Midterm Exam, Quality control 2015 Fall [+20 points]

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- **Numerical answers up to 4 digits, 0.XXXX**

+8 **[Q1] [+8]** Consider $\overline{X}$-chart with $\mu = \mu_0$ being the in-control mean

1) [+2] An engineer set ARL_0=370 and ARL_1<=2 under $\mu = \mu_1 = \mu_0 + \sigma$. What is the required sample size n?

+2

$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0027} = 370$

$ARL_1 = \frac{1}{1-\beta} \le 2 \Rightarrow 1 \le 2 - 2\beta \Rightarrow \beta \le \frac{1}{2}$

$\beta = \Phi\left(Z_{\frac{\alpha}{2}} - \frac{\sqrt{n}\delta}{\sigma}\right) - \Phi\left(-Z_{\frac{\alpha}{2}} - \frac{\sqrt{n}\sigma}{\sigma}\right)$

$= \Phi(3 - \sqrt{n}) - \Phi(-3 - \sqrt{n})$

$= \Phi(3 - \sqrt{n}) \le 0.5$

$\Rightarrow 3 - \sqrt{n} \le 0 \Rightarrow 3 \le \sqrt{n} \Rightarrow n \ge 9$ #

2) [+3] Draw the OC curve for n obtained from the previous question.

+3

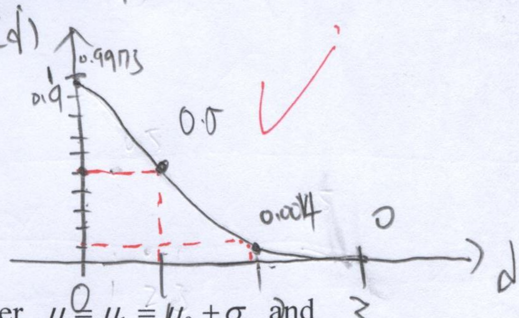
$n = 9$, Let $\beta(d) = \Phi(3 - \sqrt{n}d) - \Phi(-3 - \sqrt{n}d)$, where $d = \frac{|\delta|}{\sigma}$

$\beta(0) = \Phi(3) - \Phi(-3) = 0.9973$

$\beta(1) = \Phi(3-3) - \Phi(-3-3) = \Phi(0) = 0.5$

$\beta(2) = \Phi(3 - 3\times 2) = \Phi(-3) = 1 - 0.99865 \doteq 0.0014$

$\beta(3) = \Phi(3 - 3\times 3) = \Phi(-6) \doteq 0$



3) [+3] If the engineer set ARL_0=20, then what is ARL_1 under $\mu = \mu_1 = \mu_0 + \sigma$ and n=16?

+3

$\frac{1}{\alpha} = 20 \Rightarrow \alpha = \frac{1}{20} = 0.05$

$\beta = \Phi\left(Z_{0.025} - \frac{4\times\sigma}{\sigma}\right) - \Phi\left(-Z_{0.025} - \frac{4\times\sigma}{\sigma}\right)$

$= \Phi(1.96 - 4) - \Phi(-1.96 - 4)$

$= \Phi(-2.04)$

$= 1 - 0.9793$

$= 0.0207$

$ARL_1 = \frac{1}{1 - 0.0207} \doteq 1.0211$ #

+5 **[Q3] [+5]:** Let $X \sim Bin(n,p)$.

+1 1) [+1] State the central limit theorem for independent random variables $X_i, i=1,\cdots,n$.

$E\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} EX_i$, $Var\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} Var(X_i)$ ($\because X_1,\cdots,X_n$ are indep.)

$$\frac{\sum_{i=1}^{n} X_i - \sum_{i=1}^{n} EX_i}{\sqrt{\sum_{i=1}^{n} Var(X_i)}} \xrightarrow{d} N(0,1) \text{ as } n \to \infty$$

2) [+2] Explain how to apply the central limit theorem to $X \sim Bin(n,p)$.

+2 (e.g. how to define $X_i, i=1,\cdots,n$; how do you use independence)

Let $X_i \overset{i.i.d.}{\sim} Bernoulli(p)$, $i=1,2,\cdots,n$

Hence, $\sum_{i=1}^{n} X_i = X \sim Bin(n,p)$, $EX = np$, $Var(X) = np(1-p)$

When $n \to \infty$, $\frac{X-np}{\sqrt{np(1-p)}} \xrightarrow{d} N(0,1)$, by Central limit theorem.

3) [+2]The criterion to the approximation to $X \sim Bin(n,p)$ hold for the cases below?

+2

Write the criterion *Here* (↓)	$p = 0.231$ $n = 50$	$p = 0.023$ $n = 150$	$p = 0.059$ $n = 100$	$p = 0.008$ $n = 500$	$p = 0.164$ $n = 100$
$0.1 \le p \le 0.9$, $np > 10$	Yes	no	no	no	Yes
np	11.55(v)	3.45 (X)	5.9 (X)	4 (X)	16.4 (v)
$0.1 \le p \le 0.9$	v	X	X	X	v

Let hold → Yes → v

not hold → No → X

+7 **[Q2] [+7]:** Data, $X_{ij}, i=1,\cdots,m, \quad j=1,\cdots,n \overset{iid}{\sim} N(\mu,\sigma^2)$, are collected as follows:

					Mean	Range
i=1	12	16	10	10	12	6
i=2	22	20	22	16	20	6
i=3	15	16	12	11	13.5	5
i=4	15	13	12	12	13	3
i=5	14	14	9	11	12	5

$\bar{R}=5$

1) [+2] Calculate $\hat{\mu}$

+2

$$\hat{\mu}=\bar{\bar{X}}=\frac{12+20+13.5+13+12}{5}=14.1 \checkmark$$

2) [+2] Calculate $\hat{\sigma}$ by the range method.

+2

$$\hat{\sigma}=\frac{\bar{R}}{d_2}=\frac{5}{2.059}\doteq 2.4284 \checkmark$$

3) [+3] Draw $\overline{X}$ -Chart (3-sigma quality performance), and state your conclusion

+3

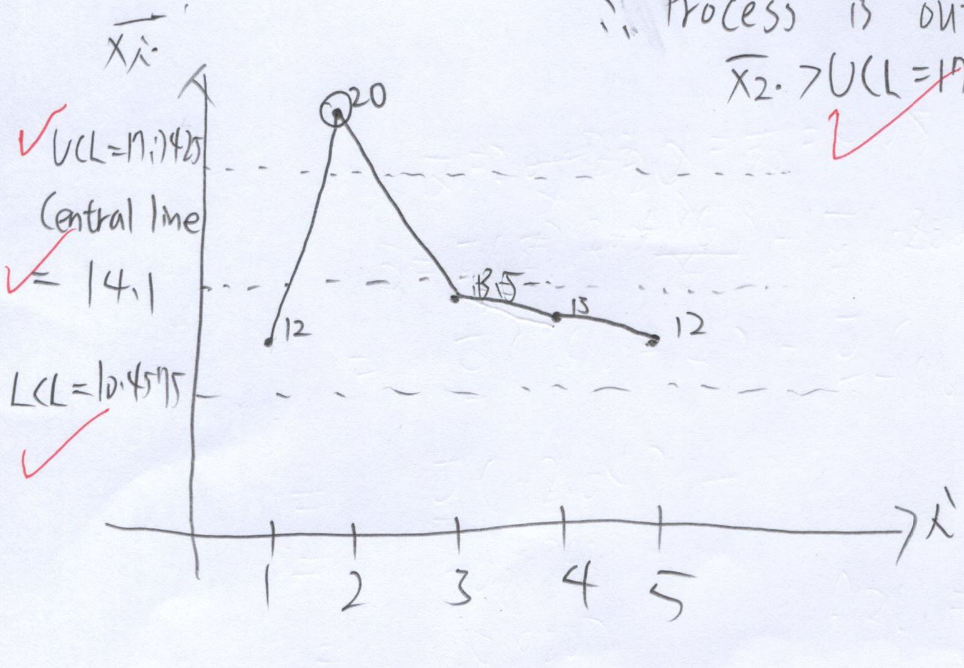
Central line $=\bar{\bar{X}}=14.1$

$$A_2=\frac{3}{\sqrt{n}\,d_2}=\frac{3}{2\times 2.059}=0.7285$$

$$UCL = 14.1+0.7285\times 5 = 17.7425 \checkmark$$

$$LCL = 14.1-0.7285\times 5 = 10.4575 \checkmark$$

$\therefore$ Process is out of control due to $\bar{X}_{2\cdot} > UCL = 17.7425$ ✓



Tables

Table: the normal distribution $p = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$

z	0	0.5	1	1.5	1.96	2	2.04	2.5	3	3.5	4
P	0.5	0.6915	0.8413	0.9332	0.975	0.9772	0.9793	0.9938	0.99865	0.99977	0.99997

Table: Conversion of range to standard deviation

n	2	3	4	5	6
d_2	1.128	1.683	2.059	2.326	2.534