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6.3.

$f(x;\mu,\sigma)=\frac{1}{\sigma}e^{-\frac{(x-\mu)}{\sigma}}$, $\mu<x<\infty$. $0<\sigma<\infty$.

$$f(x_1,\dots,x_n;\mu,\sigma)=\prod_{i=1}^{n}f(x_i;\mu,\sigma)$$
$$=\prod_{i=1}^{n}\frac{1}{\sigma}e^{-\frac{x_i-\mu}{\sigma}}I_{(\mu,\infty)}(x_i)$$
$$=\frac{1}{\sigma^n}e^{-\frac{\sum_{i=1}^{n}x_i-n\mu}{\sigma}}I_{(-\infty,x_{(1)})}(\mu)\times 1$$

$$\prod_{i=1}^{n}I_{(\mu,\infty)}(x_i)=\begin{cases}1 & ;(x_1>\mu)\cap(x_2>\mu)\cap\cdots(x_n>\mu)\\ 0 & ;\text{otherwise}\end{cases}$$
$$=\begin{cases}1 & ;\mu<x_{(1)}<x_{(2)}\cdots<x_{(n)}\\ 0 & ;\text{otherwise}\end{cases}$$

$\therefore g(\sum x_i,x_{(1)},\mu,\sigma)=\frac{1}{\sigma^n}e^{-\frac{\sum_{i=1}^{n}x_i-n\mu}{\sigma}}I_{(-\infty,x_{(1)})}(\mu)$

$h(\underset{\sim}{x})=1$

$\therefore$ By the Factorization Theorem, $(\sum_{i=1}^{n}X_i, X_{(1)})$ is the sufficient statistic for (μ,σ). #

6.4. $X_1,\dots,X_n$ be iid r.v. with pdf or pmf $f(x|\theta)$

$f(x|\theta)=h(x)c(\theta)\exp\left(\sum_{i=1}^{k}w_i(\theta)t_i(x)\right)$,

where $\theta=(\theta_1,\theta_2,\dots,\theta_d)$, $d\le k$. Then,

$T(X)=\left(\sum_{j=1}^{n}t_1(X_j),\sum_{j=1}^{n}t_2(X_j),\dots,\sum_{j=1}^{n}t_k(X_j)\right)$ is sufficient statistic for θ.

<proof>

$$f(\underset{\sim}{x}|\theta)=\prod_{i=1}^{n}f(x_i|\theta)=\prod_{j=1}^{n}h(x_j)c(\theta)\exp\left(\sum_{i=1}^{k}w_i(\theta)t_i(x_j)\right)$$
$$=\prod_{j=1}^{n}h(x_j)(c(\theta))^n\exp\left(\sum_{i=1}^{k}w_i(\theta)\sum_{j=1}^{n}t_i(x_j)\right)$$

$\therefore g(T(\underset{\sim}{x}),\theta)=(c(\theta))^n\exp\left(\sum_{i=1}^{k}w_i(\theta)\sum_{j=1}^{n}t_i(x_j)\right)$

$h(\underset{\sim}{x})=\prod_{j=1}^{n}h(x_j)$

$\therefore$ By the Factorization Theorem, $\left(\sum_{j=1}^{n}t_1(X_j),\sum_{j=1}^{n}t_2(X_j),\dots,\sum_{j=1}^{n}t_k(X_j)\right)$ is sufficient statistic for θ. #

6.5 $X_1,\dots,X_n$ be independent r.v. with pdfs

$$f(x_i|\theta)=\begin{cases}\frac{1}{2i\theta} & ;-i(\theta-1)<x_i<i(\theta+1)\\ 0 & ;\text{otherwise}\end{cases}$$

$$f(\underset{\sim}{x};\theta)=\prod_{i=1}^{n}f(x_i;\theta)=\prod_{i=1}^{n}\frac{1}{2i\theta}I_{(-i(\theta-1),i(\theta+1))}(x_i)$$
$$=\frac{1}{(2\theta)^n n!}I_{(-\frac{x_{(1)}}{i}+1,\frac{x_{(n)}}{i}-1)}(\theta)$$

$\because \prod_{i=1}^{n}I_{(-i(\theta-1),i(\theta+1))}(x_i)$
$$=\begin{cases}1 & ;(-i(\theta-1)<x_1<i(\theta+1))\cap\cdots\cap(-i(\theta-1)<x_n<i(\theta+1))\\ 0 & ;\text{otherwise}\end{cases}$$
$$=\begin{cases}1 & ;-i(\theta-1)<x_{(1)}<x_{(2)}<\cdots<x_{(n)}<i(\theta+1)\\ 0 & ;\text{otherwise}\end{cases}$$

$\therefore g\left(\frac{x_{(1)}}{i},\frac{x_{(n)}}{i},\theta\right)=\frac{1}{(2\theta)^n n!}I_{(-\frac{x_{(1)}}{i}+1,\frac{x_{(n)}}{i}-1)}(\theta)$

$h(\underset{\sim}{x})=1$, where $X_{(1)}=\min X_i$. $X_{(n)}=\max X_i$

$\therefore$ By the Factorization Theorem, $\left(\min\frac{X_i}{i},\max\frac{X_i}{i}\right)$ is the sufficient statistic for θ. #