Quality control 2013 Spring, Quiz 1. [+10 points] +10/10

Not only answer but also calculation

+1 **(A)** [+1] We obtained sizes of chips (43, 44, 47, 46, 45) (cm). Find an estimate of the standard deviation by "range method".

$R = 47 - 43 = 4$, $n=5$

$\hat{\sigma} = \frac{R}{d_2} = \frac{4}{2.236} = 1.789$ ✓ #

+3 **(B)** [+3]We obtained the following data (n=4)

Group					Mean	Range
i=1	12	16	10	10	12	6
i=2	22	20	22	16	20	6
i=3	13	15	12	12	13	3
i=4	15	13	12	12	13	3
i=5	14	14	9	11	12	5

1) Find $\overline{\overline{X}}$ and $\overline{R}$

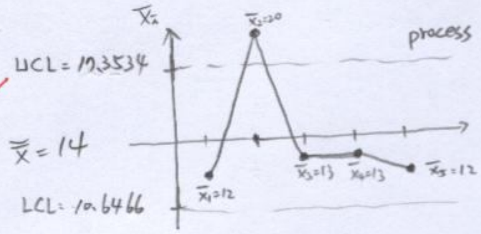
$\overline{\overline{X}} = \frac{12+20+13+13+12}{5} = 14$, $\overline{R} = \frac{6+6+3+3+5}{5} = 4.6$

2) Draw $\overline{X}$ -Chart

when $n=4 \Rightarrow A_2 = 0.729$ ✓

$UCL = \overline{\overline{X}} + A_2\overline{R} = 14 + 0.729 \cdot 4.6 = 17.3534$

$LCL = \overline{\overline{X}} - A_2\overline{R} = 14 - 0.729 \cdot 4.6 = 10.6466$



process is out-of-control

✓

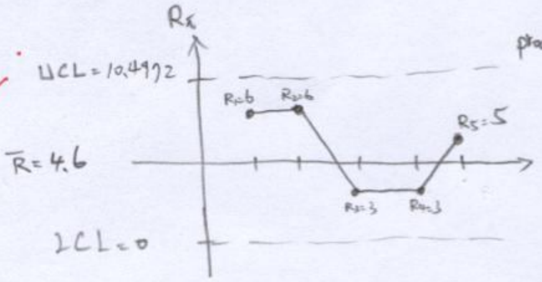
3) Draw R-chart

when $n=4$ $D_4 = 2.282$ ✓

$D_3 = 0$

$UCL = D_4\overline{R} = 2.282 \times 4.6 = 10.4972$

$LCL = D_3\overline{R} = 0$



process is in-control

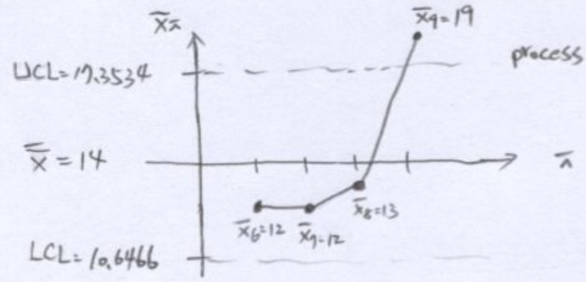
✓

+2

(C) [+2] In addition to the previous data (Phase I data), we also obtained the Phase II data as follows:

Group					Mean	Range
i=6	12	11	10	15	12	5
i=7	14	14	9	11	12	5
i=8	10	15	12	15	13	5
i=9	20	20	20	16	19	4

4) Draw $\overline{X}$ -Chart based on Phase II data when Phase I data is available



process is out-of-control

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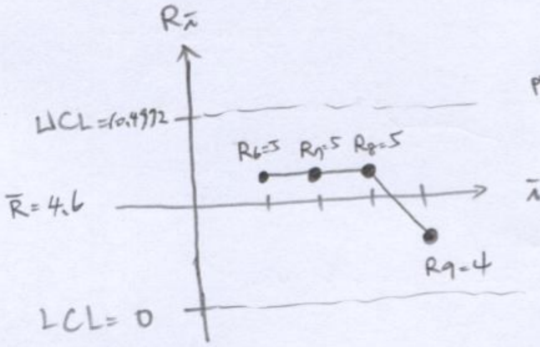
5) Draw R-chart based on Phase II data when Phase I data is available



process is in-control

✓

+4

(D) [+4] Suppose $X_{ij}, i=1,..,m, j=1,...,n \overset{iid}{\sim} N(\mu,\sigma^2)$. An engineer designs a 3-sigma limit for $\overline{X}$-Chart for monitoring $H_0:\mu=\mu_0$ vs. $H_1:\mu\neq\mu_0$.

1) Derive the producer's risk (Type I error)

2) Derive the consumer's risk (Type II error) under $H_1:\mu=\mu_0+\delta,\ \delta>0$

3) Draw OC curve for $n=4$ using some approximation

4) Suppose the engineer wish to keep the consumer's risk below 0.20. Using some approximation, derive the formula of sample size n in terms of δ and σ.

1. (reject H_0)

$\alpha = \Pr\left(\overline{X} > \mu_0 + 3\frac{\sigma}{\sqrt{n}} \text{ or } \overline{X} < \mu_0 - 3\frac{\sigma}{\sqrt{n}} \mid H_0\right) = \Pr\left(\frac{(\overline{X}-\mu)\sqrt{n}}{\sigma} > 3 \text{ or } \frac{(\overline{X}-\mu)\sqrt{n}}{\sigma} < -3\right)$

$= \Pr(Z>3 \text{ or } Z<-3) = 1-[\Phi(3)-\Phi(-3)]$

$= 1-[\Phi(3)-\{1-\Phi(3)\}]$

$= 2-2\Phi(3) = 0.0027$

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2. (accept H_0)

$\beta = \Pr\left(\mu_0 - 3\frac{\sigma}{\sqrt{n}} < \overline{X} < \mu_0 + 3\frac{\sigma}{\sqrt{n}} \mid H_1\right) = \Pr\left(-3 < \frac{(\overline{X}-\mu_0)\sqrt{n}}{\sigma} < 3 \mid H_1\right)$

$= \Pr\left(-3 < \frac{\sqrt{n}(\overline{X}-\mu)}{\sigma} + \sqrt{n}\frac{\delta}{\sigma} < 3\right)$

$= \Pr\left(-3-\sqrt{n}\frac{\delta}{\sigma} < Z < 3-\sqrt{n}\frac{\delta}{\sigma}\right) = \Phi(3-\sqrt{n}k) - \Phi(-3-\sqrt{n}k)$

(where $k=\frac{\delta}{\sigma}$, $\Phi(\cdot)$ is the c.d.f of $N(0,1)$)

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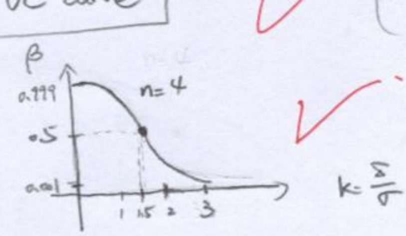
$\boxed{\text{OC curve}}$



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3. $n=4$ and $k=\frac{\delta}{\sigma}$

$\beta = \Phi(3-\sqrt{n}k) - \overset{\approx 0}{\Phi(-3-\sqrt{n}k)}$

$\approx \Phi(3-\sqrt{n}k) = \Phi(3-2k)$

k	0	1	1.5	2	3
β	$\Phi(3)$ = 0.99865	$\Phi(1)$ = 0.84134	$\Phi(0)$ = 0.5	$\Phi(-1)$ = 0.15866	$\Phi(-3)$ = 0.001

4. $\beta \le 0.20$ and $k=\frac{\delta}{\sigma}$

$\Phi(0.84)=0.8$

$0.20 = \Phi(3-\sqrt{n}k) \Rightarrow \Phi^{-1}(0.20) = 3-\sqrt{n}k$

$\Rightarrow n = \left(\frac{3-\Phi^{-1}(0.20)}{k}\right)^2 = \left(\frac{3-\Phi^{-1}(0.20)}{\frac{\delta}{\sigma}}\right)^2$

$= \left(\frac{3-(-0.84)}{\frac{\delta}{\sigma}}\right)^2$

$= \left(\frac{3.84\sigma}{\delta}\right)^2$

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