+27

Midterm exam, High-dimensional Data Analysis, 2018 Spring [+30 points]

Not only answer but also calculation **Name:** [redacted]

+8 **Q1 [+8] Data are given below:**

y_i	x_i	h_i	$\hat{y}_i$
3	-1	$\frac{1}{3}$	$\frac{5}{3}$
4	1	$\frac{1}{3}$	$\frac{13}{3}$
2	-1	$\frac{1}{3}$	$\frac{5}{3}$
1	1	$\frac{1}{3}$	$\frac{13}{3}$
0	-1	$\frac{1}{3}$	$\frac{5}{3}$
8	1	$\frac{1}{3}$	$\frac{13}{3}$

$$X=\begin{bmatrix}1&-1\\1&1\\1&-1\\1&1\\1&-1\\1&1\end{bmatrix},\quad X^TX=\begin{bmatrix}1&1&1&1&1&1\\-1&1&-1&1&-1&1\end{bmatrix}\begin{bmatrix}1&-1\\1&1\\1&-1\\1&1\\1&-1\\1&1\end{bmatrix}=\begin{bmatrix}6&0\\0&6\end{bmatrix}$$

$$(X^TX)^{-1}=\frac{1}{36}\begin{bmatrix}6&0\\0&6\end{bmatrix}=\begin{bmatrix}\frac{1}{6}&0\\0&\frac{1}{6}\end{bmatrix}$$

$$(X^Ty)=\underbrace{\begin{bmatrix}1&1&1&1&1&1\\-1&1&-1&1&-1&1\end{bmatrix}}_{2\times6}\underbrace{\begin{bmatrix}3\\4\\2\\1\\0\\8\end{bmatrix}}_{6\times1}=\begin{bmatrix}18\\8\end{bmatrix}$$

$$h_1=\begin{bmatrix}1&-1\end{bmatrix}\begin{bmatrix}\frac{1}{6}&0\\0&\frac{1}{6}\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}=\frac{1}{3}=h_3=h_5$$

$$h_2=\begin{bmatrix}1&1\end{bmatrix}\begin{bmatrix}\frac{1}{6}&0\\0&\frac{1}{6}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\frac{1}{3}=h_4=h_6$$

+2 (1) [+2] Compute the training MSE

$$\text{training MSE}=\frac{1}{6}\sum_{i=1}^{6}(y_i-\hat{y}_i)^2$$

$$\hat{\beta}=(X^TX)^{-1}X^Ty=\begin{bmatrix}\frac{1}{6}&0\\0&\frac{1}{6}\end{bmatrix}\begin{bmatrix}18\\8\end{bmatrix}=\begin{bmatrix}3\\\frac{4}{3}\end{bmatrix}$$

$$\hat{y}_i=3+\frac{4}{3}x_i,\quad \text{training MSE}=\frac{1}{6}\left[\left(\frac{4}{3}\right)^2+\left(-\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(-\frac{10}{3}\right)^2+\left(-\frac{5}{3}\right)^2+\left(\frac{11}{3}\right)^2\right]=\frac{44}{9}\ \#$$

+2 (2) [+2] Compute the testing MSE by LOOCV

$$\text{testing MSE}=\frac{1}{6}\sum_{i=1}^{6}\left(\frac{y_i-\hat{y}_i}{1-h_i}\right)^2=\frac{1}{6}\left[(2)^2+\left(-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+(-5)^2+\left(-\frac{5}{2}\right)^2+\left(\frac{11}{2}\right)^2\right]$$

$$=\frac{1}{6}\left(4+\frac{1}{4}+\frac{1}{4}+25+\frac{25}{4}+\frac{121}{4}\right)=11\ \#$$

+4 (3) [+4] Compute the testing MSE by 3-fold CV, including MSE_1, MSE_2, and MSE_3

MSE_1

training data

y_i	x_i	$\hat{y}_i$
2	-1	
1	1	
0	-1	
8	1	

testing data

y_i	x_i	$\hat{y}_i$
3	-1	1
4	1	$\frac{9}{2}$

$$X=\begin{bmatrix}1&-1\\1&1\\1&-1\\1&1\end{bmatrix},\quad X^TX=\begin{bmatrix}4&0\\0&4\end{bmatrix}$$

$$(X^TX)^{-1}=\frac{1}{16}\begin{bmatrix}4&0\\0&4\end{bmatrix}=\begin{bmatrix}\frac{1}{4}&0\\0&\frac{1}{4}\end{bmatrix}$$

$$X^Ty=\begin{bmatrix}11\\7\end{bmatrix}$$

$$\hat{\beta}=(X^TX)^{-1}X^Ty=\begin{bmatrix}\frac{1}{4}&0\\0&\frac{1}{4}\end{bmatrix}\begin{bmatrix}11\\7\end{bmatrix}=\begin{bmatrix}\frac{11}{4}\\\frac{7}{4}\end{bmatrix}$$

$$\hat{y}_i=\frac{11}{4}+\frac{7}{4}x_i$$

$$MSE_1=\frac{1}{2}\left((2)^2+\left(-\frac{1}{2}\right)^2\right)=\frac{17}{8}\ \#$$

MSE_2

training data

y_i	x_i
3	-1
4	1
0	-1
8	1

testing data

y_i	x_i	$\hat{y}_i$
2	-1	$\frac{3}{2}$
1	1	6

$$X^Ty=\begin{bmatrix}15\\9\end{bmatrix}$$

$$\hat{\beta}=\begin{bmatrix}\frac{15}{4}\\\frac{9}{4}\end{bmatrix}$$

$$\hat{y}_i=\frac{15}{4}+\frac{9}{4}x_i$$

$$MSE_2=\frac{1}{2}\left(\left(\frac{1}{2}\right)^2+(-5)^2\right)=\frac{101}{8}\ \#$$

MSE_3

training data

y_i	x_i
3	-1
4	1
2	-1
1	1

testing data

y_i	x_i	$\hat{y}_i$
0	-1	$\frac{5}{2}$
8	1	$\frac{5}{2}$

$$X^Ty=\begin{bmatrix}10\\0\end{bmatrix}$$

$$\hat{\beta}=\begin{bmatrix}\frac{5}{2}\\0\end{bmatrix}$$

$$\hat{y}_i=\frac{5}{2}$$

$$MSE_3=\frac{1}{2}\left(\left(-\frac{5}{2}\right)^2+\left(\frac{11}{2}\right)^2\right)=\frac{73}{4}\ \#$$

Hence, testing MSE

$$=\frac{1}{3}\left(\frac{17}{8}+\frac{101}{8}+\frac{73}{4}\right)=11\ \#$$

+10

Q2 [+10] Consider a linear model $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i$, $\varepsilon_i \overset{iid}{\sim} N(0,\sigma^2)$, $i = 1, \ldots, n$.

+1 (1) [+1] The design matrix is $X = \begin{bmatrix} 1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{n-1,1} & x_{n-1,2} \\ 1 & x_{n,1} & x_{n2} \end{bmatrix}_{n\times 3}$

+2 (2) [+2] The LSE is $\begin{bmatrix} \hat\beta_0 \\ \hat\beta_1 \\ \hat\beta_2 \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^n x_{i1} & \sum_{i=1}^n x_{i2} \\ \sum_{i=1}^n x_{i1} & \sum_{i=1}^n x_{i1}^2 & \sum_{i=1}^n x_{i1}x_{i2} \\ \sum_{i=1}^n x_{i2} & \sum_{i=1}^n x_{i1}x_{i2} & \sum_{i=1}^n x_{i2}^2 \end{bmatrix}^{-1}_{3\times 3} \begin{bmatrix} \sum_{i=1}^n Y_i \\ \sum_{i=1}^n x_{i1}Y_i \\ \sum_{i=1}^n x_{i2}Y_i \end{bmatrix}_{3\times 1}$

+1 (3) [+1] Define $RSS = \sum_{i=1}^n (Y_i - \hat Y_i)^2 = (Y - X\hat\beta)^T(Y - X\hat\beta) = \sum_{i=1}^n (Y_i - \hat\beta_0 - \hat\beta_1 x_{i1} - \hat\beta_2 x_{i2})^2$ #

+1 (4) [+1] Define an unbiased estimator of σ^2.

Since $\frac{\sum_{i=1}^n \varepsilon_i^2}{\sigma^2} \sim \chi^2_n \Rightarrow \frac{\sum_{i=1}^n (Y_i - \hat\beta_0 - \hat\beta_1 x_{i1} - \hat\beta_2 x_{i2})^2}{\sigma^2} \sim \chi^2_{n-3} \Rightarrow E\left[\frac{\sum_{i=1}^n (Y_i - \hat\beta_0 - \hat\beta_1 x_{i1} - \hat\beta_2 x_{i2})^2}{\sigma^2}\right] = n-3$

Hence, $\hat\sigma^2 = \frac{1}{n-3}\sum_{i=1}^n (Y_i - \hat\beta_0 - \hat\beta_1 x_{i1} - \hat\beta_2 x_{i2})^2 = \frac{1}{n-p-1} RSS$ #

+1 (5) [+1] Define $R^2 =$

$R^2 = 1 - \frac{RSS}{TSS}$ #

+2 (6) [+2] Derive a test for $H_0: \beta_1 = \beta_2 = 0$.

Since $\frac{TSS-RSS}{\sigma^2} = \underbrace{\frac{\sum_{i=1}^n (Y_i - \bar Y)^2}{\sigma^2}}_{\chi^2_{n-1}} - \underbrace{\frac{\sum_{i=1}^n (Y_i - \hat Y_i)^2}{\sigma^2}}_{\chi^2_{n-3}} \sim \chi^2_2$

$F = \frac{(TSS-RSS)/2}{RSS/n-3} \sim F_{2,n-3}$

If $F > F_{2,n-3;\alpha=0.05}$

$\Rightarrow$ Reject Ho at $\alpha = 0.05$ #

+1 (7) [+1] Let v_{ij} be the (i,j) element of $(X^TX)^{-1}$. Derive a test for $H_0: \beta_0 = 0$

since $\varepsilon \sim N(0, \sigma^2) \Rightarrow Y \sim N(X\beta, \sigma^2)$, $\hat\beta \sim$ Normal

$E(\hat\beta) = E[(X^TX)^{-1}X^TY] = (X^TX)^{-1}X^TX\beta = \beta$

$Var(\hat\beta) = Var((X^TX)^{-1}X^TY) = (X^TX)^{-1}X^T Var(Y) X (X^TX)^{-1} = \sigma(X^TX)^{-1}$

① If σ unknown, $SE(\hat\beta_0) = \hat\sigma\sqrt{V_{1,1}}$

$t = \frac{\hat\beta_0 - \beta_0}{SE(\hat\beta_0)} = \frac{\hat\beta_0}{\hat\sigma\sqrt{V_{1,1}}} = \frac{\frac{\hat\beta_0}{\sigma/\sqrt{V_{1,1}}}}{\sqrt{\frac{\hat\sigma^2}{\sigma}}} \sim t_{n-3}$

If $|t| > |t_{n-3;\alpha=0.025}|$

$\Rightarrow$ Reject Ho at $\alpha = 0.05$ #

② If σ known, $SE(\hat\beta_0) = \sigma\sqrt{V_{1,1}}$

$\frac{\hat\beta_0}{\sigma\sqrt{V_{1,1}}} = Z$

If $|Z| > |Z_{\alpha=0.025}| = 1.96$

$\Rightarrow$ Reject Ho at $\alpha = 0.05$

+1 (8) [+1] Derive a test for $H_0: \beta_1 = 0$

① If σ unknown, $SE(\hat\beta_1) = \hat\sigma\sqrt{V_{2,2}}$

$t = \frac{\hat\beta_1 - \beta_0}{SE(\hat\beta_0)} = \frac{\hat\beta_1}{\hat\sigma\sqrt{V_{2,2}}} = \frac{\frac{\hat\beta_1}{\sigma/\sqrt{V_{2,2}}}}{\sqrt{\frac{\hat\sigma^2}{\sigma^2}}} \sim t_{n-3}$

If $|t| > |t_{n-3;\alpha=0.05}|$

$\Rightarrow$ Reject Ho at $\alpha = 0.05$

② If σ known, $SE(\hat\beta_1) = \sigma\sqrt{V_{2,2}}$

$\frac{\hat\beta_1}{\sigma\sqrt{V_{2,2}}} = Z$

If $|Z| > |Z_{\alpha=0.025}| = 1.96$

$\Rightarrow$ Reject Ho at $\alpha = 0.05$ #

The simple regression of x_{i1} and x_{i2} is

$Y_i = \beta_1 x_{i1} + \varepsilon$

$Y_i = \beta_2 x_{i2} + \varepsilon$

$CC_i = \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2}$

+5

Q3 [+6] Consider a model without an intercept:

$$Y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i, \quad \varepsilon_i \overset{iid}{\sim} N(0,\sigma^2), \quad i = 1, \ldots, n.$$

Assume $\sum_{i=1}^{n} x_{i1} = \sum_{i=1}^{n} x_{i2} = 0$ and $\sum_{i=1}^{n} x_{i1}^2 = \sum_{i=1}^{n} x_{i2}^2 = n$. Let $\mathrm{CC}_i = \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2}$ be a compound covariate.

+2 (1) [+2] Derive $\hat{\beta}_1$ and $\hat{\beta}_2$ in the CC.

$RSS(\beta_1) = \sum_{i=1}^{n} (Y_i - \beta_1 x_{i1})^2, \quad \frac{\partial RSS(\beta_1)}{\partial \beta_1} = -2\sum_{i=1}^{n} (Y_i - \beta_1 x_{i1})(x_{i1}) \overset{set}{=} 0, \quad \sum_{i=1}^{n} x_{i1} Y_i - \beta_1 \sum_{i=1}^{n} x_{i1}^2 = 0$

$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_{i1} Y_i}{\sum_{i=1}^{n} x_{i1}^2} = \frac{\sum_{i=1}^{n} x_{i1} Y_i}{n}$ ✓ ※

$RSS(\beta_2) = \sum_{i=1}^{n} (Y_i - \beta_2 x_{i2})^2, \quad \frac{\partial RSS(\beta_2)}{\partial \beta_2} = -2\sum_{i=1}^{n} (Y_i - \beta_2 x_{i2})(x_{i2}) \overset{set}{=} 0, \quad \sum_{i=1}^{n} x_{i2} Y_i - \beta_2 \sum_{i=1}^{n} x_{i2}^2 = 0$

$\hat{\beta}_2 = \frac{\sum_{i=1}^{n} x_{i2} Y_i}{\sum_{i=1}^{n} x_{i2}^2} = \frac{\sum_{i=1}^{n} x_{i2} Y_i}{n}$ ※ ✓

+1 (2) [+1] Calculate $\sum_{i=1}^{n} \mathrm{CC}_i$.

$\sum_{i=1}^{n} CC_i = \sum_{i=1}^{n} \left[\left(\frac{\sum_{j=1}^{n} (x_{j1} Y_j)}{n} \right) x_{i1} + \left(\frac{\sum_{j=1}^{n} (x_{j2} Y_j)}{n} \right) x_{i2} \right] = \sum_{i=1}^{n} (\hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2}) = \hat{\beta}_1 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{i2} = 0$ ✓ ※

+2 (3) [+3] Let $\hat{Y}_i^{CC} = \hat{\gamma}\mathrm{CC}_i$ be a predictor of Y_i. Derive $\hat{\gamma}$ in terms of $\hat{\beta}_1$, $\hat{\beta}_2$, and $r_{12} = \frac{1}{n}\sum_{i=1}^{n} x_{i1} x_{i2}$.

$Y_i^{cc} = \gamma CC_i + \varepsilon \Rightarrow \hat{Y}_i^{cc} = \hat{\gamma} CC_i$

$RSS(\gamma) = \sum_{i=1}^{n} (Y_i^{cc} - \gamma CC_i)^2, \quad \frac{\partial RSS(\gamma)}{\partial \gamma} = -2\sum_{i=1}^{n} (Y_i^{cc} - \gamma CC_i)(-CC_i) \overset{set}{=} 0$

$\Rightarrow \sum_{i=1}^{n} Y_i^{cc} CC_i - \gamma \sum_{i=1}^{n} CC_i^2 = 0 \Rightarrow \gamma \sum_{i=1}^{n} CC_i^2 = \sum_{i=1}^{n} Y_i^{cc} CC_i$

$\hat{\gamma} = \frac{\sum_{i=1}^{n} Y_i^{cc} CC_i}{\sum_{i=1}^{n} CC_i^2} = \frac{\sum_{i=1}^{n} Y_i^{cc} (\hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2})}{\sum_{i=1}^{n} (\hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2})^2} = \frac{\sum_{i=1}^{n} Y_i^{cc} (\hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2})}{\sum_{i=1}^{n} (\hat{\beta}_1^2 x_{i1}^2 + \hat{\beta}_2^2 x_{i2}^2 + 2\hat{\beta}_1 \hat{\beta}_2 x_{i1} x_{i2})}$

$= \frac{\sum_{i=1}^{n} Y_i^{cc} (\hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2})}{\hat{\beta}_1^2 \sum_{i=1}^{n} x_{i1}^2 + \hat{\beta}_2^2 \sum_{i=1}^{n} x_{i2}^2 + 2\hat{\beta}_1 \hat{\beta}_2 \sum_{i=1}^{n} x_{i1} x_{i2}} = \frac{\sum_{i=1}^{n} Y_i^{cc} (\hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2})}{n(\hat{\beta}_1^2 + \hat{\beta}_2^2) + 2\hat{\beta}_1 \hat{\beta}_2 \sum_{i=1}^{n} x_{i1} x_{i2}}$

$= \frac{\frac{1}{n}\sum_{i=1}^{n} [Y_i^{cc} (\hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2})]}{(\hat{\beta}_1^2 + \hat{\beta}_2^2) + 2\hat{\beta}_1 \hat{\beta}_2 r_{12}} = \frac{\hat{\beta}_1 \overset{\hat{\beta}_1}{\left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{cc} x_{i1}\right)} + \hat{\beta}_2 \overset{\hat{\beta}_2}{\left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{cc} x_{i2}\right)}}{(\hat{\beta}_1^2 + \hat{\beta}_2^2) + 2\hat{\beta}_1 \hat{\beta}_2 r_{12}}$ -1

Let $r_{cc,1} = \frac{1}{n}\sum_{i=1}^{n} Y_i^{cc} x_{i1}$ and $r_{cc,2} = \frac{1}{n}\sum_{i=1}^{n} Y_i^{cc} x_{i2}$ $\quad \frac{\hat{\beta}_1 \overset{\hat{\beta}_1}{r_{cc,1}} + \hat{\beta}_2 \overset{\hat{\beta}_2}{r_{cc,2}}}{(\hat{\beta}_1^2 + \hat{\beta}_2^2) + 2\hat{\beta}_1 \hat{\beta}_2 r_{12}}$ ※

$$X=\begin{bmatrix}1&1\\1&-1\\1&1\\-1&-1\\-1&1\\-1&-1\end{bmatrix},\quad (X^TX)=\begin{bmatrix}1&1&1&-1&-1&-1\\1&-1&1&-1&1&-1\end{bmatrix}\begin{bmatrix}1&1\\1&-1\\1&1\\-1&-1\\-1&1\\-1&-1\end{bmatrix}=\begin{bmatrix}6&2\\2&6\end{bmatrix}$$

+4

Q4 [+6] Data are given below:

y_i	x_{i1}	x_{i2}	$\hat{y}_i$
3	1	1	-1
4	1	-1	2
2	1	1	-1
1	-1	-1	1
0	-1	1	-2
8	-1	-1	1

$$(X^TX)^{-1}=\frac{1}{32}\begin{bmatrix}6&-2\\-2&6\end{bmatrix}=\frac{1}{16}\begin{bmatrix}3&-1\\-1&3\end{bmatrix}$$

$$\hat{\beta}_\lambda=(X^TX+\lambda I_p)^{-1}X^Ty,\quad X^Ty=\begin{bmatrix}1&1&1&-1&-1&-1\\1&-1&1&-1&1&-1\end{bmatrix}\begin{bmatrix}3\\4\\2\\1\\0\\8\end{bmatrix}=\begin{bmatrix}0\\-8\end{bmatrix}$$

$$\hat{\beta}=(X^TX)^{-1}X^Ty=\frac{1}{16}\begin{bmatrix}3&-1\\-1&3\end{bmatrix}\begin{bmatrix}0\\-8\end{bmatrix}=\frac{1}{16}\begin{bmatrix}8\\-24\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\-\frac{3}{2}\end{bmatrix},\quad \hat{y}_i=\frac{1}{2}x_{i1}-\frac{3}{2}x_{i2}.$$

+2 (1) [+2] Compute the ridge estimators (without an intercept):

$$\hat{\beta}_{\lambda 1}=\frac{16}{(6+\lambda)^2-4}$$

$$\hat{\beta}_{\lambda 2}=\frac{-8(6+\lambda)}{(6+\lambda)^2-4} \quad \#$$

$$\hat{\beta}_\lambda=(X^TX+\lambda I_p)^{-1}X^Ty=\left(\begin{bmatrix}6&2\\2&6\end{bmatrix}+\begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix}\right)^{-1}\begin{bmatrix}0\\-8\end{bmatrix}=\begin{bmatrix}6+\lambda&2\\2&6+\lambda\end{bmatrix}^{-1}\begin{bmatrix}0\\-8\end{bmatrix}$$

$$=\frac{1}{(6+\lambda)^2-4}\begin{bmatrix}6+\lambda&-2\\-2&6+\lambda\end{bmatrix}\begin{bmatrix}0\\-8\end{bmatrix}=\frac{1}{(6+\lambda)^2-4}\begin{bmatrix}16\\-48-8\lambda\end{bmatrix} \quad \#$$

$$RSS(\lambda)=(y-X\beta)^T(y-X\beta)+\lambda\beta^T\beta,\ \frac{\partial RSS(\lambda)}{\partial\beta}=-2X^T(y-X\beta)+2\lambda\beta\overset{set}{=}0\Longrightarrow\hat{\beta}_\lambda=(X^TX+\lambda I_p)^{-1}X^Ty$$

+2 (2) [+2] Compute the effective degree of freedom

$$df_\lambda=tr\left[X(X^TX+\lambda I_p)^{-1}X^T\right]=tr\left[(X^TX+\lambda I_p)^{-1}X^TX\right]=tr\left[\frac{1}{(6+\lambda)^2-4}\begin{bmatrix}6+\lambda&-2\\-2&6+\lambda\end{bmatrix}\begin{bmatrix}6&2\\2&6\end{bmatrix}\right]$$

$$=\frac{1}{(6+\lambda)^2-4}\,tr\begin{bmatrix}6\lambda+32&2\lambda\\2\lambda&32+6\lambda\end{bmatrix}=\frac{2(6\lambda+32)}{(6+\lambda)^2-4}=\frac{4(3\lambda+16)}{(6+\lambda)^2-4} \quad \#$$

+0 (3) [+2] Solve $df_\lambda=1$.

$$\frac{12\lambda+64}{(6+\lambda)^2-4}=1\Rightarrow\frac{12\lambda+64}{36+12\lambda+\lambda^2-4}=1\Rightarrow\frac{12\lambda+64}{\lambda^2+12\lambda+32}=1$$

$$\Rightarrow 12\lambda+64=\lambda^2+12\lambda+32\Rightarrow\lambda^2=36\Rightarrow\lambda=\pm6 \quad \#$$