High-dimensional data analysis (HW#7)

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1) Derive B-spline bases with order M=3 under a knot sequence $\xi_0 < \xi_1 < \xi_2$ with an equally spaced mesh $\Delta = \xi_1 - \xi_0 = \xi_2 - \xi_1$.

<u>solution</u>:

m=1,

$$\mathrm{B}_{1,1}(x) = \mathrm{B}_{2,1}(x) = \mathrm{B}_{5,1}(x) = \mathrm{B}_{6,1}(x) = 0\,; \quad \mathrm{B}_{3,1}(x) = I(\xi_0 \le x < \xi_1)\,;$$

$$\mathrm{B}_{4,1}(x) = I(\xi_1 \le x < \xi_2)\,.$$

m=2,

$$\mathrm{B}_{1,2}(x) = \mathrm{B}_{5,2}(x) = 0\,; \quad \mathrm{B}_{2,2}(x) = \frac{\xi_1 - x}{\Delta} I(\xi_0 \le x < \xi_1)\,;$$

$$\mathrm{B}_{3,2}(x) = \frac{x - \xi_0}{\Delta} I(\xi_0 \le x < \xi_1) + \frac{\xi_2 - x}{\Delta} I(\xi_1 \le x < \xi_2)\,;$$

$$\mathrm{B}_{4,2}(x) = \frac{x - \xi_1}{\Delta} I(\xi_1 \le x < \xi_2)\,.$$

m=3,

$$\mathrm{B}_{1,3}(x) = \left(\frac{\xi_1 - x}{\Delta}\right)^2 I(\xi_0 \le x < \xi_1)\,;$$

$$\mathrm{B}_{2,3}(x) = \left\{\frac{(x-\xi_0)(\xi_1 - x)}{\Delta^2} + \frac{(\xi_2 - x)(x - \xi_0)}{2\Delta^2}\right\} I(\xi_0 \le x < \xi_1) + \frac{(\xi_2 - x)^2}{2\Delta^2} I(\xi_1 \le x < \xi_2)$$

$$\mathrm{B}_{3,3}(x) = \left\{\frac{(x-\xi_1)(\xi_2 - x)}{\Delta^2} + \frac{(\xi_2 - x)(x - \xi_0)}{2\Delta^2}\right\} I(\xi_1 \le x < \xi_2) + \frac{(x - \xi_0)^2}{2\Delta^2} I(\xi_0 \le x < \xi_1)$$

$$\mathrm{B}_{4,3}(x) = \left(\frac{\xi_1 - x}{\Delta}\right)^2 I(\xi_1 \le x < \xi_2)\,.$$

2) Express the above B-spline bases in terms of $z_i(t) = \dfrac{t - \xi_{\mathrm{i}}}{\Delta}$ for $i = 0, 1, 2$.

<u>solution</u>:

m=1,

$$\mathrm{B}_{1,1}(x) = \mathrm{B}_{2,1}(x) = \mathrm{B}_{5,1}(x) = \mathrm{B}_{6,1}(x) = 0\,; \quad \mathrm{B}_{3,1}(x) = I(\xi_0 \le x < \xi_1)\,;$$

$$\mathrm{B}_{4,1}(x) = I(\xi_1 \le x < \xi_2)\,.$$

m=2,

$$\mathrm{B}_{1,2}(x) = \mathrm{B}_{5,2}(x) = 0\,; \quad \mathrm{B}_{2,2}(x) = \{1 - z_0(x)\} I(\xi_0 \le x < \xi_1)\,;$$

$B_{3,2}(x) = z_0(x)I(\xi_0 \le x < \xi_1) - z_2(x)I(\xi_1 \le x < \xi_2)$;

$B_{4,2}(x) = \{z_2(x)+1\}I(\xi_1 \le x < \xi_2)$.

m=3,

$B_{1,3}(x) = z_1(x)^2 I(\xi_0 \le x < \xi_1)$;

$$B_{2,3}(x) = \left\{2z_0(x) - \frac{3}{2}z_0(x)^2\right\}I(\xi_0 \le x < \xi_1) + \frac{z_2(x)^2}{2}I(\xi_1 \le x < \xi_2)$$

$$B_{3,3}(x) = \frac{z_0(x)^2}{2}I(\xi_0 \le x < \xi_1) - \left\{2z_2(x) + \frac{3}{2}z_2(x)^2\right\}I(\xi_1 \le x < \xi_2)$$

$B_{4,3}(x) = z_1(x)^2 I(\xi_1 \le x < \xi_2)$.

3) Depicts the B-spline basis function with knots $\xi_0 = 0$, $\xi_1 = 1$ and $\xi_2 = 2$ using R; All the basis functions plotted in one figure. Distinguish them by color.

solution:

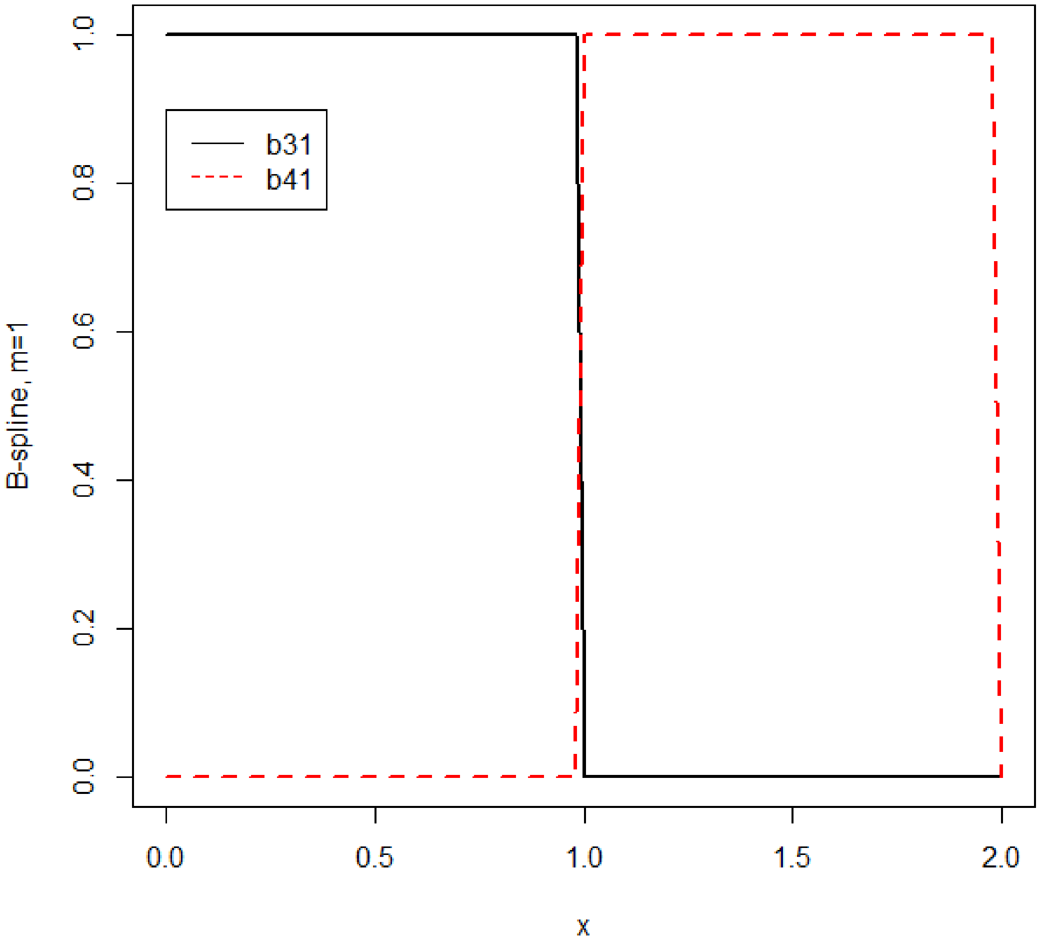

Figure 1. Order m=1.

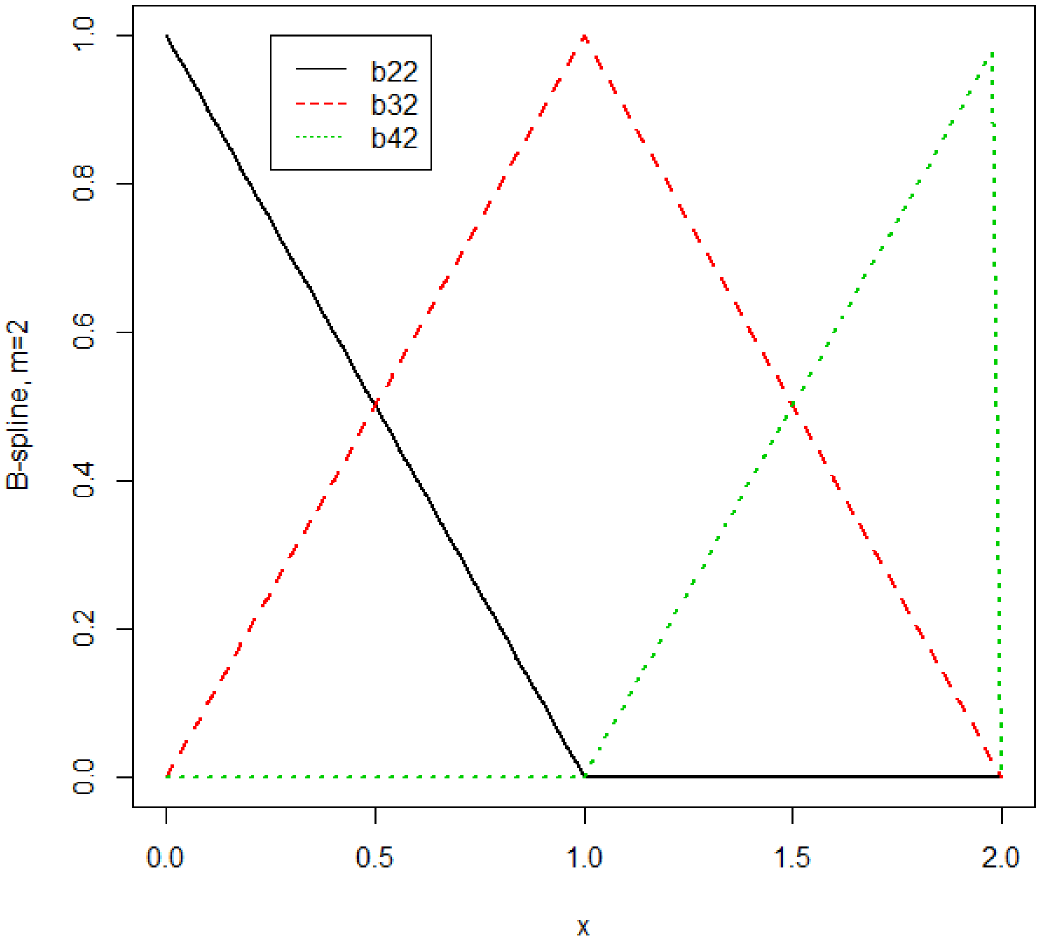

Figure 2. Order m=2.

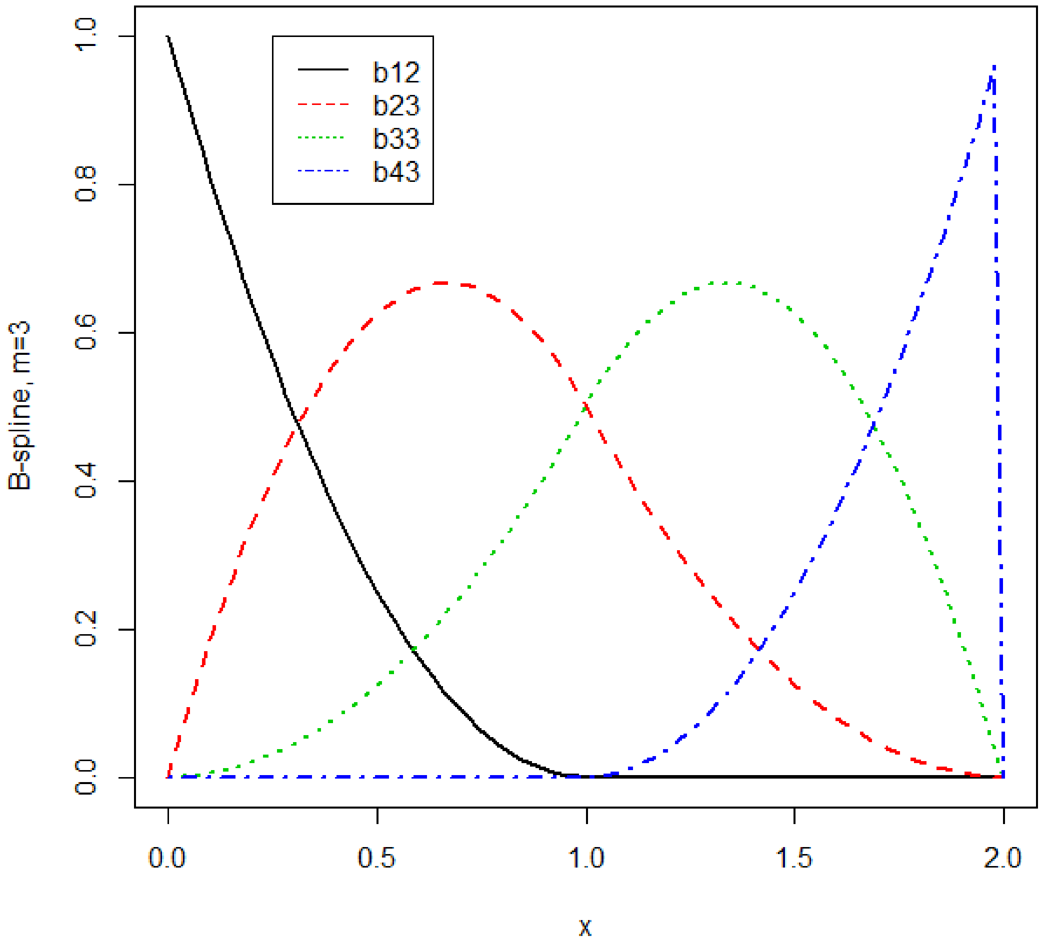

Figure 3. Order m=3.

R code

```
#high dim HW
#Q3
z0 = function(x){x}
z1 = function(x){x-1}
z2 = function(x){x-2}

#m=1
b11=0
b21=0
b31=function(x){(x>=0 & x<1)}
b41=function(x){(x>=1 & x<2)}
b51=0
b61=0
#m=2
```

```
b12=0
b22=function(x){(1-x)*(x>=0 & x<1)}
b32=function(x){x*(x>=0 & x<1)-(x-2)*(x>=1 & x<2)}
b42=function(x){(1+(x-2))*(x>=1 & x<2)}
b52=0
#m=3
b13=function(x){(x-1)^2*(x>=0 & x<1)}
b23=function(x){(2*x-3/2*x^2)*(x>=0 & x<1)+(x-2)^2*(x>=1 & x<2)/2}
b33=function(x){x^2*(x>=0 & x<1)/2-(2*(x-2)+3/2*(x-2)^2)*(x>=1 &
x<2)}
b43=function(x){(x-1)^2*(x>=1 & x<2)}

#m=1
curve(b31,from=0, to=2, col=1, lty=1, ylab="B-spline, m=1",lwd=2)
curve(b41,from=0, to=2, col=2, lty=2,lwd=2, add=TRUE)
legend(x=0,y=.9,legend = c("b31","b41"), lty=c(1:2), col=c(1:2))
#m=2
curve(b22,from=0, to=2, col=1, lty=1, ylab="B-spline, m=2",lwd=2)
curve(b32,from=0, to=2, col=2, lty=2,lwd=2, add=TRUE)
curve(b42,from=0, to=2, col=3, lty=3,lwd=2, add=TRUE)
legend(x=.25,y=1,legend = c("b22","b32","b42"), lty=c(1:3), col=c(1:3))
#m=3
curve(b13,from=0, to=2, col=1, lty=1, ylab="B-spline, m=3",lwd=2)
curve(b23,from=0, to=2, col=2, lty=2,lwd=2, add=TRUE)
curve(b33,from=0, to=2, col=3, lty=3,lwd=2, add=TRUE)
curve(b43,from=0, to=2, col=4, lty=4,lwd=2, add=TRUE)
legend(x=.25,y=1,legend = c("b12","b23","b33","b43"), lty=c(1:4), col=c(1:4))
```