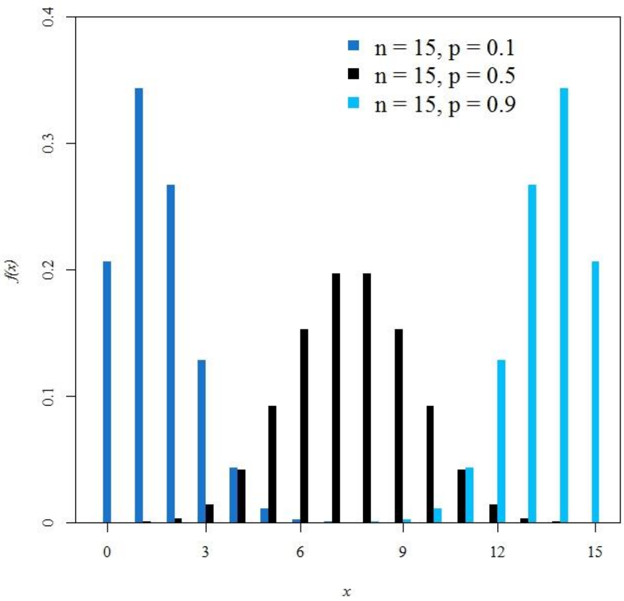
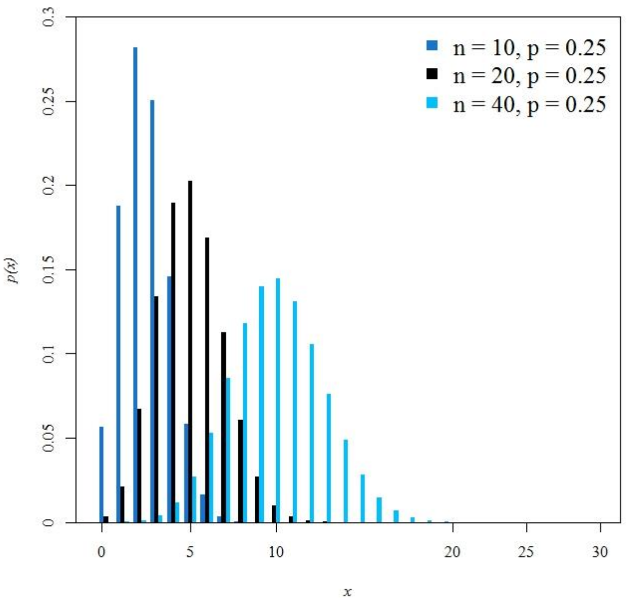
**Homework #1**

**#1**

The mean of the binomial distribution is $np$, the variance is $np(1-p)$, and the skewness is $\frac{1-2p}{\sqrt{np(1-p)}}$, where $n$ is the number of trials and $p$ is the successful probability in each trial. Fix $n$ = 15, the means of the binomial distribution are 1.5, 7.5, and 13.5 for $p$ = 0.1, 0.5, and 0.9, respectively. The variances are 1.35, 3.75, and 1.35. The values of the skewness are 0.689, 0.000, and -0.689. These results indicate that the binomial distribution tends to be left-skewed as $p$ increases when $n$ is fixed. In other words, the binomial distribution is right-skewed if $p$ is smaller than 0.5. When $p$ = 0.5, the binomial distribution is symmetric. On the other hand, fix $p$ = 0.25, the means of the binomial distribution are 2.5, 5, and 10 for $n$ = 10, 20, and 40, respectively. The variances are 1.875, 3.750, and 7.500. The values of the skewness are 0.365, 0.258, and 0.183. Theses results indicate that the binomial distribution becomes more symmetric when $n$ increases. The value of skewness decreases to 0 as the $n$ increases when $p$ is fixed. In conclusion, the binomial distribution is symmetric when $p$ is 0.5, and is right-skewed when $p$ is smaller than 0.5. The following figures shows these properties.




《**CODE**》

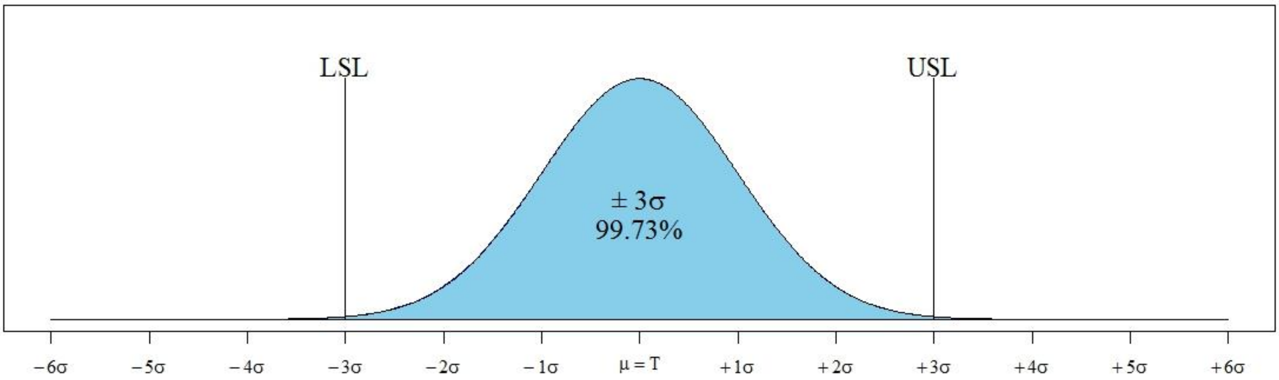
```
rm(list=ls(all=TRUE))
layout(matrix(1:2,1,2))
######## (a) ########
na = 15
P = c(0.1,0.5,0.9)
p1 = c()
for(i in 1:16){
```

```
    z =  i - 1
    p1[i] = dbinom(z, size=na, prob=P[1])
}
p2 = c()
for(i in 1:16){
    z =  i - 1
    p2[i] = dbinom(z, size=na, prob=P[2])
}
p3 = c()
for(i in 1:16){
    z =  i - 1
    p3[i] = dbinom(z, size=na, prob=P[3])
}
X = data.frame(p1,p2,p3)
par(family="serif")
barplot(t(as.matrix(X)),beside=TRUE,col=c("dodgerblue3","black","deepskyblue"),axes=FALSE,
border=NA,ylim=c(0,0.4),xlim=c(0,64))
axis(1,at=c(1.5,14,26,39,51,63.5),lab=c("0","3","6","9","12","15"),font.axis=5)
axis(2,at=c(0,0.1,0.2,0.3,0.4),lab=c("0","0.1","0.2","0.3","0.4"),font.axis=5)
box()
legend(30,0.4,c("n = 15, p = 0.1","n = 15, p = 0.5","n = 15, p =
0.9"),pch=c(15,15,15),col=c("dodgerblue3","black","deepskyblue"),bty = "n",cex=1.5)
title(xlab="x",ylab="f(x)",font.lab=3)
######## (b) ########
rm(list=ls(all=TRUE))
na = c(10,20,40)
P = rep(0.25,3)
p1 = c()
for(i in 1:(na[1]+1)){
    z =  i - 1
    p1[i] = dbinom(z, size=na[1], prob=P[1])
}
p2 = c()
for(i in 1:(na[2]+1)){
    z =  i - 1
    p2[i] = dbinom(z, size=na[2], prob=P[2])
}
p3 = c()
for(i in 1:(na[3]+1)){
    z =  i - 1
    p3[i] = dbinom(z, size=na[3], prob=P[3])
}
X = data.frame(c(p1,rep(0,30)),c(p2,rep(0,20)),c(p3[1:21],rep(0,20)))
par(family="serif")
barplot(t(as.matrix(X)),beside=TRUE,col=c("dodgerblue3","black","deepskyblue"),axes=FALSE,
border=NA,ylim=c(0,0.3),xlim=c(0,120))
axis(1,at=c(1.5,22.5,43,85,102.5,120),lab=c("0","5","10","20","25","30"),font.axis=5)
axis(2,at=c(0,0.05,0.1,0.15,0.2,0.25,0.3),lab=c("0","0.05","0.1","0.15","0.2","0.25","0.3"),font.axi
s=5)
box()
legend(75,0.3,c("n = 10, p = 0.25","n = 20, p = 0.25","n = 40, p =
0.25"),pch=c(15,15,15),col=c("dodgerblue3","black","deepskyblue"),bty = "n",cex=1.5)
title(xlab="x",ylab="p(x)",font.lab=3)
```

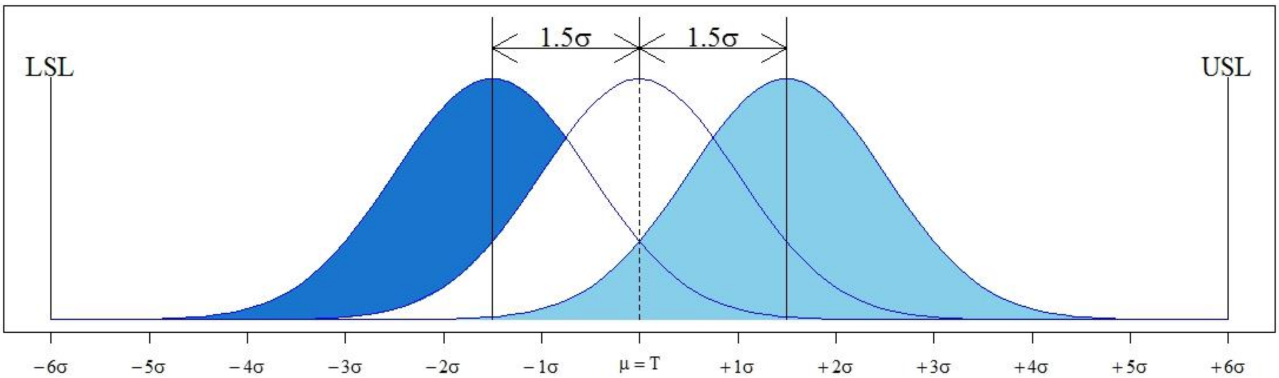
**#2**

Let $X \sim N(\mu, \sigma^2)$. $P(\mu-\sigma \le X \le \mu+\sigma) = P(\frac{\mu-\sigma-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{\mu+\sigma-\mu}{\sigma}) = P(-1 \le Z \le 1)$ $= P(Z \le 1) - P(Z \le -1) = \Phi(1) - \Phi(-1)$ where Z is the standard normal distribution, and $\Phi(\cdot)$ is the cumulative density function of standard normal distribution. In addition, " $pnorm(k, \mu, \sigma)$ " can calculate the $P(X \le k)$, in *R* language and the ppm Defective is $\{1 - P(-\sigma - \mu \le X \le \sigma + \mu)\} \times 10^6$.

Therefore, we could use the *R* language to determine the percentage inside specs and ppm Defective with different Spec. Limits.




| Spec. Limit | Percentage Inside Specs | ppm Defective |
|---|---|---|
| ± 1 Sigma | 68.27 | 317310.508 |
| ± 2 Sigma | 95.45 | 45500.264 |
| ± 3 Sigma | 99.73 | 2699.796 |
| ± 4 Sigma | 99.9937 | 63.342 |
| ± 5 Sigma | 99.999943 | 0.573 |
| ± 6 Sigma | 99.9999998 | 0.002 |



| Spec. Limit | Percentage Inside Specs | ppm Defective |
|---|---|---|
| ± 1 Sigma | 30.233 | 697672.127 |
| ± 2 Sigma | 69.123 | 308770.168 |
| ± 3 Sigma | 93.319 | 66810.599 |
| ± 4 Sigma | 99.3790 | 6209.684 |
| ± 5 Sigma | 99.97670 | 232.629 |
| ± 6 Sigma | 99.999660 | 3.398 |

**《CODE》**

```
rrm(list=ls(all=TRUE))
######## Determination of percentage inside specs and ppm defective ########
options(digits=10)
a = c(1:6)
spec.limit = c()
for(i in 1:6){
	spec.limit[i] = pnorm(a[i])- pnorm(-a[i])
}
ppmdef = (1-spec.limit)*10^6
round(spec.limit, digits = 10)*100
round(ppmdef, digits = 3)

options(digits=10)
b = c(1:6)
spec.limitb = c()
for(i in 1:6){
	spec.limitb[i] = pnorm(b[i],1.5,1)- pnorm(-b[i],1.5,1)
}
ppmdefb = (1-spec.limitb)*10^6
round(spec.limitb, digits = 10)*100
round(ppmdefb, digits = 3)######## (a) ########
layout(matrix(1:2,2,1))
stnorm = function(x){
	(1/sqrt(2*pi))*exp(-(x^2/2))
}
par(family="serif")
curve(stnorm,from=-6,to=6,ylim=c(0,0.5),xlab="",ylab="",col="blue",axes=FALSE)
axis(1,at=c(-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6),
	lab=c(expression(-6*sigma),expression(-5*sigma),expression(-4*sigma),expression(-
3*sigma),
	expression(-2*sigma),expression(-1*sigma),expression(mu==T),expression(+1*sigma),
	expression(+2*sigma),expression(+3*sigma),expression(+4*sigma),expression(+5*sigma),e
xpression(+6*sigma)),font.axis=5)
box()
polygon(c(-6,seq(-6,6,0.01),6) ,c(0,dnorm(seq(-6,6,0.01)),0) ,col='skyblue')
par(new=TRUE)
plot(rep(-
```

```
3,length(seq(0,0.45,by=0.1))),seq(0,0.45,by=0.1),type="l",xlab="",ylab="",axes=FALSE,xlim=c(-
6,6),ylim=c(0,0.5))
par(new=TRUE)
plot(rep(3,length(seq(0,0.45,by=0.1))),seq(0,0.45,by=0.1),type="l",xlab="",ylab="",axes=FALSE,
xlim=c(-6,6),ylim=c(0,0.5))
text(-3,0.42,"LSL",cex=1.5)
text(3,0.42,"USL",cex=1.5)
text(0,0.2,"±"~3*sigma,cex=1.5)
text(0,0.15,"99.73%",cex=1.5)
######## (b) ########
rm(list=ls(all=TRUE))
stnorm1 = function(x){
	(1/sqrt(2*pi))*exp(-(x^2/2))
}
stnorm2 = function(x){
	(1/sqrt(2*pi))*exp(-((x-1.5)^2/2))
}
stnorm3 = function(x){
	(1/sqrt(2*pi))*exp(-((x+1.5)^2/2))
}
par(family="serif")
curve(stnorm2,from=-6,to=6,ylim=c(0,0.5),xlab="",ylab="",col="blue",axes=FALSE)
polygon(c(-6,seq(-6,6,0.01),6) ,c(0,dnorm(seq(-6,6,0.01),1.5,1),0) ,col='skyblue', border
='skyblue')
axis(1,at=c(-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6),
	lab=c(expression(-6*sigma),expression(-5*sigma),expression(-4*sigma),expression(-
3*sigma),
	expression(-2*sigma),expression(-1*sigma),expression(mu==T),expression(+1*sigma),
	expression(+2*sigma),expression(+3*sigma),expression(+4*sigma),expression(+5*sigma),e
xpression(+6*sigma)),font.axis=5)
box()
polygon(c(-3.5,seq(-3.5,-1.5,0.01),-1.5) ,c(0,dnorm(seq(-3.5,-1.5,0.01),-1.5,1),dnorm(-
1.5)) ,col='dodgerblue3', border ='dodgerblue3')
polygon(c(-6,seq(-6,-3.5,0.01),-3.5) ,c(0,dnorm(seq(-6,-3.5,0.01),-1.5,1),0) ,col='dodgerblue3',
border ='dodgerblue3')
polygon(c(-3.5,seq(-3.5,-1.5,0.01),-1.5) ,c(dnorm(-3.5),dnorm(seq(-3.5,-1.5,0.01)),stnorm1(-
1.5)) ,col='dodgerblue3', border ='dodgerblue3')
polygon(c(-1.5,seq(-1.5,-1,0.01),-1) ,c(stnorm3(-0.5),dnorm(seq(-1.5,-1,0.01),-1.5,1),stnorm3(-
0.5)) ,col='dodgerblue3', border ='dodgerblue3')
polygon(c(-1.5,seq(-1.5,-1,0.01),-1) ,c(stnorm1(-0.5),dnorm(seq(-1.5,-1,0.01)),stnorm1(-
0.5)) ,col='dodgerblue3', border ='dodgerblue3')
polygon(c(-1.5,seq(-1.5,-1,0.01),-9/12) ,c(stnorm3(-0.5),dnorm(seq(-1.5,-1,0.01),-1.5,1),stnorm3(-
9/12)) ,col='dodgerblue3', border ='dodgerblue3')
polygon(c(-1.5,seq(-1.5,-1,0.01),-9/12) ,c(stnorm3(-0.5),dnorm(seq(-1.5,-1,0.01)),stnorm3(-
9/12)) ,col='dodgerblue3', border ='dodgerblue3')
curve(stnorm2,from=-6,to=6,ylim=c(0,0.5),xlab="",ylab="",col="blue",axes=FALSE,add=TRUE)
curve(stnorm1,from=-6,to=6,ylim=c(0,0.5),axes=FALSE,xlab="",ylab="",col="blue",add=TRUE)
curve(stnorm3,from=-6,to=6,ylim=c(0,0.5),xlab="",ylab="",col="blue",axes=FALSE,add=TRUE)
par(new=TRUE)
plot(rep(-
6,length(seq(0,0.45,by=0.1))),seq(0,0.45,by=0.1),type="l",xlab="",ylab="",axes=FALSE,xlim=c(-
6,6),ylim=c(0,0.5))
par(new=TRUE)
```

```
plot(rep(6,length(seq(0,0.45,by=0.1))),seq(0,0.45,by=0.1),type="l",xlab="",ylab="",axes=FALSE,
xlim=c(-6,6),ylim=c(0,0.5))
text(-6,0.42,"LSL",cex=1.5)
text(6,0.42,"USL",cex=1.5)
par(new=TRUE)
plot(rep(-
1.5,length(seq(0,0.55,by=0.1))),seq(0,0.55,by=0.1),type="l",xlab="",ylab="",axes=FALSE,xlim=
c(-6,6),ylim=c(0,0.5))
par(new=TRUE)
plot(rep(1.5,length(seq(0,0.55,by=0.1))),seq(0,0.55,by=0.1),type="l",xlab="",ylab="",axes=FALS
E,xlim=c(-6,6),ylim=c(0,0.5))
par(new=TRUE)
plot(rep(0,length(seq(0,0.45,by=0.1))),seq(0,0.45,by=0.1),type="l",xlab="",ylab="",axes=FALSE,
xlim=c(-6,6),ylim=c(0,0.5),lty=2)
par(new=TRUE)
plot(rep(0,length(seq(0.4,0.55,by=0.1))),seq(0.4,0.55,by=0.1),type="l",xlab="",ylab="",axes=FAL
SE,xlim=c(-6,6),ylim=c(0,0.5))
arrows(-1.5,0.45,0,0.45,code=3)
arrows(0,0.45,1.5,0.45,code=3)
text(-0.75,0.47,expression(1.5*sigma),cex=1.5)
text(0.75,0.47,expression(1.5*sigma),cex=1.5)
```

**《OUTPUT》**

```
> spec.limit
[1] 0.6826894921 0.9544997361 0.9973002039 0.9999366575 0.9999994267 0.9999999980
> ppmdef
[1] 3.173105079e+05 4.550026390e+04 2.699796063e+03 6.334248367e+01 5.733031438e-01
1.973175401e-03
> spec.limitb
[1] 0.3023278734 0.6912298322 0.9331894011 0.9937903157 0.9997673709 0.9999966023
> ppmdefb
[1] 6.976721266e+05 3.087701678e+05 6.681059894e+04 6.209684315e+03 2.326291192e+02
3.397673157e+00
```