Homework#4, Mathematical Statistics I, 2013 Fall

1. Revise the midterm exam.
2. Use two different ways (using mgf and integration) to answer Question 7.
3. Use R to simulate a large number of binomial data using the answer to Question 6. Compare the frequency probabilities of 0, 1, 2, 3 with their theoretical values.
4. Use R to compute the simulated value of $E(S^2-\sigma^2)^2$. Compare the simulated values with the answer to Question 4. You can choose selected values of *n*, μ and σ^2.

HW4

```
> #3
> n=1000
> U=c()
> Y=c()
> for(i in 1:n){
+ U=runif(1,0,1)
+ if(0<=U &U<1/27){
+ Y[i]=0
+ }else{
+ if (1/27<=U&U<7/27){
+ Y[i]=1
+ }else{
+ if (7/27<=U&U<19/27){
+ Y[i]=2
+ }else{Y[i]=3}
+ }}}
> m=tabulate(Y)
> y_0=(n-m[1]-m[2]-m[3])/n
> y_1=m[1]/n
> y_2=m[2]/n
> y_3=m[3]/n
> y_0
[1] 0.039
> y_1
[1] 0.221
> y_2
[1] 0.434
> y_3
[1] 0.306
> 1/27
[1] 0.03703704
> 2/9
[1] 0.2222222
> 4/9
[1] 0.4444444
> 8/27
[1] 0.2962963
```

```
> #4
> n=1000
> m=1000
> mu=0
> sig2=1
> for(i in 1:m){
+ x=rnorm(n,mu,sig2)
+ x
+ s2[i]=(var(x)-sig2)^2
+
+ }
> mean(s2)
[1] 0.002159744
> (2*n-2)/((n-1)^2*sig2^4)
[1] 0.002002002
```

Q3、

$Y \sim bin(3,\frac{2}{3})$

$$Y=\begin{cases} 0\text{，}0 \le U < \frac{1}{27} \\ 1\text{，}\frac{1}{27} \le U < \frac{7}{27} \\ 2\text{，}\frac{7}{27} \le U < \frac{19}{27} \\ 3\text{，}\frac{19}{27} \le U \le 1 \end{cases}$$

P(Y=0)= 1/27= 0.03703704(true value) ≅0.039 (simulated value)

P(Y=1)= 2/9=0.2222222(true value) ≅0.221 (simulated value)

P(Y=2)= 4/9=0.4444444(true value) ≅0.434 (simulated value)

P(Y=3)= 8/27=0.2962963(true value) ≅0.306 (simulated value)

+3

Q4、

x~norm(0,1)

n=1000

mu=0

sig2=1

$E(s^2-\sigma^2)^2$

$=Var(s^2)$

$=\frac{2n-2}{\sigma^4(n-1)^2}$

$=\frac{998}{(999)^2}$

=0.002002002(true value)

≅0.002159744(simulated value)