Quiz#2 Quality Control, Fall 2014

10/10

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Answer the questions about profile monitoring for the data in Table 10.3. The value of α is chosen so that the in-control ARL is 200. Provide all necessary information (UCL, LCL, center, calculation of α, etc.; missing information leads to the reduction in your score.

Quantile values

```
> qnorm(0.00167)
[1] -2.934579
> qnorm(0.000835)
[1] -3.143396
> qnorm(1-0.00167)
[1] 2.934579
> qnorm(1-0.000835)
[1] 3.143396
>
> qchisq(0.00167,df=1)
[1] 4.3808e-06
> qchisq(0.000835,df=1)
[1] 1.095199e-06
> qchisq(0.00167,df=2)
[1] 0.003342792
> qchisq(0.000835,df=2)
[1] 0.001670698
> qchisq(1-0.00167,df=1)
[1] 9.880936
> qchisq(1-0.000835,df=1)
[1] 11.16177
> qchisq(1-0.00167,df=2)
[1] 12.78986
> qchisq(1-0.000835,df=2)
[1] 14.17616
```

$\sigma = 0.06826$

$Y_{ij} = \beta_0 + \beta_1 x_j + \varepsilon_{ij} = \overset{\beta_0^*}{\overset{\|}{\boxed{\beta_0 + \beta_1 \bar{x}}}} + \beta_1 (x_j - \bar{x}) + \varepsilon_{ij}$

+1 1) [+1]Write down the standardized model

$\bar{x} = \frac{1}{3}(0.76 + 3.29 + 8.89) \doteqdot 4.3133$

$Y_{ij} = \beta_0 + \beta_1 \bar{x} + \beta_1 (x_j - \bar{x}) + \varepsilon_{ij} = 0.2817 + \overset{4.2128}{0.9767 \times 4.3133} + 0.9767(x_j - 4.3133) + \varepsilon_{ij}$

$= 4.4945 + 0.9767(x_j - 4.3133) + \varepsilon_{ij}$

+3 2) [+3]Calculate the regression estimates

	✓ $\hat{\beta}_{i0}^*$	✓ $\hat{\beta}_{i1}$	✓ $\hat{\sigma}_i^2$
i=1	4.5733	0.9862	0.0086
i=2	4.47	0.9693	0.0042
i=3	4.51	0.9824	0.0031
i=4	4.6033	1.0406	0.0703.
i=5	4.5133	0.9935	0.0018
i=6	4.5233	0.9827	0.00000817

$\hat{\beta}_{i0}^* = \bar{Y}_i = \frac{1}{n}\sum_{j=1}^{n} Y_{ij}$

$\hat{\beta}_{i1} = \dfrac{\sum_{j=1}^{n}(x_j - \bar{x})Y_{ij}}{\underset{S_{xx}}{\sum_{j=1}^{n}(x_j - \bar{x})^2}} = 34.6193$

$\hat{\sigma}_i^2 = \underset{\overset{\|}{1}}{\boxed{\frac{1}{n-2}}} \sum_{j=1}^{n} \{Y_{ij} - \hat{\beta}_{i0}^* - \hat{\beta}_{i1}(x_j - \bar{x})\}^2$

$\Rightarrow \beta_0^* = 4.4945$

$\beta_1 = 0.9767$

+2 3) [+2] Derive the variances of $\hat{\beta}_{i0}^*$, $\hat{\beta}_{i1}$, and $\hat{\sigma}_i^2$.

$Var(\hat{\beta}_{i0}^*) = Var(\bar{Y}_i) = Var(\frac{1}{n}\sum_{j=1}^{n} Y_{ij}) = \frac{1}{n^2}\sum_{j=1}^{n} Var(Y_{ij}) = \frac{\sigma^2}{n} = \frac{(0.06826)^2}{3} = 0.0016$

$Var(\hat{\beta}_{i1}) = Var\left(\frac{\sum_{j=1}^{n}(x_j - \bar{x})Y_{ij}}{S_{xx}}\right) = \frac{1}{S_{xx}^2}\sum_{j=1}^{n}(x_j - \bar{x})^2 Var(Y_{ij}) = \frac{S_{xx}}{S_{xx}^2}\sigma^2 = \frac{\sigma^2}{S_{xx}} = \frac{(0.06826)^2}{34.6193} = 0.0001$

$\because \frac{(n-2)\hat{\sigma}_i^2}{\sigma^2} \sim \chi^2_{n-2} \Rightarrow Var\left(\frac{(n-2)\hat{\sigma}_i^2}{\sigma^2}\right) = 2(n-2) \Rightarrow \frac{(n-2)^2}{\sigma^4} Var(\hat{\sigma}_i^2) = 2(n-2)$

$\Rightarrow Var(\hat{\sigma}_i^2) = \frac{2\sigma^4}{n-2} = \frac{2 \times (0.06826)^4}{1} = 0.0000434.$

+4 4) [+4] Perform the profile monitoring

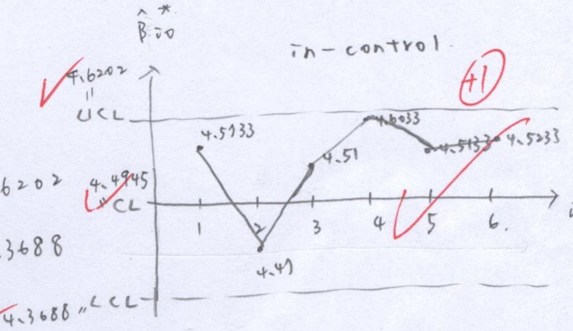
$\alpha = 1 - (1 - \frac{1}{ARL})^{\frac{1}{3}} = 1 - (1 - \frac{1}{200})^{\frac{1}{3}} \doteqdot 0.00167$ +1

* chart for β_0^*

Center = $\beta_0^* = 4.4945$

UCL = $\beta_0^* + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}} = 4.4945 + \underset{\overset{\|}{3.143396}}{Z_{\frac{0.00167}{2}}}\sqrt{0.0016} = 4.6202$

LCL = $\beta_0^* - Z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}} = 4.4945 - 3.143396\sqrt{0.0016} = 4.3688$

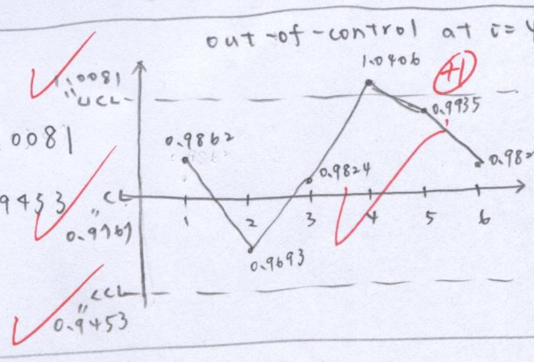


* chart for β_1

center = $\beta_1 = 0.9767$.

UCL = $\beta_1 + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{xx}}} = 0.9767 + 3.143396\sqrt{0.0001} = 1.0081$

LCL = $\beta_1 - Z_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{xx}}} = 0.9767 - 3.143396\sqrt{0.0001} = 0.9453$

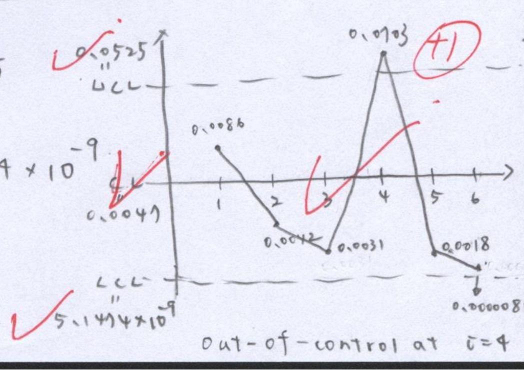


* chart for σ^2

center = $\sigma^2 = 0.0047$

UCL = $\frac{\sigma^2 \chi^2_{n-2,\frac{\alpha}{2}}}{n-2} = \frac{0.0047 \times \overset{11.16177}{\overset{\|}{\chi^2_{1,\frac{0.00167}{2}}}}}{1} = 0.0525$

LCL = $\frac{\sigma^2 \chi^2_{n-2,1-\frac{\alpha}{2}}}{n-2} = \frac{0.0047 \times \overset{1.095199 \times 10^{-6}}{\overset{\|}{\chi^2_{1,1-\frac{0.00167}{2}}}}}{1} = 5.1474 \times 10^{-9}$



where $\chi^2_{\alpha/2,\,(n-2)}$ and $\chi^2_{(1-\alpha/2),\,(n-2)}$ are the upper and lower $\alpha/2$ percentage points of the chi-square distribution with $n - 2$ degrees of freedom [see Montgomery, Peck, and Vining (2006) for details]. The value of $\alpha_{overall}$ is calculated using the equation $\alpha_{overall} = 1 - (1 - \alpha)^3$ and the in-control ARL is computed by taking the reciprocal of $\alpha_{overall}$. The value of α can be chosen to obtain a desired ARL value.

We will use the data presented in the *NIST/SEMATECH e-Handbook of Statistical Methods* to illustrate the method. The dataset consists of line widths of photo masks reference standards on 10 units (40 measurements) used for monitoring linear calibration profiles of an optical imaging system. The line widths are used to estimate the parameters of the linear calibration profile, $\hat{y}_{ij} = 0.2817 + +0.9767x'_i$ with a residual standard deviation of 0.06826 micrometers. This is the phase I profile. A monitoring scheme is established to monitor measurements on units for upper, middle, and lower end of the relevant measurement range from the estimated phase I profile. The dataset is provided in Table 10.3 and plotted in Fig. 10.27. In the plot, the in-control line is the established phase I profile. On careful observation of the measurements for the fourth sample, the plotted values seem to be slightly offset from the in-control line. We employ both the Shewhart control charts defined in equations (10.38), (10.39), and (10.40) to monitor the phase II line width data. The control charts are as shown in Fig. 10.28. In the control charts, the three horizontal lines indicate the upper control limit, the center line, and the lower control limit, respectively. The numerical values for these quantities are (4.62, 4.49, 4.37), (1.01, 0.98, 0.94), and (0.0087, 0.0046, 0.002), respectively. To achieve the overall in-control ARL of 200, the value of α for the control charts was adjusted to be 0.00167. Note that the measurements on the fourth day are out of control. The error variance values on the fifth and sixth days are below the lower control limit with the values 0.0018 and 0.0000, respectively.

■ **TABLE 10.3**
Line-Width Measurements

Day	Position	*x*	*y*
1	L	0.76	1.12
1	M	3.29	3.49
1	U	8.89	9.11
2	L	0.76	0.99
2	M	3.29	3.53
2	U	8.89	8.89
3	L	0.76	1.05
3	M	3.29	3.46
3	U	8.89	9.02
4	L	0.76	0.76
4	M	3.29	3.75
4	U	8.89	9.3
5	L	0.76	0.96
5	M	3.29	3.53
5	U	8.89	9.05
6	L	0.76	1.03
6	M	3.29	3.52
6	U	8.89	9.02