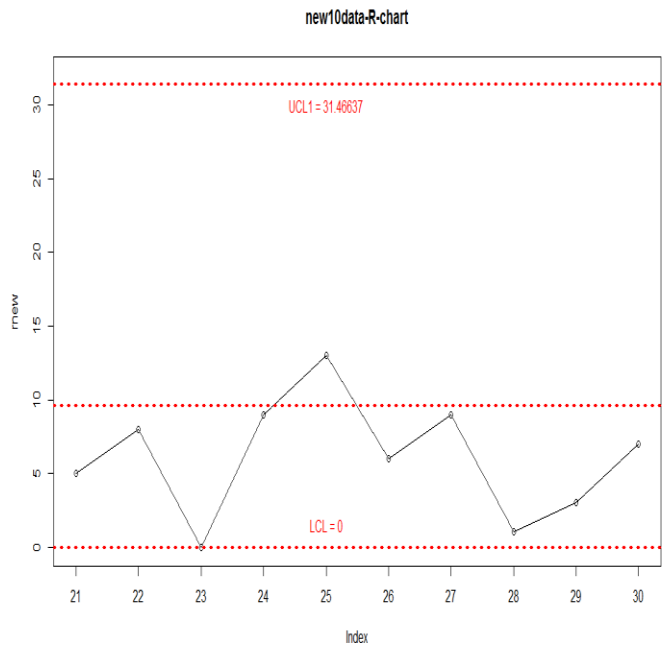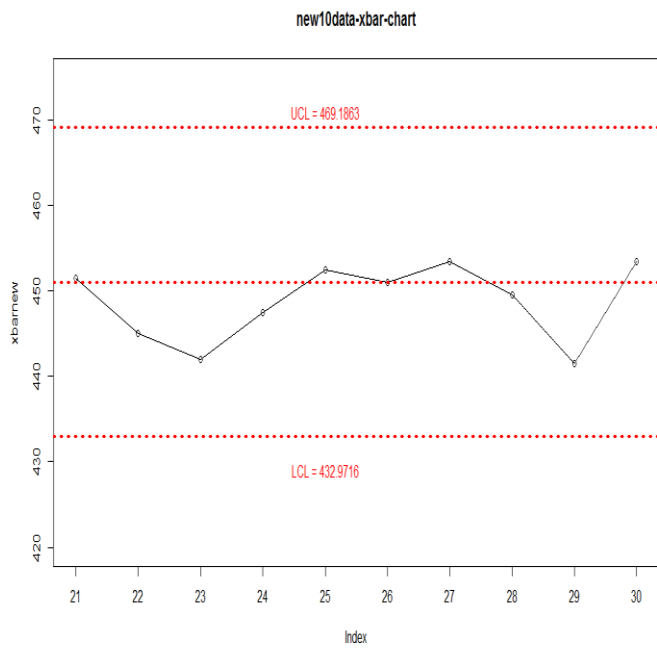


We need to revised. Take out the out-control-point, number 18, then reproduce again:



We can find that all the points are in between the UCL and LCL, the process is in-control.

Then, by the topic, add the new 10 thickness data, we can plot:



There is no point exceed the UCL and LCL, the process is in statistical control.

**Code**

```
> # (6.36)
> rm(list = ls())
> x1 =
c(459,443,457,469,443,444,445,446,444,432,445,456,459,441,460,453,451,42
2,444,450)
> x2 =
c(449,440,444,463,457,456,449,455,452,463,452,457,445,465,453,444,460,43
1,446,450)
> #x1 = c(454,449,442,443,446,454,458,450,443,457)
> #x2 = c(449,441,442,452,459,448,449,449,440,450)
> data = rbind(x1, x2)
> # find X and R
> xbar = rep(0, 20)
> for(i in 1:20){
+   xbar[i] = mean(data[,i])
+ }
> xbar
 [1] 454.0 441.5 450.5 466.0 450.0 450.0 447.0 450.5 448.0 447.5 448.5 456.5
452.0
[14] 453.0 456.5 448.5 455.5 426.5 445.0 450.0
> A2 = 1.88
> xbarsum = sum(xbar)
> xbarsum
[1] 8997
> r = rep(0, 20)
> for (i in 1:20){
+   r[i] = max(data[,i]) - min(data[,i])
+ }
> r
 [1] 10  3 13  6 14 12  4  9  8 31  7  1 14 24  7  9  9  9  2  0
> rbar = mean(r)
> D4 = 3.267 ; D3 = 0
> sum(r) ; rbar
[1] 192
[1] 9.6
> # xbar-chart
> center = xbarsum / 20
> UCL = center + A2 * rbar
> LCL = center - A2 * rbar
> center ; UCL ; LCL
```

```
[1] 449.85
[1] 467.898
[1] 431.802
> plot(xbar, ylim = c(420, 475), xaxt = "n", main = "xbar-chart")
> axis(1, at = seq(0, 20, 1), labels = seq(0, 20, 1))
> lines(xbar)
> abline(a = center, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = UCL, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = LCL, b = 0, lty = 3, lwd = 3, col = "red")
> text(5, 470, "UCL = 467.898", col = "red")
> text(5, 429, "LCL = 431.802", col = "red")
> # R-chart
> center1 = sum(r) / 20
> UCL1 = D4 * center1
> LCL1 = D3 * center1
> center1 ; UCL1 ; LCL1
[1] 9.6
[1] 31.3632
[1] 0
> plot(r, xaxt = "n", main = "R-chart")
> axis(1, at = seq(0, 20, 1), labels = seq(0, 20, 1))
> lines(r)
> abline(a = center1, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = UCL1, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = LCL1, b = 0, lty = 3, lwd = 3, col = "red")
> text(5, 30, "UCL1 = 31.3632", col = "red")
> text(5, 1.5, "LCL = 0", col = "red")
> # revise
> rm(list = ls())
> x1 =
c(459,443,457,469,443,444,445,446,444,432,445,456,459,441,460,453,451,44
4,450)
> x2 =
c(449,440,444,463,457,456,449,455,452,463,452,457,445,465,453,444,460,44
6,450)
> data = rbind(x1, x2)
> # find X and R
> xbar = rep(0, 19)
> for(i in 1:19){
+   xbar[i] = mean(data[,i])
+ }
> xbar
```

```
 [1] 454.0 441.5 450.5 466.0 450.0 450.0 447.0 450.5 448.0 447.5
[11] 448.5 456.5 452.0 453.0 456.5 448.5 455.5 445.0 450.0
> A2 = 1.88
> xbarsum = sum(xbar)
> xbarsum
[1] 8570.5
> r = rep(0, 19)
> for (i in 1:19){
+   r[i] = max(data[,i]) - min(data[,i])
+ }
> r
 [1] 10  3 13  6 14 12  4  9  8 31  7  1 14 24  7  9  9  2  0
> rbar = mean(r)
> D4 = 3.267 ; D3 = 0
> sum(r) ; rbar
[1] 183
[1] 9.631579
> # xbar-chart
> center = xbarsum / 19
> UCL = center + A2 * rbar
> LCL = center - A2 * rbar
> center ; UCL ; LCL
[1] 451.0789
[1] 469.1863
[1] 432.9716
> plot(xbar, ylim = c(420, 475), xaxt = "n", main = "xbar-chart")
> axis(1, at = seq(0, 19, 1), labels = seq(0, 19, 1))
> lines(xbar)
> abline(a = center, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = UCL, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = LCL, b = 0, lty = 3, lwd = 3, col = "red")
> text(5, 467, "UCL = 469.1863", col = "red")
> text(5, 430, "LCL = 432.9716", col = "red")
> # R-chart
> center1 = sum(r) / 19
> UCL1 = D4 * center1
> LCL1 = D3 * center1
> center1 ; UCL1 ; LCL1
[1] 9.631579
[1] 31.46637
[1] 0
> plot(r, xaxt = "n", main = "R-chart")
```

```
> axis(1, at = seq(0, 19, 1), labels = seq(0, 19, 1))
> lines(r)
> abline(a = center1, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = UCL1, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = LCL1, b = 0, lty = 3, lwd = 3, col = "red")
> text(5, 30, "UCL1 = 31.46637", col = "red")
> text(5, 1.5, "LCL = 0", col = "red")
> # add new 10 data
> x1 = c(454,449,442,443,446,454,458,450,443,457)
> x2 = c(449,441,442,452,459,448,449,449,440,450)
> data = rbind(x1, x2)
> # find X and R
> xbarnew = rep(0, 10)
> for(i in 1:10){
+   xbarnew[i] = mean(data[,i])
+ }
> xbarnew
 [1] 451.5 445.0 442.0 447.5 452.5 451.0 453.5 449.5 441.5 453.5
> xbarbarnew = mean(xbarnew)
> rnew = rep(0, 10)
> for(i in 1:10){
+   rnew[i] = max(data[,i]) - min(data[,i])
+ }
> rnew
 [1]  5  8  0  9 13  6  9  1  3  7
> rbarnew = mean(r)
> # xbar-chart
> center = 451.0789
> UCL = 469.1863
> LCL = 432.9716
> plot(xbarnew, ylim = c(420, 475), xaxt = "n", main = "new10data-xbar-chart")
> axis(1, at = seq(1, 10, 1), labels = seq(21, 30, 1))
> lines(xbarnew)
> abline(a = center, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = UCL, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = LCL, b = 0, lty = 3, lwd = 3, col = "red")
> text(5, 471, "UCL = 469.1863", col = "red")
> text(5, 429, "LCL = 432.9716", col = "red")
> # R-chart
> center1 = 9.631579
> UCL1 = 31.46637
> LCL1 = 0
```

```
> plot(rnew, xaxt = "n", ylim = c(0, 32), main = "new10data-R-chart")
> axis(1, at = seq(1, 10, 1), labels = seq(21, 30, 1))
> lines(rnew)
> abline(a = center1, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = UCL1, b = 0, lty = 3, lwd = 3, col = "red")
> abline(a = LCL1, b = 0, lty = 3, lwd = 3, col = "red")
> text(5, 30, "UCL1 = 31.46637", col = "red")
> text(5, 1.5, "LCL = 0", col = "red")
```

**(6.40) the data in Table 6E.17 were collected from a process manufacturing power supplies. The variable of interest is output voltage, n=5.**

**(a). Compute center lines and control limits suitable for controlling future production.**

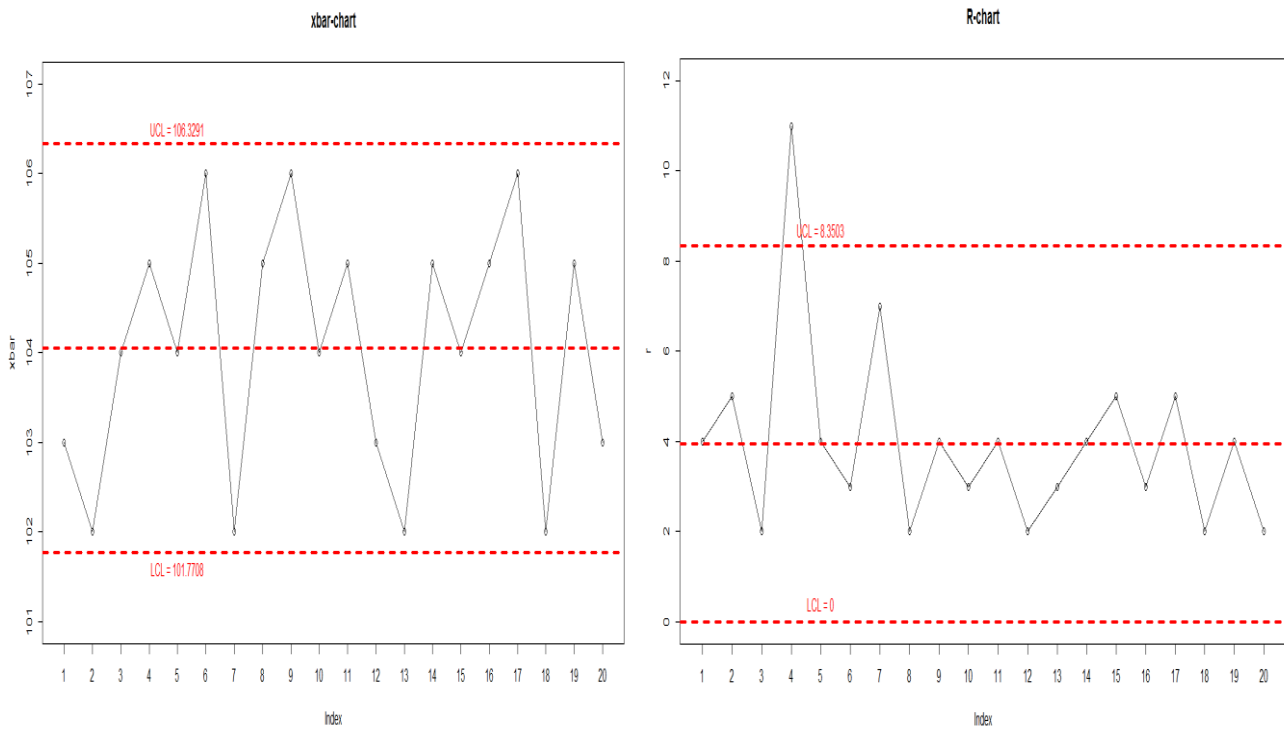By the topic, we have to find center, UCL and LCL:

$$A_2 = 0.577$$

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{20} X_i}{20} = \frac{2081}{20} = 104.05$$

$$\bar{R} = \frac{\sum_{i=1}^{20} R_i}{20} = \frac{79}{20} = 3.95$$

$$UCL = 104.05 + 0.577 \cdot 3.95 = 106.3291$$

$$LCL = 104.05 - 0.577 \cdot 3.95 = 101.7708$$

Then plot this data:



There are no points exceed the UCL and LCL in $\bar{x}$-chart, so it is ok. But in $R$-chart, there is one point exceed UCL.

**(b). Assume that the quality characteristic is normally distributed. Estimate the process standard deviation.**

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{3.95}{2.326} = 1.6982$$

**(c). What are the apparent three-sigma natural tolerance limits of the process?**

$$UNTL = \overline{\overline{x}} + 3\hat{\sigma} = 104.05 + 3 \cdot 1.6982 = 109.1446$$

$$LNTL = \overline{\overline{x}} - 3\hat{\sigma} = 104.05 - 3 \cdot 1.6982 = 98.9554$$

**(d). What would be your estimate of the process fraction nonconforming if the specifications on the characteristic were** $103 \pm 4$ **?**

$$p = P(x < 99) + P(x > 107) = \Phi\left(\frac{99 - 104.05}{1.6982}\right) + 1 - \Phi\left(\frac{107 - 104.05}{1.6982}\right)$$

$$= \Phi(-2.9737) + 1 - \Phi(1.7371) = 1 - \Phi(2.9737) + 1 - \Phi(1.7371)$$

$$= 2 - 0.99851 - 0.95907 = 0.04242$$

**(e). What approaches to reducing the fraction nonconforming can you suggest?**
According to the result of (d), we can suggest that there is about 4.242%
of the manufacturing power supplies will be outside of the specifications.