Quiz #1, Quality control 2015 Fall

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1. A factory produces defective items with probability 0.015.

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   1) What is the probability that the number of defective items exceed 5 (X > 5) in 200 items? By using Poisson approximation to Binomial, your answer should be a simplified formula using e .
   2) Is the criterion of the Poisson approximation held?

X : defective items

$n=200$, $p=0.015$

(1) $\lambda = np = 200 \times 0.015 = 3$

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$$P(X>5) = 1 - P(X \le 5) = 1 - \sum_{x=0}^{5} \frac{\lambda^x e^{-\lambda}}{x!} = 1 - e^{-3}\left(1 + \frac{3}{1} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!}\right)$$

$$= 1 - e^{-3}\left(1 + 3 + \frac{9}{2} + \frac{\cancel{27}\,9}{\cancel{6}\,2} + \frac{\cancel{81}\,27}{\cancel{24}\,8} + \frac{\cancel{243}\,81}{\cancel{120}\,40}\right)$$

$$= 1 - e^{-3}\left(\frac{40+120+180+180+135+81}{40}\right)$$

$$= 1 - e^{-3}\left(\frac{736}{40}\right) = 1 - e^{-3}\left(\frac{92}{5}\right) \#$$

+2 (2) Yes, it is held.

$\because$ $n=200 > 100$ and $p = 0.015 < 0.1$

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2. Let X be the number of nonconforming items in n=100 samples and p = 0.01 be the fraction nonconforming. Let $\hat{p}$ be an unbiased estimator of p. Calculate
   1) $P(\hat{p} \le 0.005)$
   2) $P(\hat{p} \le 0.01)$
   3) $P(\hat{p} \le 0.02)$

   Answer up to 2 digits 0.<u>xx</u> using the approximation below:

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round(0.99^(98),2)  = 0.37
round(0.99^(99),2)  = 0.37
round(0.99^(100),2) = 0.37
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X : # of nonconforming items.

$n=100$, $p=0.01$, $Bin(100, 0.01)$

$\hat{p} = \frac{X}{n}$

(1) $P(\hat{p} \le 0.005) = P(X \le \lfloor an \rfloor) = P(X \le \lfloor 0.005 \times 100 \rfloor) = P(X \le \lfloor 0.5 \rfloor) = P(X \le 0)$

$= P(X=0) = \binom{100}{0} (0.01)^0 \cdot (0.99)^{100} \simeq 0.37$ #

(2) $P(\hat{p} \le 0.01) = P(X \le \lfloor 0.01 \times 100 \rfloor) = P(X \le 1) = P(X=0) + P(X=1)$

$= \binom{100}{0} (0.01)^0 (0.99)^{100} + \binom{100}{1} (0.01)^1 \cdot (0.99)^{99}$

$\simeq 0.37 + 0.37 = 0.74$ #

(3) $P(\hat{p} \le 0.02) = P(X \le \lfloor 0.02 \times 100 \rfloor) = P(X \le 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \binom{100}{0} (0.01)^0 (0.99)^{100} + \binom{100}{1} (0.01)^1 (0.99)^{99} + \binom{100}{2} (0.01)^2 (0.99)^{98}$

$\simeq 0.37 + 0.37 + \underbrace{50 \times 99 \times 0.0001 \times 0.37}_{= 0.18315} \simeq 0.37 + 0.37 + 0.18 = 0.92$ #

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3. Draw the detailed graph of

1) p.d.f. of $N(0,1)$

Let $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ is the p.d.f of $N(0,1)$

✓ $\phi'(x) = -x\,\phi(x)$

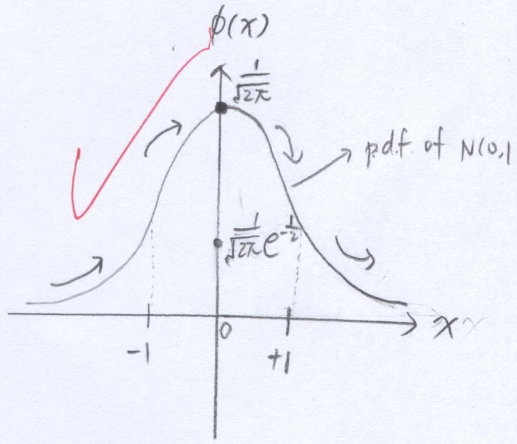
✓ $\phi''(x) = -\phi(x) - x(-x\phi(x)) = -\phi(x) + x^2\phi(x) = \phi(x)(x^2-1)$

$\phi'(x) = 0 \Rightarrow x = 0$

$\phi''(x) = 0 \Rightarrow x = 1$ or -1

x	$x<-1$	-1	$-1<x<0$	0	$0<x<1$	1	$x>1$
$\phi'(x)$	+	+	+	0	−	−	−
$\phi''(x)$	+	0	−	−	−	0	+
$\phi(x)$	↗	$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}}$	↗	$\frac{1}{\sqrt{2\pi}}$	↘	$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}}$	↘

✓



2) c.d.f of $N(0,1)$

Let $\Phi(x) = \int_{-\infty}^{x} \phi(u)\,du$ is the c.d.f of $N(0,1)$

✓ $\Phi'(x) = \phi(x)$

✓ $\Phi''(x) = -x\,\phi(x)$

$\Phi'(x) > 0$, for $\forall x$

$\Phi''(x) = 0 \Leftrightarrow x = 0$

x	$x<0$	0	$x>0$
$\Phi'(x)$	+	+	+
$\Phi''(x)$	+	0	−
$\Phi(x)$	↗	$\Phi(0) = \frac{1}{2}$	↗

✓