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Quiz #1, Quality control 2014 Fall [10 points]

Name:

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1. [+2] A factory produces defective items with probability 0.015 What is the probability that the number of defective items exceed 5 in 200 items? By using Poisson approximation to Binomial, your answer should be a simplified formula using e.

X: defective items , $X \sim Bin(200, 0.015)$

$\Rightarrow \lambda = np = 200 \cdot 0.015 = 3$

$$P(X>5) = 1 - P(X \le 5) \simeq 1 - \sum_{x=0}^{5} \frac{\lambda^x e^{-\lambda}}{x!} = 1 - \sum_{x=0}^{5} \frac{3^x e^{-3}}{x!}$$

$$= 1 - e^{-3}\left(1 + \frac{3}{1} + \frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \frac{243}{120}\right)$$

$$= 1 - e^{-3}\left(\frac{40+120+180+180+135+81}{40}\right) = 1 - e^{-3}\left(\frac{736}{40}\right) = 1 - e^{-3} \cdot \frac{92}{5} \#$$

✓

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2. [+2] Let $X \sim \text{Bin}(n=150,\ p=0.08)$. For a = 2.03205 and b = 21.96795, the exact value by R is $\alpha = \Pr(a \le X \le b)$ =0.9954. Find the approximate value $\alpha^* = \Pr(a \le X \le b)$ by the normal approximation, and calculate the error. [Exactly calculate up to 5 digits: 0.00000]

$np = 150 \cdot 0.08 = 12$, $np(1-p) = 11.04 \Rightarrow \sqrt{np(1-p)} = 3.32265$

$$\alpha^* = \Pr(a \le X \le b) \simeq \Pr\left(\frac{a-np}{\sqrt{np(1-p)}} \le Z \le \frac{b-np}{\sqrt{np(1-p)}}\right)$$

$$= \Phi\left(\frac{21.96795-12}{3.32265}\right) - \Phi\left(\frac{2.03205-12}{3.32265}\right)$$

$$= \Phi(3) - \Phi(-3) = \Phi(3) - (1-\Phi(3))$$

$$= 2\Phi(3) - 1 = 2 \cdot 0.99865 - 1 = 1.99730 - 1 = 0.99730$$

✓

$\therefore$ error $= 0.99730 - 0.9954 = 0.00190$ #

✓

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3. [+2] Chips of an engineering plastic have average size 45mm and standard deviation 2mm. Upper and lower specification limits for the chips are 48mm and 41mm, respectively.
   1) What is the probability that a chip gets outside the specification? [up to 4 digits: 0.0000]
   2) We obtained sizes of chips (43, 44, 47, 46, 45). Find an estimate of the standard deviation by "range method". [up to 2 digits: 0.00]

1) $\Pr(X<41) + \Pr(X>48) = \Pr\left(\frac{X-45}{2} < \frac{41-45}{2}\right) + \Pr\left(\frac{X-45}{2} > \frac{48-45}{2}\right)$

$\simeq \Phi(-2) + 1 - \Phi(1.5) = 2 - \Phi(2) - \Phi(1.5) = 0.0896$ #

✓

(2) By range method

$\Rightarrow \hat{\sigma} = \frac{R}{d_2}$, $n=5$

$= \frac{47-43}{2.326}$

$= 1.72$ #

✓

+1

4. [+1] Derive the value of d_2 when n = 2. Calculate more accurate value for d_2 than the table's value [Exactly calculate up to 5 digits: 0.00000].

✓ Let $Z_1, Z_2 \sim N(0,1) \Rightarrow Z_1 - Z_2 \sim N(0,2)$ $\therefore \frac{Z_1 - Z_2}{\sqrt{2}} \sim N(0,1)$

$$d_2 = E|Z_1 - Z_2|$$

$$= \sqrt{2}\, E\left|\frac{Z_1 - Z_2}{\sqrt{2}}\right| = \sqrt{2} E|Z| = \sqrt{2}\int_{-\infty}^{\infty} |z| \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{z^2}{2}\right)} dz$$

$$= \int_0^{\infty} 2z \frac{1}{\sqrt{\pi}} e^{\left(-\frac{z^2}{2}\right)} dz \quad (dz^2 = 2z\,dz)$$

$$= \frac{1}{\sqrt{\pi}} e^{\left(-\frac{z^2}{2}\right)} \cdot -2 \Big|_0^{\infty}$$

$$= -\frac{2}{\sqrt{\pi}}(0-1) = \frac{2}{\sqrt{\pi}} = \frac{2}{1.772454} = 1.12838 \#$$

5. We obtained the following data (n=4)

					Mean	Range
i=1	12	16	10	10	12	6
i=2	22	20	22	16	20	6
i=3	13	15	12	12	13	3
i=4	15	13	12	12	13	3
i=5	14	14	9	11	12	5

+1 1) [+1]Calculate $\overline{\overline{X}}$ and $\overline{R}$

$$\overline{\overline{X}} = \frac{12+20+13+13+12}{5} = 14 \quad , \quad \overline{R} = \frac{6+6+3+3+5}{5} = 4.6$$

+2 2) [+2] Draw $\overline{X}$ -Chart

$CL : \overline{\overline{X}} = 14$

$UCL : \overline{\overline{X}} + \frac{3}{\sqrt{n}d_2} \cdot \overline{R}$

$LCL : \overline{\overline{X}} - \frac{3}{\sqrt{n}d_2} \cdot \overline{R}$

$A_2 = \frac{3}{\sqrt{n}d_2} = \frac{3}{\sqrt{4}d_2} = \frac{3}{2\cdot(2.059)} = 0.7285$

$\fallingdotseq 0.729$

$\Rightarrow UCL : 14 + (0.729)4.6$

$= 14 + 3.3434$

$= 17.3434$

$LCL : 14 - 3.3434$

$= 10.6566$

$4.118 \overline{)3.0000}$ = 0.7285

28826

11740

8236

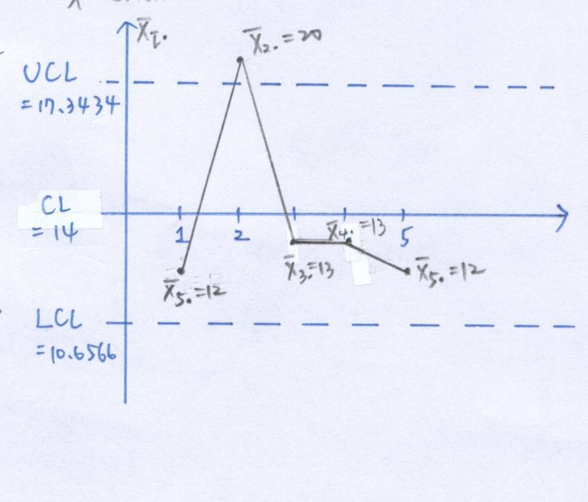
35040

32944

20960

20590

$\overline{X}$-chart



Process is out-of control.

($\overline{X}_{2\cdot} > UCL$)

Tables

Table: the normal distribution $p = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$

z	0	0.5	1	1.5	2	2.5	3	3.5	4
P	0.5	0.6915	0.8413	0.9332	0.9772	0.9938	0.99865	0.99977	0.99997

Table: Conversion of range to standard deviation

n	2	3	4	5	6
d_2	1.128	1.683	2.059	2.326	2.534

Table: Square, square-root table

number	square	square root
3.141593	9.869604	1.772454
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3.000
10	100	3.162
11	121	3.317
11.04	121.8816	3.32265
12	144	3.464

A_2

$\frac{3}{\sqrt{n}\, d_2}$