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Approximate Tolerance Limits under Log-location-scale Regression Models in Presence of Censoring

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Outlines

Part I: Tolerance limit: Review

- Tolerance limit - definition
- Tolerance limit - motivating example (data)

Part II: Proposed Tolerance limit

- Log-location scale regression model
- Proposed tolerance limit
- Computation & Asymptotic validity
- Simulations, compare with existing one
- Data analysis
- Summary & Future work

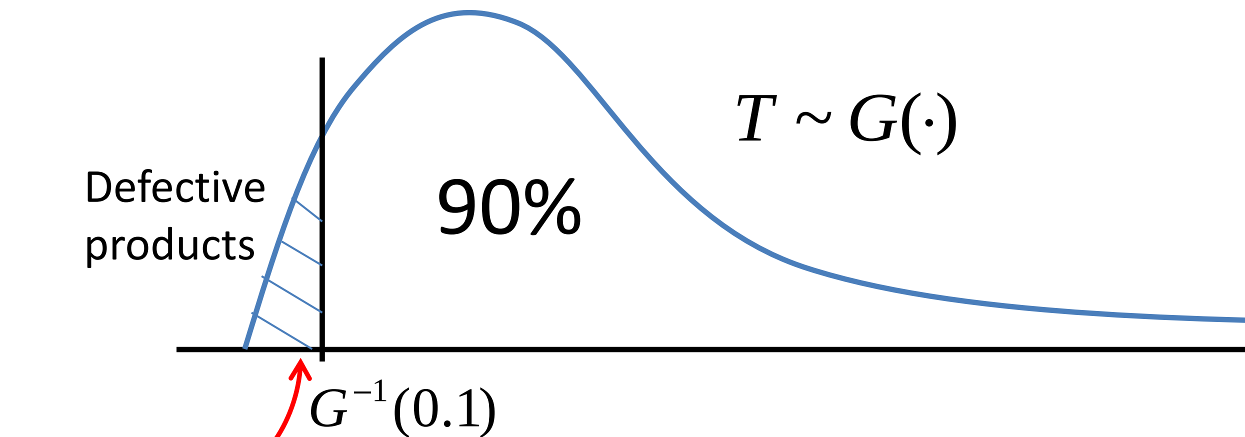
Part I

Tolerance limit: Review

Tolerance limit

Role of tolerance limit

Life time of motorette = T , with $G(t)$=Pr($T \le t$)



- Engineers require that lifetime of the motorettes is at least 3 years in 90% of them. That is, $G^{-1}(0.1) = 3$

- Engineers need to find an tolerance limit $L(data)$ so that $L(data) \le G^{-1}(0.1)$ with probability 95%.

Tolerance limit

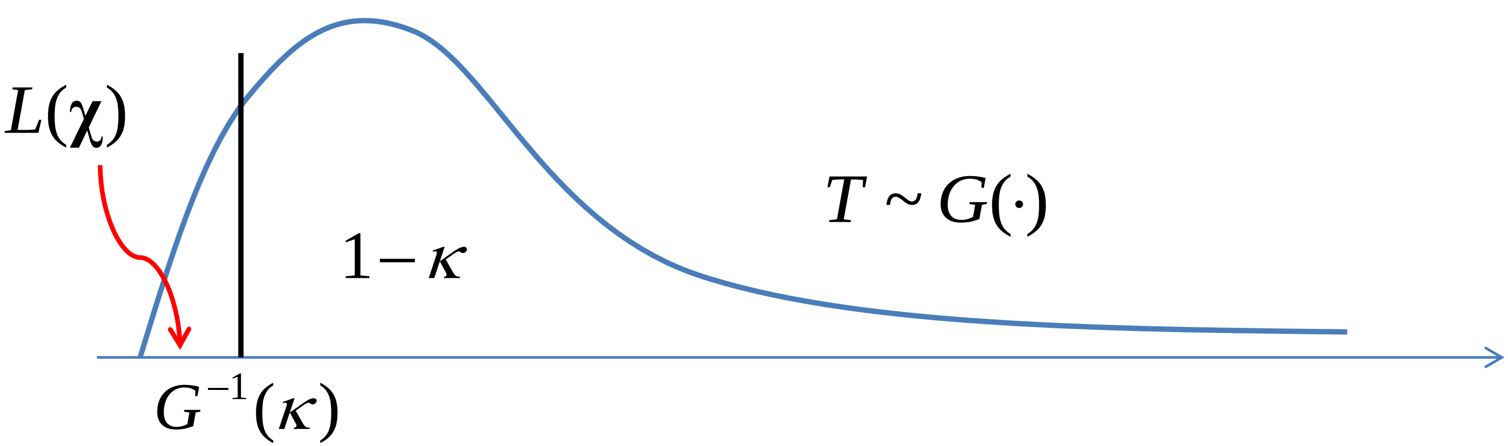
Let $\boldsymbol{\chi} = \{(X_j, \delta_j) : j = 1, \ldots, n\}$,

where $X_j = \min(T_j, C_j), \delta_j = I(T_j \leq C_j), T_j \sim G(\cdot)$

Definition

$L(\boldsymbol{\chi})$ is said to be (1-κ)-content, (1-α)-confidence lower tolerance limit if

$$\Pr(\, L(\boldsymbol{\chi}) \leq G^{-1}(\kappa)\,) = 1 - \alpha$$



Tolerance limit

$L(\chi)$ is obtained by solving

$$\Pr(\, L(\chi) \le G^{-1}(\kappa)\,) = 1 - \alpha$$

(1) Exact method (commonly used*)

e.g., T ~ Exponential, C ~ Type II censoring

$$\Rightarrow \quad L(\chi) = -2\sum_i \delta_i \log(1-\kappa)\hat{\lambda} / \chi_{2\sum_i \delta_i}(1-\alpha)$$

(2) Approximate method (not commonly used*)

e.g., Wald-type approximate tolerance limit

$$\Rightarrow \quad L(\chi) = \hat{G}^{-1}(\kappa)\exp[-z_\alpha V\{\log \hat{G}^{-1}(\kappa)\}]$$

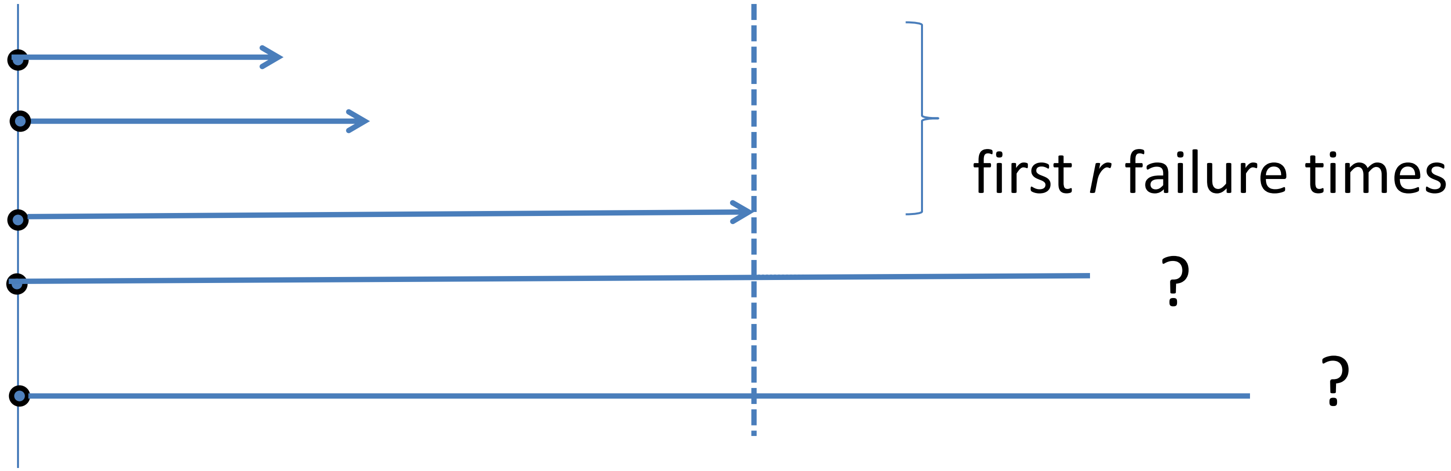
(poor performance, p.214 of Lawless, 2003)

Tolerance limit

Example (Exponential, Type II censoring)

$$T \sim G(t) = 1 - \exp(-\lambda t)$$

$$C = T_{(r)},\ 1 < r < n \qquad T_{(1)} < \cdots < T_{(r)} < \cdots < T_{(n)}$$



$$L(\chi) = \frac{2}{\chi^2_{2r}(1-\alpha)} \{-\log(1-\kappa)\} \sum_j X_j$$

Exact, optimal tolerance limit (Goodman & Madansky, 1962)

Tolerance limit

- Construction of tolerance limits has been mostly discussed under i.i.d. setting.

  ◊ Exponential (Goodman & Madansky, 1962), Exact method

  ◊ Weibull (Bain, 1978), Exact method

  ◊ Generalized Gamma (Bain & Weeks, 1965), Exact method

  * Small sample accuracy: even *n*=100 is still too large in practice

- Only one paper ever discussed tolerance limit in the presence of covariates

  ◊ Jones et al. (1985, Technometrics), Approximate method

  ◊ Above method is not applied for censored data

Tolerance limit

- Many accelerated life-test experiments involve covariates

  T : Lifetime of the product

  $\mathbf{z}$: Stress level (e.g. temperature)

Hours to failure for motorettes and tolerance limits

Temperature	Sample size	Observed failure times	Censoring time	Tolerance Limit
150°C	10	None	8064	?
170°C	10	1764, 2772, 3444, 3542, 3780, 4860, 5196	5448	?
190°C	10	408, 408, 1344, 1440	1680	?
220°C	10	408, 408, 504, 504, 504	528	?

From Kalbfleisch & Prentice (2002), *Statistical Analysis of Failure Time Data*

Tolerance limit

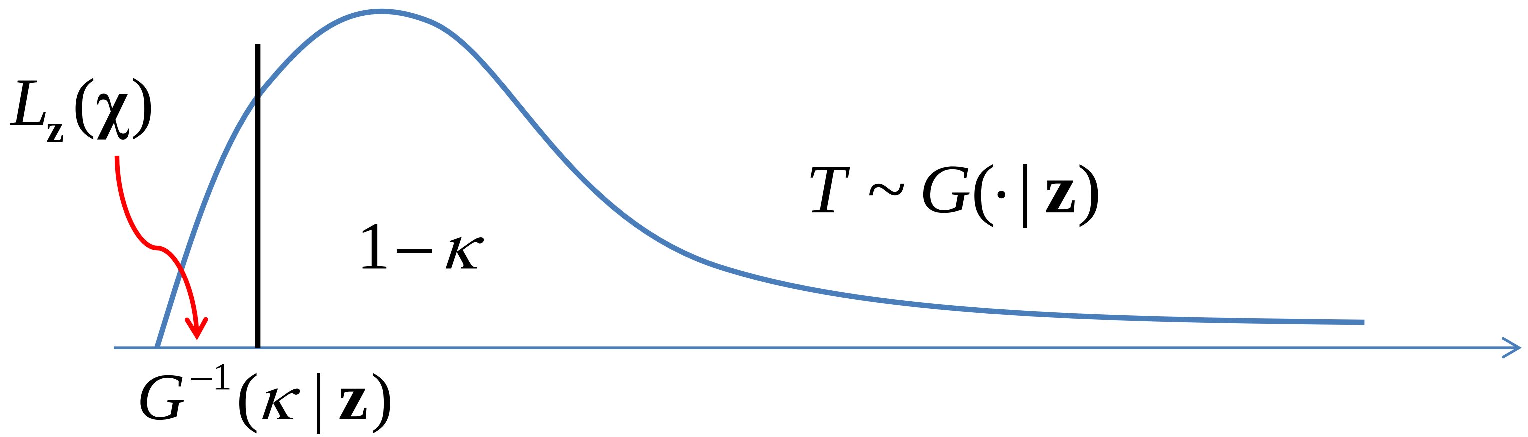
Let $\chi = \{(X_j, \delta_j, \mathbf{z}_i) : j = 1, \ldots, n\}$,

where $X_j = \min(T_j, C_j), \delta_j = I(T_j \le C_j), T_j \sim$ CDF of $G(\cdot \mid \mathbf{z}_i)$

Definition (Jones et al., 1985)

For a given $\mathbf{z}$, $L_{\mathbf{z}}(\chi)$ is said to be (1-κ)-content, (1-α)-confidence lower tolerance limit if

$$\Pr(\, L_{\mathbf{z}}(\chi) \le G^{-1}(\kappa \mid \mathbf{z}) \,) = 1 - \alpha$$



Tolerance limit

- Our goal is to find a tolerance limit $L_z(\chi)$, which allows censoring and covariates
- Small sample accuracy is fundamental in industrial application (e.g., n=25, 50, 75)

Part II

Proposed tolerance limit

Proposed method

Log-location-scale regression model

$$\log(T_j) = \mathbf{Z}'_j\boldsymbol{\beta} + \sigma W_j \qquad W_j \sim \text{known p.d.f.} \;\; f(\cdot)$$

Example (Weibull regression)

$$f(w) = \exp\{w - e^w\} \;\Rightarrow T_j \sim \text{Weibull}\begin{cases} \text{Scale parameter} = \exp(\mathbf{Z}'_j\boldsymbol{\beta}) \\ \text{Shape parameter} = \sigma \end{cases}$$

◊ Most widely used parametric regression in survival analysis (Kalbfleish & Prentice, 2002; Lawless, 2003)

◊ T_j has $\begin{cases} \text{CDF} \quad G(t \mid \mathbf{Z}_j) = F\{\,(\log(t) - \mathbf{Z}'_j\boldsymbol{\beta})/\sigma\,\} \\ \text{Quantile} \quad G^{-1}(\kappa \mid \mathbf{Z}_j) = \exp\{\mathbf{Z}'_j\boldsymbol{\beta} + \sigma F^{-1}(\kappa)\} \end{cases}$

Proposed method

MLE under location-scale regression model

Given $\chi = \{(X_j, \delta_j, \mathbf{z}_i) : j = 1, \ldots, n\}$, maximize

$$L(\boldsymbol{\beta}, \sigma) = \prod_j \left\{ \frac{1}{\sigma} f\left(\frac{\log(X_j) - \mathbf{Z}'_j \boldsymbol{\beta}}{\sigma} \right) \right\}^{\delta_j} \left\{ 1 - F\left(\frac{\log(X_j) - \mathbf{Z}'_j \boldsymbol{\beta}}{\sigma} \right) \right\}^{1-\delta_j}$$

◊Implemented in R (`survreg`) and SAS (`LIFEREG`)

◊ $(\hat{\boldsymbol{\beta}}, \hat{\sigma}) \sim N((\boldsymbol{\beta}, \sigma), \mathbf{i}_n(\hat{\boldsymbol{\beta}}, \hat{\sigma})^{-1})$: via Martingale CLT

◊ MLE for κth Quantile: $\hat{G}^{-1}(\kappa \mid \mathbf{Z}_j) = \exp\{\mathbf{Z}'_j \hat{\boldsymbol{\beta}} + \hat{\sigma} F^{-1}(\kappa)\}$

Proposed method

Problem of interest

Find $L_{\mathbf{z}}(\boldsymbol{\chi})$ such that

$$\Pr(\, L_{\mathbf{z}}(\boldsymbol{\chi}) \le G^{-1}(\kappa \mid \mathbf{z})\,) = 1-\alpha \qquad (1)$$

Under the log-location-scale model

$$G(t \mid \mathbf{Z}_j) = F\{\, (\log(t) - \mathbf{Z}'_j\boldsymbol{\beta}) / \sigma \,\} \qquad (2)$$

How to solve (1) under (2)?

- *Step 1:* Set $L_Z(\boldsymbol{\chi}) = K^Z_{\alpha,n} \cdot \{\hat{G}^{-1}(\kappa \mid \mathbf{Z}) - B^Z_n\}$

  where $\hat{G}^{-1}(\kappa \mid \mathbf{Z}_j) = \exp\{\mathbf{Z}'_j\hat{\boldsymbol{\beta}} + \hat{\sigma}F^{-1}(\kappa)\,\}$

  and $B^Z_n = E\{\hat{G}^{-1}(\kappa \mid \mathbf{Z}) - G^{-1}(\kappa \mid \mathbf{Z})\}$
- *Step 2*: Find $K^Z_{\alpha,n}$ under (1) and (2)

Proposed method

Motivation of the proposed tolerance limit

Consider from a simple setting:

$$\mathbf{Z}'_j = 1, \boldsymbol{\beta} = \beta_0 \in R^1,\ f(w) = \exp\{w - e^w\}, \sigma = 1, C_j = T_{(r)} : \text{Type II censoring}$$

$$\Rightarrow \log(T_j) = \beta_0 + W_j \ \Rightarrow \Pr(T_j > t) = \exp\left(-\frac{t}{e^{\beta_0}}\right)$$

The MLE for the *κ*th quantile is

$$\hat{G}^{-1}(\kappa \mid \mathbf{Z} = 1) = \{-\log(1-\kappa)\}\exp(\hat{\beta}_0) = \{-\log(1-\kappa)\}\sum_j X_j / r$$

On the other hand, the optimal tolerance limit is

$$L(\boldsymbol{\chi}) = \frac{2}{\chi^2_{2r}(1-\alpha)}\{-\log(1-\kappa)\}\sum_j X_j$$

$$= \frac{2r}{\chi^2_{2r}(1-\alpha)}\hat{G}^{-1}(\kappa \mid \mathbf{Z} = 1)$$

$$= \underbrace{K_{\alpha,n}}_{\text{Coefficient}} \times \underbrace{\hat{G}^{-1}(\kappa \mid \mathbf{Z} = 1)}_{\text{Unbiased estimator of quantile}}$$

Proposed Formula

$$L_Z(\chi) = \underbrace{\hat{K}^Z_{\alpha,n}}_{\text{Adjustment term}} \times \underbrace{\{\hat{G}^{-1}(\kappa \mid \mathbf{Z}) - \hat{B}^Z_n\}}_{\text{Unbiased estimator for quantile}} \quad ,$$

where $$\hat{B}^Z_n = (n-1)\left\{\frac{1}{n}\sum_i \hat{G}^{-1}_{(-i)}(\kappa \mid \mathbf{Z}) - \hat{G}^{-1}(\kappa \mid \mathbf{Z})\right\}$$

$$\hat{K}^Z_{\alpha,n} = \exp[-z_{1-\alpha}\{\mathbf{A}' \underbrace{\mathbf{i}_n(\hat{\boldsymbol{\beta}},\hat{\sigma})}_{\text{Observed Fisher}}{}^{-1}\mathbf{A}\}^{1/2}]$$

$$\mathbf{A}' = (\mathbf{Z}', F^{-1}(\kappa))$$

◊ Bias $\hat{B}^Z_n$ is very close to 0 (consistency of MLE).

- Ignore bias: Simply set $\hat{B}^Z_n = 0$, and

$$L^0_Z(\chi) = \hat{K}^Z_{\alpha,n} \times \hat{G}^{-1}(\kappa \mid \mathbf{Z}): \quad \text{Wald-type lower conf. limit}$$

Computation

$$L_Z(\boldsymbol{\chi}) = \hat{K}^Z_{\alpha,n}\{\hat{G}^{-1}(\kappa \mid \mathbf{Z}) - \hat{B}^Z_n\}$$

1. $\hat{G}^{-1}(\kappa \mid \mathbf{Z}_j) = \exp\{\mathbf{Z}'_j\hat{\boldsymbol{\beta}} + \hat{\sigma}F^{-1}(\kappa)\}$

where $(\hat{\boldsymbol{\beta}}, \hat{\sigma})$ obtained by `survreg` in R (package "survival")

2. $\hat{K}^Z_{\alpha,n} = \exp[-z_{1-\alpha}\{\mathbf{A}'\mathbf{i}_n(\hat{\boldsymbol{\beta}}, \hat{\sigma})^{-1}\mathbf{A}\}^{1/2}]$

where $\mathbf{i}_n(\hat{\boldsymbol{\beta}}, \hat{\sigma})$ obtained by `survreg` in R (package "survival")

3. $\hat{B}^Z_n$: Jackknife is easy to program

```
fit=survreg(Surv(X,d)~z,dist="weibull")
hat.beta=fit$coefficients
hat.sigma=fit$scale
hat.var.log=fit$var
```

*Similar computation may be possible in SAS using LIFEREG procedure

Asymptotic validity

Assumption A (Independent Right-censoring):

$$\lim_{\Delta \to 0} \Pr(t \le T_j < t + \Delta, \delta_j = 1;\ j = 1, \ldots, n \mid R(t), \mathbf{Z}_j) / \Delta = \prod_{j \in R(t)} h(t \mid \mathbf{Z}_j)$$

where $R(t) = \{ j ; T_j \ge t, C_j \ge t \}$ and $h(t \mid \mathbf{Z}_j) = -\frac{d}{dt} \log\{1 - G(t \mid \mathbf{Z}_j)\}$

Various censoring types in Engineering: C_j not $\perp$ T_j

(Type I, Type II, progressive Type II, Random)

Theorem: Under Assumption A and if $B_n^Z = O(n^{-1})$,

$$\lim_{n \to \infty} \Pr(L_{\mathbf{z}}(\chi) \le G^{-1}(\kappa \mid \mathbf{z})) = 1 - \alpha$$

Details: Emura and Wang (2010; technometrics)

Simulation setting (I)

- Purpose:

a) Check the validity (correctness of coverage prob.)

b) Effect of bias correction (propose vs. Wald-type)

- Regression model:

--Model 1: T_j follows the Weibull regression model with $\mathbf{Z}_j = (1, z_{1j})'$

--Model 2: T_j follows the Weibull regression model with $\mathbf{Z}_j = (1, z_{1j}, z_{2j})'$

- Censored proportion: 0% or 50%

$$\Pr(C_j < T_j \mid \mathbf{Z}_j) = 0 \text{ or } 0.5$$

*50%: Generate C_j from the same distribution as T_j

Simulation setting (I)

- Data generation:

$$\chi = \{(X_j, \delta_j, \mathbf{z}_i) : j = 1, \ldots, n\}, \quad n = 25, 50, 75, \ldots, 275, 300$$

$$z_{1i} \sim \text{Bernoulli}(0.5), \quad z_{2i} \sim U(0,1)$$

from Model 1 or 2, for 1000 times

- Evaluation criteria = coverage probability

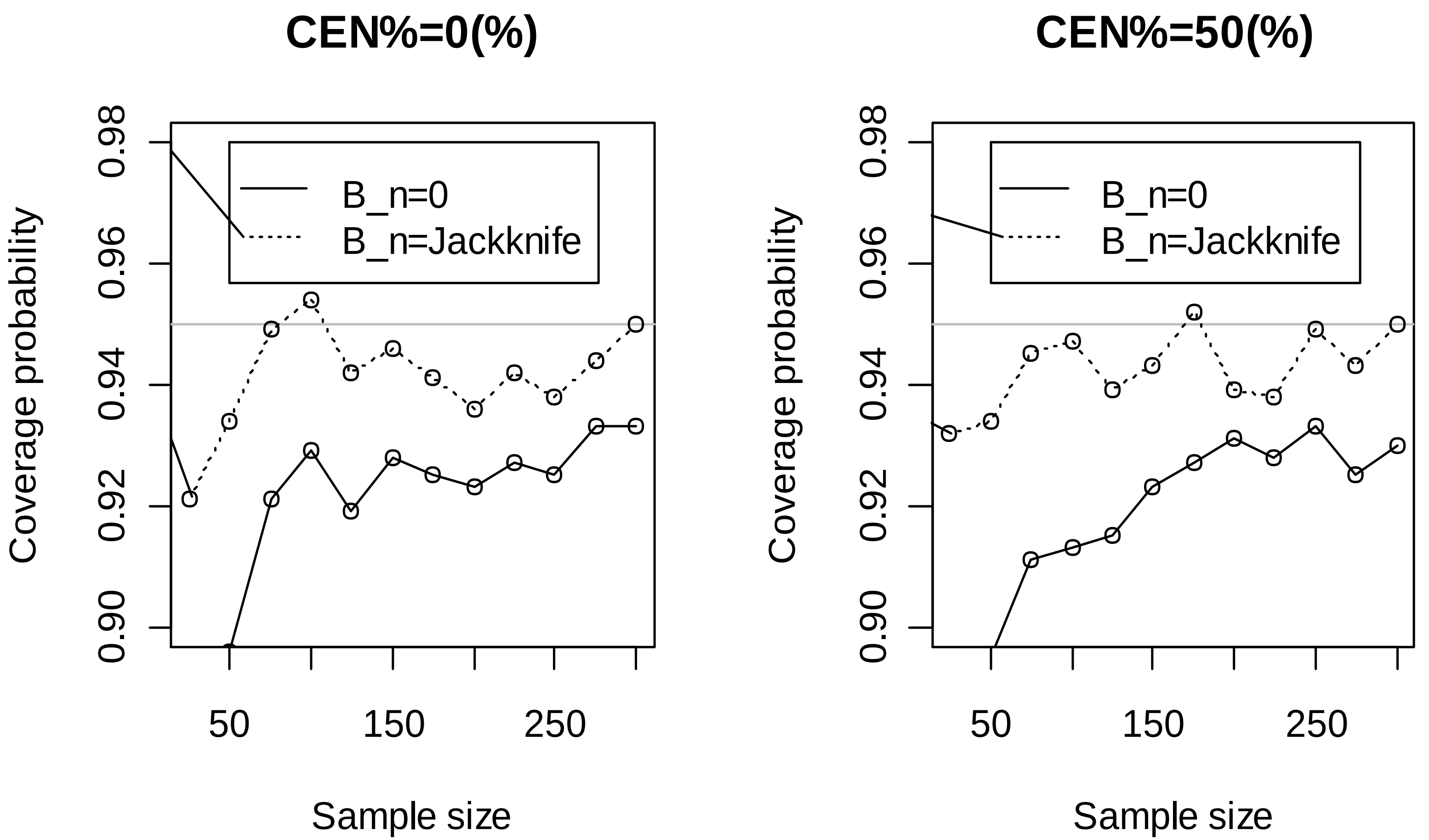
At $\mathbf{z} = 1$ (Model 1) and $\mathbf{z} = (0.5, 0.5)$ (Model 2),

Nominal: $\Pr(L_{\mathbf{z}}(\chi) \le G^{-1}(\kappa \mid \mathbf{z})) = 1 - \alpha = 0.95$

Empirical:

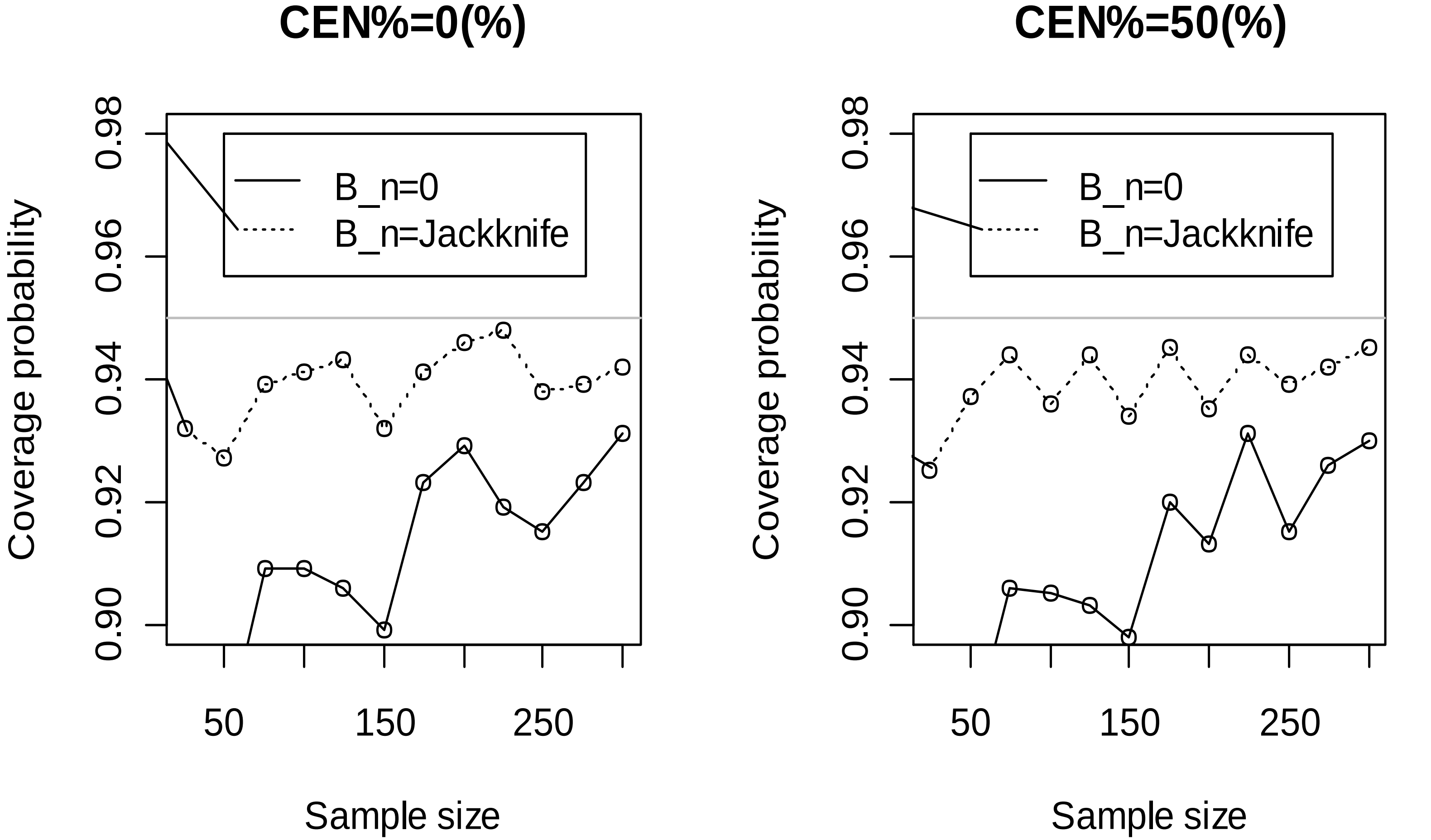
$$\frac{\text{The number of tolerance limit satisfying } \{ L_{\mathbf{z}}(\chi) \le G^{-1}(\kappa \mid \mathbf{z}) \}}{\text{The number of simulation runs (1000)}}$$

Results under Model 1



(2A) Weibull regression with $\mathbf{Z}_i'\boldsymbol{\beta} = \beta_0 + z_{1i}\beta_1$, where $(\beta_0, \beta_1, \sigma) = (0, 1, 1)$

Results under Model 2



2B) Weibull regression with $\mathbf{Z}_i'\boldsymbol{\beta} = \beta_0 + z_{1i}\beta_1 + z_{2i}\beta_2$, where $(\beta_0, \beta_1, \beta_2, \sigma) = (0, 1, 1, 1$

Simulation setting (II)

Purpose: Ours vs. Jones et al. (1986)

Comparison is possible under no censoring and under log-normal regression

- Jones et al.

$$\log(T_j) = \mathbf{Z}'_j\boldsymbol{\beta}^* + \sigma^* W_j^*, \quad E(W_j^*) = 0, \quad Var(W_j^*) = 1, \quad W_j^* \sim f^* : \text{known}$$

- Ours

$$\log(T_j) = \mathbf{Z}'_j\boldsymbol{\beta} + \sigma W_j, \quad W_j \sim f : \text{known}$$

Two regression models are equivalent when

$$f(u) = 1/\sqrt{2\pi}\exp(-u^2/2)$$

Simulation setting (II)

- Jones et al. : $L_Z^*(\chi) = \exp\left[-z_{1-\alpha} \frac{\hat{\sigma}}{(n-p-1)^{1/2}} \frac{\{\tau^2 + z_\kappa^2/2 - z_{1-\alpha}^2 \tau^2/(2n)\}^{1/2}}{1 - z_{1-\alpha}^2/(2n)} - \hat{\sigma}\left\{ z_\kappa - \left(\frac{n}{n-p-1} \right)^{1/2} \frac{z_\kappa}{1 - z_{1-\alpha}^2/(2n)} \right\} \right] \cdot \hat{G}^{-1}(\kappa \mid \mathbf{Z}),$

where $\tau^2 = 1 + \mathbf{z}'\{\sum_j \mathbf{z}_j \mathbf{z}_j'/n\}^{-1}\mathbf{z}$, $\hat{G}^{-1}(\kappa \mid \mathbf{Z}) = \exp\{\mathbf{Z}'\hat{\boldsymbol{\beta}} + \hat{\sigma} \cdot z_\kappa\}$

- Ours : $L_Z(\chi) = \exp[-z_{1-\alpha}\{\mathbf{A}'\mathbf{i}_n(\hat{\boldsymbol{\beta}}, \hat{\sigma})^{-1}\mathbf{A}\}^{1/2}][\hat{G}^{-1}(\kappa \mid \mathbf{Z}) - \hat{B}_n]$

Data generation:

Model 3: T_j follows the log-normal regression model with $\mathbf{Z}_j = (1, z_{1j})'$

Model 4: T_j follows the log-normal regression model with $\mathbf{Z}_j = (1, z_{1j}, z_{2j})'$

*Here, data do not include censoring

Results under Model 3

$1-\alpha=0.95$	$\beta_0=0,\ \beta_1=0,\ \sigma=1$					
	$\mathbf{Z}'=(1,0)$			$\mathbf{Z}'=(1,1)$		
	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones
$n=25$	0.932	0.880	0.908	0.927	0.888	0.958
$n=50$	0.939	0.899	0.904	0.947	0.910	0.963
$n=100$	0.945	0.912	0.911	0.957	0.938	0.974
$n=200$	0.952	0.934	0.916	0.942	0.927	0.968

	$\beta_0=0,\ \beta_1=1,\ \sigma=1$					
	$\mathbf{Z}'=(1,0)$			$\mathbf{Z}'=(1,1)$		
	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones
$n=25$	0.914	0.874	0.885	0.933	0.872	0.957
$n=50$	0.931	0.911	0.896	0.933	0.916	0.961
$n=100$	0.934	0.925	0.895	0.942	0.935	0.964
$n=200$	0.948	0.930	0.906	0.955	0.927	0.978

-NOTE: Coverage probabilities for 0.90-content, $100(1-\alpha)\%$-confidence tolerance-limits are calculated under the lognormal regression models:

$$\log(T_j)=\beta_0+\beta_1 z_{1j}+\sigma W_j\,.$$

Results under Model 4

$1-\alpha = 0.95$				$\beta_0 = 0, \beta_1 = 0, \beta_2 = 0, \sigma = 1$					
	$\mathbf{Z}' = (1, 0, 0)$			$\mathbf{Z}' = (1, 0.5, 0)$			$\mathbf{Z}' = (1, 0.5, 0.5)$		
	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones
$n = 25$	0.935	0.871	0.818	0.949	0.875	0.892	0.919	0.857	0.960
$n = 50$	0.941	0.924	0.828	0.949	0.918	0.881	0.933	0.894	0.967
$n = 100$	0.959	0.923	0.820	0.960	0.939	0.888	0.944	0.910	0.969
$n = 200$	0.948	0.943	0.820	0.947	0.934	0.886	0.944	0.937	0.970
				$\beta_0 = 0, \beta_1 = 1, \beta_2 = 1, \sigma = 1$					
	$\mathbf{Z}' = (1, 0, 0)$			$\mathbf{Z}' = (1, 0.5, 0)$			$\mathbf{Z}' = (1, 0.5, 0.5)$		
	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones	$L_Z(\chi)$	$L_Z^0(\chi)$	Jones
$n = 25$	0.946	0.891	0.836	0.938	0.894	0.879	0.910	0.871	0.963
$n = 50$	0.942	0.905	0.824	0.959	0.909	0.905	0.943	0.898	0.980
$n = 100$	0.946	0.922	0.820	0.939	0.923	0.870	0.956	0.901	0.973
$n = 200$	0.954	0.932	0.829	0.943	0.913	0.874	0.946	0.918	0.980

NOTE: Empirical coverage probabilities for 0.90-content, $100(1-\alpha)\%$-confidence tolerance limits are calculated under the lognormal regression models:

$$\log(T_j) = \beta_0 + \beta_1 z_{1j} + \beta_2 z_{2j} + \sigma W_j .$$

Data analysis

T : Lifetime of the motorettes

$\mathbf{z}$: Temperature

Hours to failure for motorettes and tolerance limits

Temperature	Sample size	Observed failure times	Censoring time	Tolerance Limit
150°C	10	None	8064	?
170°C	10	1764, 2772, 3444, 3542, 3780, 4860, 5196	5448	?
190°C	10	408, 408, 1344, 1440	1680	?
220°C	10	408, 408, 504, 504, 504	528	?

From Kalbfleisch & Prentice (2002), *Statistical Analysis of Failure Time Data*

Data analysis

T : Lifetime of the motorettes

$\mathbf{z}$: Temperature

- $z_j = 1000/(273.2 + {}^\circ\mathrm{C})$: see Nelson & Hahn (1972)

$$\log(T_j) = \beta_0 + \beta_1 z_j + \sigma W_j$$

- Error distribution = Generalized Gamma

$$W_j \sim f(w) = \frac{m_1^{m_1}}{\Gamma(m_1)} \exp(m_1 w - m_1 e^w)$$

$$q = \begin{cases} 0 & \text{log-normal} \\ 1 & \text{Weibull} \end{cases}, \text{ where } \quad q = m_1^{-1/2}.$$

Data analysis

- Likelihood ratio test (computed by nlm in R)

$$q=\begin{cases} H_0: q=0 & \text{log-normal} \quad (\text{p-value}=0.310) \\ H_0: q=1 & \text{Weibull} \quad (\text{p-value}=0.018) \end{cases}$$

Hence, choose log-normal model.

$$\log(T_j)=\beta_0+\beta_1 z_j+\sigma W_j,$$

$$\text{where} \quad W_j \sim f(u)=1/\sqrt{2\pi}\exp(-u^2/2).$$

- Estimates (computed by survreg in R)

$$\hat{\beta}_0=-13.36, \quad \hat{\beta}_1=9.730, \quad \hat{\sigma}=0.325$$

Data analysis

- 0.90-content, 0.95-confidence Tolerance limit

Data and results for hours to failure for motorettes and tolerance limits

Temperature	Sample size	Observed failure times	Censoring time	Tolerance Limit
150°C	10	None	8064	5193.9
170°C	10	1764, 2772, 3444, 3542, 3780, 4860, 5196	5448	1977.2
190°C	10	408, 408, 1344, 1440	1680	778.3
220°C	10	408, 408, 504, 504, 504	528	203.9

At 170°C, empirical content = 9/10 = 0.90

Data analysis

Tolerance limit tells what information ?

Suppose that the engineers need to guarantee that, with 95% confidence, 90% of the motorettes can run at least 2000 hours at 170 ℃.

- The engineers can guarantee the above statement since the tolerance limit at 170°C is 1977.2, which precedes 2000.

Summary: Proposed method

- We proposed an approximate tolerance limit under the log-location-scale regression model that allows <u>censoring</u>

- Good small sample accuracy e.g., n=50 or 75

  (the bias-correction technique is the key)

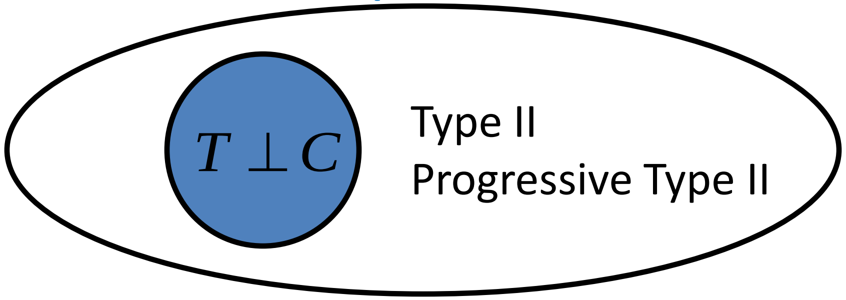
- Even without censoring, the proposed method has better performance than the existing one (simulation).

- Easy to compute (e.g., in R). R code is available from me.

Future work (1)

Present work:

Parametric log-location-scale model with "independent censoring"



Extension to different models:

Semi-parametric AFT or transformation model, Piecewise-constant hazard model, etc.

Challenge:

- The small sample accuracy, which is fundamental in engineering & industry
- "Real" independence between lifetime and censoring does not hold in engineering (Type II, progressive Type II)

Future work (2)

Simultaneous tolerance limits:

Find tolerance limits $\{L_{Z_l}(\chi); l = 1, \ldots, L\}$ that satisfy

$$\Pr(L_{Z_l}(\chi) \le G^{-1}(\kappa \,|\, \mathbf{Z}_l); l = 1, \ldots, L) = 1 - \alpha$$

Data and results for hours to failure for motorettes and tolerance limits

Temperature	Sample size	Observed failure times	Censoring time	Tolerance Limit
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190°C	10	408, 408, 1344, 1440	1680	778.3
220°C	10	408, 408, 504, 504, 504	528	203.9

Thank you for your kind attention