Mathematical Statistics I

Homework (due on 2013.10.25)

1 (40%)If Y is a Poisson random variable with parameter $\lambda = 10$, and $X \mid Y$ is a binomial with parameter $(Y, p = 0.7)$,

(a) (20%)Estimate $E[X]$ by generating 1000 values of X.

(b) (20%)What do you think is the value of $E[X]$?

2 (60%)If P is a Beta random variable with parameter $(\alpha = 1, \beta = 3)$, and $X \mid \mathrm{P}$ is a binomial with parameter $(n = 10, \mathrm{P})$,

(a) (30%)Estimate $E[X]$ and $Var(X)$ by generating 1000 values of X.

(b) (30%)What do you think is the value of $E[X]$ and $Var(X)$?

Remark

Group discussion is strongly encouraged but do not just copy! In addition, late work is not allowed.

Please hand the homework with code.

10/10

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1(a)

R code:

```
> x = c(1000)
> for(i in 1:1000){
+
+ y=rpois(1,10)
+ z=rbinom(1,y,0.7)
+ x[i]=z
+ }
> mean(x)
[1] 7.03
```

1(b). $X|Y \sim$ binomial $(Y, 0.7)$, $Y \sim$ Poisson (10) ∵ $E(X), E(Y)$ exist.

By Thm 4.4.3 ⇒ $E[X] = E[E[X|Y]] = E(Y \cdot 0.7) = 0.7E(Y) = 0.7 \times 10 = 7$ ✱

2(a)

R code:

```
> rm(list=ls())
> x = c(1000)
> for(i in 1:1000){
+
+ b=rbeta(1,1,3)
+ z=rbinom(1,10,b)
+ x[i]=z
+ }
> mean(x)
[1] 2.407
> var(x)
[1] 5.33869
```

2(b)

$X|P \sim$ binomial $(10, P)$, $P \sim$ beta $(1, 3)$ ∵ $E(X), E(Y)$ exist.

$E[X] = E(E[X|P]) = E(10 \cdot P) = 10 \cdot E(P) = 10 \cdot \frac{1}{1+3} = 2.5$

By Thm 4.4.7: $Var(X) = E(Var(X|Y)) + Var(E(X|Y))$

$= E(10 \cdot P \cdot (1-P)) + Var(10 \cdot P)$

$= 10[E(P) - E(P^2)] + 10^2 Var(P)$

$= 10[\frac{1}{4} - \frac{1}{10}] + 100 \cdot \frac{3}{80}$

$= \frac{6}{4} + \frac{15}{4} = \frac{21}{4} = 5.25$

$Var(P) = E(P^2) - E(P)^2$

$\Rightarrow E(P^2) = Var(P) + E(P)^2$

$= \frac{1 \cdot 3}{(1+3)^2(1+3+1)} + (\frac{1}{1+3})^2$

$= \frac{3}{80} + \frac{1}{16} = \frac{8}{80}$