A robust change point estimator for binomial CUSUM control charts

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Presenter: Yi Ting Ho
Outline

- Introduction

- Background
  1. Binomial CUSUM control chart
  2. Maximum likelihood estimator
  3. Page’s last zero estimator

- Method
  Combine CUSUM estimator and MLE
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- Simulations
  Design closely follows those of Perry and Pignatiello (2005)

- Data analysis
  Jewelry manufacturing data by Burr (1979)

- Conclusion
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Introduction

- What is the change point?

- We consider observations come from binomial distribution with the same fraction nonconforming $p$.

CUSUM Chart

- $h=12.043$
- Out-of-control

Change point
In SPC, the $np$-chart is most famous charts used to monitor the number of nonconforming items for industrial manufactures.

When sample size is large and defective rate is not too small, the $np$-chart works well.

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Page (1954, 1955) first suggested the CUSUM chart to estimate the change point.

The binomial CUSUM control chart is a good alternative when small changes are important.
Introduction

Samuel and Pignatiello (2001) proposed maximum likelihood estimator (MLE) for the process change point using the step change Likelihood function for a binomial random variable.

Perry and Pignatiello (2005) shows that the performance of the MLE is often better than Page’s last zero estimator.
The MLE method outperforms CUSUM method when magnitudes of change is large.

In order to construct more robust in different parameter setting, this thesis combines CUSUM estimator and MLE. Furthermore, compares the new method with two estimators.
Outline

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1. Binomial CUSUM control chart
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Method

Combine CUSUM estimator and MLE
Binomial CUSUM control chart

\[ X_1, X_2, \ldots, X_\tau \sim \text{Bin}( n, p_0 ), \quad X_{\tau+1}, X_{\tau+2}, \ldots, X_T \sim \text{Bin}( n, p_a^{\text{True}} ) \]

\[ \Theta = \{ ( \tau, p_a^{\text{True}} ) | \tau \in \{ 1, 2, \ldots, T \}, p_0 \leq p_a^{\text{True}} \leq 1 \} \text{ for a fixed value } T \in \{ 1, 2, \ldots \}. \]

\[ P( X_i = x ) = \begin{cases} \binom{n}{x} p_0^x (1 - p_0)^{n-x}, & \text{if } i = 1, 2, \ldots, \tau, \\ \binom{n}{x} ( p_a^{\text{True}} )^x (1 - p_a^{\text{True}})^{n-x}, & \text{if } i = \tau + 1, \tau + 2, \ldots, T. \end{cases} \]

\( p_0 \): in-control fraction nonconforming.

\( p_a^{\text{True}} \): out-of-control fraction nonconforming.

\( \tau \): the true change point.
Binomial CUSUM control chart

\[ S_0 = 0 \text{ and } S_i = \max\{ 0, X_i - nk + S_{i-1} \}, \quad i = 1, 2, \ldots \]

where

\[
k = -\ln \left( \frac{1 - p_a^{as}}{1 - p_0} \right)
\]

\[
= \ln \left\{ \frac{p_a^{as} (1 - p_0)}{p_0 (1 - p_a^{as})} \right\}
\]

\[ k : \text{The reference value.} \]

\[ p_a^{as} : \text{out-of-control fraction nonconforming for which to design the CUSUM chart.} \]
When $S_i$ exceeds decision interval $h > 0$ the chart signals that an increase in the process fraction nonconforming has occurred.
Maximum likelihood estimator (MLE)

\[
X_1, X_2, \ldots, X_{\tau} \overset{i.i.d}{\sim} \text{Bin}(n, p_0), \quad X_{\tau+1}, X_{\tau+2}, \ldots, X_T \overset{i.i.d}{\sim} \text{Bin}(n, p_a^{\text{True}})
\]

\[\Theta = \{ (\tau, p_a^{\text{True}}) | \tau \in \{1, 2, \ldots, T\}, p_0 \leq p_a^{\text{True}} \leq 1 \} \text{ for fixed value } T \in \{1, 2, \ldots\}.\]

The likelihood function is given by

\[
L(\tau, p_a^{\text{True}} | p_0, X) = \prod_{i=1}^{\tau} \binom{n}{x_i} p_0^{x_i} (1 - p_0)^{n-x_i} \prod_{i=\tau+1}^{T} \binom{n}{x_i} (p_a^{\text{True}})^{x_i} (1 - p_a^{\text{True}})^{n-x_i}
\]

where \(X = (x_1, x_2, \ldots, x_T)\).
Maximum likelihood estimator (MLE)

The log-likelihood is

\[
\log L(\tau, p_a^{\text{True}} \mid p_0, X) = C + \log(p_0) \sum_{i=1}^{\tau} x_i + \log(1 - p_0) \sum_{i=1}^{\tau} (n - x_i) + \log(p_a^{\text{True}}) \sum_{i=\tau+1}^{T} x_i + \log(1 - p_a^{\text{True}}) \sum_{i=\tau+1}^{T} (n - x_i),
\]

where \( C \) is a constant. We can rewrite \( \log L(\tau, p_a^{\text{True}} \mid p_0, X) \) as

\[
\log L(\tau, p_a^{\text{True}} \mid p_0, X) = C^* + \log\left(\frac{p_a^{\text{True}}}{p_0}\right) \sum_{i=\tau+1}^{T} x_i + \log\left(\frac{1 - p_a^{\text{True}}}{1 - p_0}\right) \sum_{i=\tau+1}^{T} (n - x_i),
\]

where \( C^* = C + \log(p_0) \sum_{i=1}^{T} x_i + \log(1 - p_0) \sum_{i=1}^{T} (n - x_i) \) is a constant.
Maximum likelihood estimator (MLE)

If $\tau$ is given, the log-likelihood equation becomes

$$
\frac{\partial}{\partial p_a^{True}} \log L(\tau, p_a^{True} | p_0, X) = \frac{\sum_{i=\tau+1}^{T} x_i}{p_a^{True}} - \frac{\sum_{i=\tau+1}^{T} (n-x_i)}{1-p_a^{True}},
$$

$$
\sum_{i=\tau+1}^{T} x_i = \frac{\sum_{i=\tau+1}^{T} (n-x_i)}{p_a^{True}} 1-p_a^{True} \Rightarrow \hat{p}_a^{True}(\tau) = \frac{\sum_{i=\tau+1}^{T} x_i}{\sum_{i=\tau+1}^{T} n}.
$$
Maximum likelihood estimator (MLE)

\[
\frac{\partial^2}{\partial p_a^{\text{True}}^2} \log L(\tau, p_a \mid p_0, X) = -\sum_{i=\tau+1}^{T} x_i - \sum_{i=\tau+1}^{T} (n - x_i) < 0.
\]

Putting \(\hat{p}_a^{\text{True}}(\tau)\) into \(\log L(\tau, p_a^{\text{True}} \mid p_0, X)\), we have a profile log-likelihood for \(\tau\) as

\[
\log L(\tau, \hat{p}_a^{\text{True}}(\tau) \mid p_0, X) = C^* + \log\left(\frac{\hat{p}_a^{\text{True}}(\tau)}{p_0}\right) \sum_{i=\tau+1}^{T} x_i + \log\left(\frac{1 - \hat{p}_a^{\text{True}}(\tau)}{1 - p_0}\right) \sum_{i=\tau+1}^{T} (n - x_i).
\]
Maximum likelihood estimator (MLE)

Therefore, the change point estimator \( \hat{\tau}_{MLE} \) is

\[
\hat{\tau}_{MLE} = \arg \max_{\tau \in \{1, 2, \ldots, T\}} \left[ \log \left\{ \frac{\hat{P}_a^{True}(\tau)}{P_0} \right\} \sum_{i=\tau+1}^{T} x_i + \log \left\{ \frac{1 - \hat{P}_a^{True}(\tau)}{1 - p_0} \right\} \sum_{i=\tau+1}^{T} (n - x_i) \right].
\]
Page’s last zero estimator

Under CUSUM control chart we have cumulative sum

\[ S_0 = 0 \quad \text{and} \quad S_i = \max\{ 0, X_i - nk + S_{i-1} \}, \quad i = 1, 2, \ldots \]

An estimate of the change point is given by \( \hat{\tau}_{CUSUM} = \max\{ i : S_i = 0 \} \).
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Proposed method

We propose a new change point estimator that combines $\hat{\tau}_{MLE}$ and $\hat{\tau}_{CUSUM}$.

$$\hat{\tau}_{NEW}(w) = w\hat{\tau}_{CUSUM} + (1-w)\hat{\tau}_{MLE}, \quad 0 \leq w \leq 1,$$

$$w \equiv w(\frac{p^True_a}{p_a} \mid p^as_a) = \begin{cases} \left( \frac{p^True_a - p_0}{p^as_a - p_0} \right) \left( \frac{p^True_a}{p_0} \right), & \text{if } p^True_a - p_0 \leq p^as_a - p_0, \\ \left( \frac{p^as_a - p_0}{p^True_a - p_0} \right) \left( \frac{p^True_a}{p_0} \right), & \text{if } p^True_a - p_0 > p^as_a - p_0. \end{cases}$$
Proposed method

First, we estimate unknown $p_a^{true}$ by MLE.

\[
\hat{p}_a(\hat{\tau}_{MLE}) = \frac{\sum_{i=\hat{\tau}_{MLE}+1}^{T} x_i}{\sum_{i=\hat{\tau}_{MLE}+1}^{T} n},
\]

The estimator of weight function as

\[
\hat{w} \equiv w(\hat{p}_a(\hat{\tau}_{MLE})| p_a^{as}) = \begin{cases} 
\frac{\hat{p}_a(\hat{\tau}_{MLE}) - p_0}{p_a^{as} - p_0} & \left(\frac{\hat{p}_a(\hat{\tau}_{MLE})}{p_0}\right), \\
\frac{p_a^{as} - p_0}{\hat{p}_a(\hat{\tau}_{MLE}) - p_0} & \left(\frac{\hat{p}_a(\hat{\tau}_{MLE})}{p_0}\right),
\end{cases} 
\]

if $\hat{p}_a(\hat{\tau}_{MLE}) - p_0 \leq p_a^{as} - p_0$,

if $\hat{p}_a(\hat{\tau}_{MLE}) - p_0 > p_a^{as} - p_0$. 

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Proposed method

Hence, we obtain the proposed estimator of the change point as

\[ \hat{\tau}_{\text{NEW}}(\hat{w}) = \hat{w}\hat{\tau}_{\text{CUSUM}} + (1 - \hat{w})\hat{\tau}_{\text{MLE}}, \quad 0 \leq \hat{w} \leq 1. \]
Simulations
Design closely follows those of Perry and Pignatiello (2005)

Data analysis
Jewelry manufacturing data by Burr (1979)

Conclusion
Simulations

\[ X_1, X_2, \ldots, X_\tau \sim \text{Bin}( n, p_0 ) , \quad X_{\tau+1}, X_{\tau+2}, \ldots, X_T \sim \text{Bin}( n, p_a^{\text{True}} ) \]

- We assume the true change point \( \tau = 100 \).

- The in-control fraction nonconforming \( p_0 = 0.1 \).

- The out-of-control fraction nonconforming

\[ p_a^{\text{True}} \in \{ 0.11, 0.12, \ldots, 0.25, 0.30 \} . \]
Simulations

- Consider CUSUM charts designed to detect 30% increase in by setting $p_{a}^{as} = 1.3 \times p_0 = 0.13$.

- Choose $h = 6.57$ or $h = 11.42$ such that in-control process average run lengths (ARL) is close to 150 or 370.
Simulations

\[ p_0 = 0.1, \quad p^{as}_a = 0.13, \]
\[ p^{True}_a = 0.1, 0.11, 0.15, 0.20, 0.25, 0.30 \]
Simulations

\[ p_0 = 0.1, \quad p_{a}^{as} = 0.13, \]
\[ p_{a}^{True} = 0.1, 0.11, 0.15, 0.20, 0.25, 0.30 \]

Maximum likelihood estimator

\[ h = 6.57 \]
Simulations

\[ p_0 = 0.1, \quad p_a^{as} = 0.13, \]
\[ p_a^{true} = 0.1, 0.11, 0.15, 0.20, 0.25, 0.30 \]
Simulations

\[ p_0 = 0.1, \quad p_{a}^{as} = 0.13, \]
\[ p_{a}^{True} = 0.1, 0.11, 0.15, 0.20, 0.25, 0.30 \]
Table 4 Simulation results for estimating the change point $\tau$ under $n = 50$, $P_0 = 0.1$, and $p^{as}_a = 0.13$ based on 1000 simulation runs. The true change point $\tau = 100$.

<table>
<thead>
<tr>
<th>$h = 6.57$</th>
<th>$p_{sa}^{True}$</th>
<th>$E(\hat{\tau}_{MLE})$</th>
<th>$E(\hat{\tau}_{CUSUM})$</th>
<th>$E(\hat{\tau}_{NEW}(w))$</th>
<th>$E(\hat{\tau}_{NEW}(\hat{w}))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>118.90</td>
<td>115.98</td>
<td>118.03</td>
<td>117.61</td>
<td></td>
</tr>
<tr>
<td>0.12</td>
<td>108.50</td>
<td>105.51</td>
<td>106.66</td>
<td>107.32</td>
<td></td>
</tr>
<tr>
<td>0.13 (= p^{as}_a)</td>
<td>104.10</td>
<td>101.33</td>
<td>101.33</td>
<td>103.08</td>
<td></td>
</tr>
<tr>
<td>0.14</td>
<td>102.51</td>
<td>99.94</td>
<td>100.84</td>
<td>101.82</td>
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<tr>
<td>0.15</td>
<td>101.54</td>
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<tr>
<td>0.16</td>
<td>101.13</td>
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<tr>
<td>0.17</td>
<td>100.63</td>
<td>98.47</td>
<td>100.11</td>
<td>100.17</td>
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<tr>
<td>0.18</td>
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<td>98.54</td>
<td>100.21</td>
<td>100.15</td>
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</tr>
<tr>
<td>0.19</td>
<td>100.16</td>
<td>98.45</td>
<td>99.95</td>
<td>99.88</td>
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<tr>
<td>0.20</td>
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<td>98.42</td>
<td>100.09</td>
<td>99.96</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>100.11</td>
<td>98.34</td>
<td>100.08</td>
<td>99.88</td>
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</tr>
<tr>
<td>0.30</td>
<td>100.07</td>
<td>98.44</td>
<td>100.07</td>
<td>99.91</td>
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</table>
### Simulations

<table>
<thead>
<tr>
<th>$h = 11.42$</th>
<th>$p_a^{true}$</th>
<th>$E(\hat{t}_{MLE})$</th>
<th>$E(\hat{t}_{CUSUM})$</th>
<th>$E(\hat{t}_{NEW}(w))$</th>
<th>$E(\hat{t}_{NEW}(\hat{w}))$</th>
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<tbody>
<tr>
<td>0.11</td>
<td>153.07</td>
<td>147.96</td>
<td>151.54</td>
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<tr>
<td>0.12</td>
<td>115.53</td>
<td>108.84</td>
<td>111.42</td>
<td>113.33</td>
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<tr>
<td>0.13 ($= p_a^{\alpha}$)</td>
<td>107.14</td>
<td>100.97</td>
<td>100.97</td>
<td>105.20</td>
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<td>103.79</td>
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<td>100.34</td>
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<td>0.16</td>
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<td>97.26</td>
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<td>0.25</td>
<td>100.30</td>
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<td>0.30</td>
<td>100.14</td>
<td>97.30</td>
<td>100.13</td>
<td>99.98</td>
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</tbody>
</table>
Simulations

Table 5 Simulation results for estimating the mean squared error the change point $\tau$ Under $n = 50$, $p_o = 0.1$ and $p_{as} = 0.13$ based on 1000 simulation runs. The true change point $\tau = 100$.

<table>
<thead>
<tr>
<th>$h$ = 6.57</th>
<th>$p_{as}^{True}$</th>
<th>$MSE(\hat{\tau}_{MLE})$</th>
<th>$MSE(\hat{\tau}_{CUSUM})$</th>
<th>$MSE(\hat{\tau}_{NEW}(w))$</th>
<th>$MSE(\hat{\tau}_{NEW}(\hat{w}))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>814.11</td>
<td>669.15</td>
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<td>0.12</td>
<td>214.27</td>
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<td>145.85</td>
<td>158.91</td>
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<tr>
<td>0.13 (=$p_{as}$)</td>
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<td>30.862</td>
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<tr>
<td>$h = 11.42$</td>
<td>$p_{sa}^{True}$</td>
<td>$MSE(\hat{\tau}_{MLE})$</td>
<td>$MSE(\hat{\tau}_{CUSUM})$</td>
<td>$MSE(\hat{\tau}_{NEW}(w))$</td>
<td>$MSE(\hat{\tau}_{NEW}(\hat{w}))$</td>
</tr>
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</table>

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Simulations

- In most cases, the $\hat{\tau}_{NEW}(\hat{w})$ outperforms other estimators.

- Despite some of the mean squared error of $\hat{\tau}_{NEW}(\hat{w})$ in all cases are not the smallest, $\hat{\tau}_{NEW}(\hat{w})$ provides very precise estimator of change point.
Simulations
Design closely follows those of Perry and Pignatiello (2005)

Data analysis
Jewelry manufacturing data by Burr (1979)

Conclusion
We set the in-control fraction nonconforming \( p_0 = \frac{229}{2700} = 0.085 \). Each subgroup contains \( n = 50 \) beads so the number of subgroups is \( T = \frac{2700}{50} = 54 \).

\( X_i \) = the number of defective pieces in 50 beads, for \( i = 1, 2, \ldots, 54 \).
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<th>$S_i$</th>
<th>Sample subgroup</th>
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Data Analysis

- Center line: \( np = 50 \cdot 0.085 = 4.25. \)

- Upper control limit: 
  \[
  np + 3\sqrt{n \cdot p_0 \cdot (1 - p_0)} = 4.25 + 3\sqrt{50 \cdot 0.085 \cdot 0.915} = 10.165. 
  \]

- Lower control limit: 
  \[
  np - 3\sqrt{n \cdot p_0 \cdot (1 - p_0)} = 4.25 - 3\sqrt{50 \cdot 0.085 \cdot 0.915} = -1.665. 
  \]
Although $np$-chart do not detect the process out-of-control, the data slightly increase after the subgroup 40.
Data Analysis

CUSUM Chart

Subgroup number
Cumulative Sum

Out-of-control

h=12.043

Maximum likelihood estimator

Log - Likelihood

τ^=50

Change point

2014/6/24
Data Analysis

- We obtain $\hat{\tau}_{CUSUM} = 43$, $\hat{\tau}_{MLE} = 50$ and $\hat{\tau}_{NEW}(\hat{w}) = 47.70983$.

- $\hat{\tau}_{CUSUM}$ always underestimate the true change point while $\hat{\tau}_{MLE}$ slightly overestimate $\tau$ when $p_{a}^{True} > p_{a}^{as}$.

- Our proposed method may provide more unbiased estimator.
Outline

- Simulations
  Design closely follows those of Perry and Pignatiello (2005)

- Data analysis
  Jewelry manufacturing data by Burr (1979)

- Conclusion
Conclusion

- The estimator $\hat{\tau}_{NEW}(\hat{w})$ contains advantage of $\hat{\tau}_{CUSUM}$ and $\hat{\tau}_{MLE}$.

- Our proposed method are unbiasedness for estimating the true change point.

- The estimator $\hat{\tau}_{NEW}(\hat{w})$ is more robust than $\hat{\tau}_{CUSUM}$ and $\hat{\tau}_{MLE}$ under different parameter setting.
References

References

- Page ES, A test for a change in a parameter occurring at an unknown point. *Biometrika* 1955; 523-527.
References

Thanks for your attention.