Seminar at 國立清華大學統計所
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Semiparametric inference for an accelerated failure time model with dependent truncation

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Progress of the paper

- **2009-2010:** Paper developed as my post-doc project under Prof. Weijing Wang
- **2011-2013**:

  Presented at 第20回南區統計研會, 國立中正大學

  Rejected by JRSSB, Biometrika, Sinica

  Journal of Multivariate Analysis

  (all do not recognize our idea of a new model)

  (I myself feel difficult to interpret the new method)
- **Submit on 2013/10 –> Now** : the 2nd major revision at

  Annals of the Institute of Statistical Mathematics (AISM)

Outlines

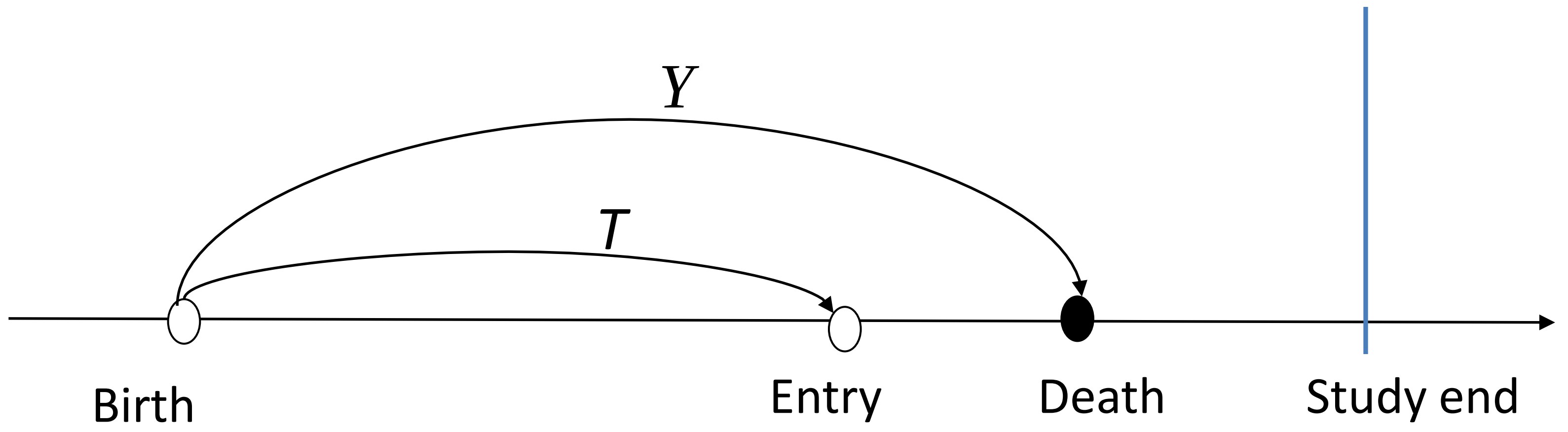
Part I: Review

- Truncated data - Channing house data -
- Regression method - AFT model -

Part II: Proposed method

- Proposed method
- Estimation procedure
- Asymptotic theory
- Simulation and data analysis
- Conclusion

Left-truncated survival data



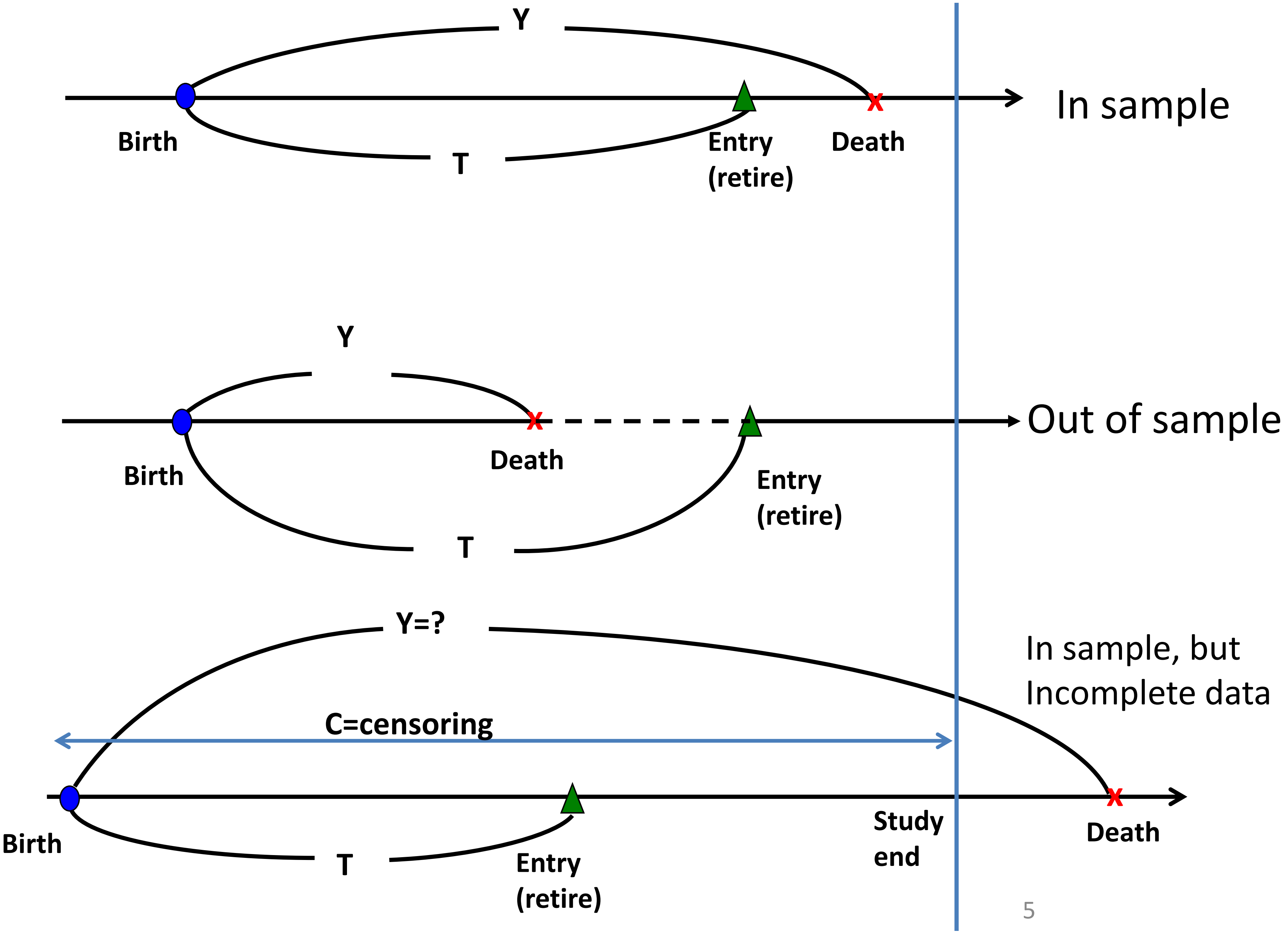
Channing House data (Hyde, 1980)

Channing house is a retirement center in California

- 462 elderly residents (97 men + 365 women)
- Age at entry = T
- Age at death = Y (possibly right-censored)

Left-truncation criterion:

$$T \leq Y \quad \Rightarrow \quad \text{Sample is available}$$


Y
Birth
T
Entry
(retire)
Death
In sample
Y
Birth
Death
T
Entry
(retire)
Out of sample
Y=?
C=censoring
Birth
T
Entry
(retire)
Study
end
Death
In sample, but
Incomplete data

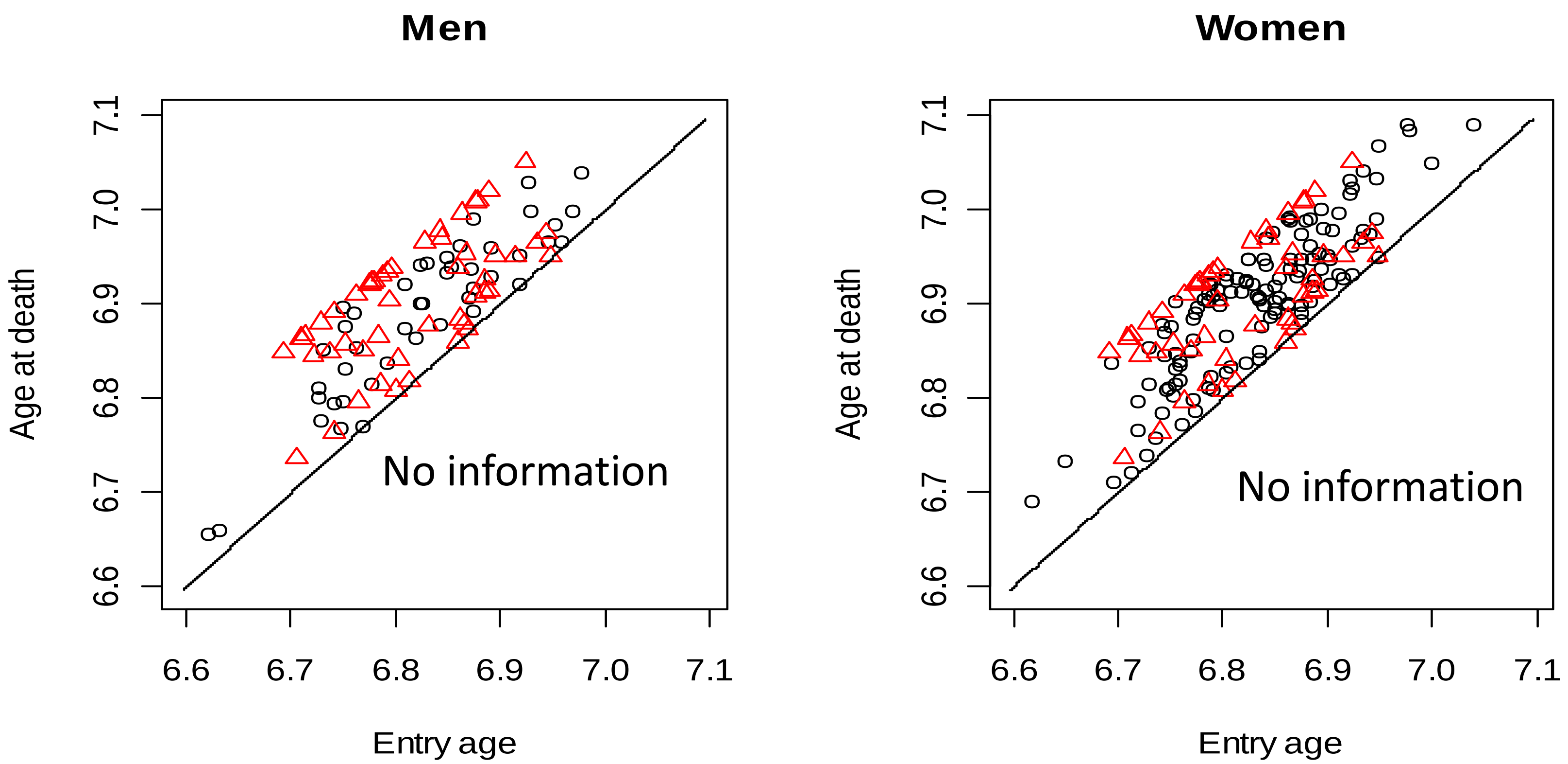
Left-truncation data

- Longer survivors have higher chance to be sampled
  - ➔ Observed lifetimes are longer than the random sampling from **the target population**
  - ➔ Ignoring left-truncation produces bias (often very serious bias)
- Many people ignore truncation or do not realize

  The famous textbook for survival analysis

  Klein & Moeschberger (2003)

  Answer to Exercise 3.7: <--ignore left-truncation
- Truncation usually occurs when
  1. scale of survival is “age” (birth <---> death).
  2. the target is “age at disease”.



Δ: Censored individual

O: Died individual

- Hyde (1980) assumed:
  knowing the person's entry age will provide no additional information about prospects for survival
- That is, $T \perp Y \mid \text{gender}$
  (*T*: Age at entry; *Y*: Age at death)

- Existing methods for left-truncated and right-censored data

➔ Rely on *independent truncation assumption*

Hyde (1977, 1980)

➔ One-sample log-rank test

Wang, Jewell & Tsai (1986)

➔ Product-limit estimator

Lai and Ying (1991):

➔ Rank regression

He and Yang (1998):

➔ Estimating truncation probability

Su and Wang (2012):

➔ Joint models with longitudinal covariates

Truncation data with covariates

Left-truncated and right-censored data:

- Y^* : Log-lifetime
- T : Log-truncation time
- C : Log-censoring time
- $\mathbf{X}$: p-dimensional covariate

Left-truncation:

A pair $(T, Y, \Delta, \mathbf{X})$ is observed only when $T \leq Y^*$, , where $Y = Y^* \wedge C$, $\Delta = I(Y^* \leq C)$

*If $T = -\infty$, this is usual right-censored data with covariates

➔ fit Cox regression (1972) or AFT regression(Tsiatis 1990)

Truncation data with covariates

Observed data:

$\{(T_i, Y_i, \Delta_i, \mathbf{X}_i); (i=1, ..., n)\}$ subject to $T_i \leq Y_i$

AFT regression (Lai and Ying, 1991 AS)

Semiparametric accelerated failure time (AFT) model:

$$Y^* = \boldsymbol{\beta}_0'\mathbf{X} + \varepsilon,$$

$$*Y^* = \text{Log-lifetime}$$

$$*\varepsilon = \text{Error (p.d.f of } \varepsilon \text{ is unspecified)}$$

$$*\boldsymbol{\beta} \in R^p$$

- X = 1 (male), X= 0 (female)

➔ β_0 : The gender difference of mean lifetime in the scale of log(lifetime)

Lai & Ying's AFT regression

Log-rank type estimating equation: $E_{\boldsymbol{\beta}}\{\mathbf{X}_i - E_{\boldsymbol{\beta}}\mathbf{X}_i\} \approx 0$

$$\mathbf{U}_n(\boldsymbol{\beta}) = \sum_{i=1}^{n} \Delta_i \phi_i(\boldsymbol{\beta}) \left\{ \mathbf{X}_i - \frac{1}{R_i(\boldsymbol{\beta})} \sum_j \mathbf{X}_j I(e_j^T(\boldsymbol{\beta}) \le e_i^Y(\boldsymbol{\beta}) \le e_j^Y(\boldsymbol{\beta})) \right\},$$

where

$e_i^T(\boldsymbol{\beta}) = T_i - \boldsymbol{\beta}'\mathbf{X}_i$, $e_i^Y(\boldsymbol{\beta}) = Y_i - \boldsymbol{\beta}'\mathbf{X}_i$: Residual

$R_i(\boldsymbol{\beta}) = \sum_j I(e_j^T(\boldsymbol{\beta}) \le e_i^Y(\boldsymbol{\beta}) \le e_j^Y(\boldsymbol{\beta}))$: Number at - risk

U-statistic form under the Gehan weight $\phi_i(\boldsymbol{\beta}) = R_i(\boldsymbol{\beta})$

$$\mathbf{U}_n^G(\boldsymbol{\beta}) = \sum_{i=1}^{n} \sum_{j=1}^{n} \Delta_i (\mathbf{X}_i - \mathbf{X}_j) I\{ e_j^T(\boldsymbol{\beta}) \le e_i^Y(\boldsymbol{\beta}) \le e_j^Y(\boldsymbol{\beta}) \}$$

$\hat{\boldsymbol{\beta}}$: $\mathbf{U}_n^G(\boldsymbol{\beta}) = \mathbf{0}$ (Lai & Ying (1991) estimator)

Lai & Ying's AFT regression

Assumptions of Lai & Ying (1991):

$$\begin{cases} Y^* = \boldsymbol{\beta}_0'\mathbf{X} + \varepsilon \\ (T, C, \mathbf{X}) \perp \varepsilon \quad \cdots (\mathrm{A}) \end{cases}$$

⇐ Interpreted as "Independent truncation"

Why (A) is independent truncation?

By (A), $Y^* - \boldsymbol{\beta}_0'\mathbf{X} \perp T$.

After adjusting for the effect of $\mathbf{X}$, the truncation variable T contains no information on survival Y^*

Motivating Example: This model satisfy (A) only under $\rho = 0$

$$\begin{bmatrix} Y^* \\ T \end{bmatrix} \sim N\left(\begin{bmatrix} \boldsymbol{\beta}_0'\mathbf{X} \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right), \quad C \sim N(0,1)$$

Independent truncation hold?

Testing quasi-independence

$$H_0: \ \Pr(L = l, Y = y \mid L \le Y) \propto dF_L(l) dF_Y(y)$$

Available test statistics:

1. Chen et al. (1996 JASA)

   - Based on the conditional Peason-correlation

2. Tsai (1990 Biometrika); Martin & Betensky (2005 JASA)

   -Based on the conditional Kendall's tau

3. Emura & Wang (2010 JMVA) - Based on weighted-logrank test

   (Optimal weight choice)

Testing quasi-indpendence

$$\mathrm{H}_0: \ \Pr(L=l, Y=y \mid L \le Y) \propto dF_L(l)dF_Y(y)$$

is rejected at 5 % level

Table 4 of Emura and Wang (2010, JMVA).

Tests of quasi-independence for the Channing House data.

	Logrank test $L_{\rho=1}$	Tsai test	Marting & Betensky test
P-value	0.048	0.043	0.040

- Quasi-independence is questionable

  → Lai & Ying's AFT regression estimate

  may be biased (later shown via simulations)
- In Channing house data, the truncation (entry age) may be dependent on survival.

  → Motivate modeling for dependent truncation

  Chaieb, Rivest, Abdous (2006 *Biometrika*)

  Beaudoin and Lakhal-Chaieb (2008 *Stat. Med.*),

  Emura and Wang (2010 *JMVA*)

  Emura and Konno (2010 *Stat Papers*; 2012 *CSDA*)

  Emura, Wang and Hung (2011 *Sinica*)

  Emura and Wang (2012 *JMVA*)

  Ding (2012 *Lifetime Data Analysis*)

  Emura and Murotani (2014 *Test*, in revision)

  (All the work is for iid case !)

Part II: Proposed method

Proposed method

Proposed model (Semi-par AFT with dependent truncation):

$$\begin{cases} Y^* = \boldsymbol{\beta}_0'\mathbf{X} + \gamma_0 T + \varepsilon \\ (T, C, \mathbf{X}) \perp \varepsilon \end{cases} \quad \text{....} \quad \text{(B)}$$

NOTE: Special case of $\gamma_0 = 0$, ➔ Lai & Ying's AFT model

Eample: Bivariate normal model

$$\begin{bmatrix} Y^* \\ T \end{bmatrix} \sim N\left(\begin{bmatrix} \boldsymbol{\beta}_0'\mathbf{X} \\ \mu_L \end{bmatrix}, \begin{bmatrix} \sigma_{Y^*}^2 & \rho\sigma_{Y^*}\sigma_T \\ \rho\sigma_{Y^*}\sigma_T & \sigma_T^2 \end{bmatrix} \right), \quad C \sim N(1, 1)$$

$$\Rightarrow Y^* \mid \mathbf{X}, T \sim N\left(\boldsymbol{\beta}_0'\mathbf{X} + \rho\frac{\sigma_{Y^*}}{\sigma_T}(T - \mu_L), \sigma_{Y^*}^2(1-\rho^2) \right)$$

This model satisfy (B) with

$$\gamma = \rho\frac{\sigma_{Y^*}}{\sigma_T}, \quad \varepsilon \sim N\left(-\rho\frac{\sigma_{Y^*}}{\sigma_T}\mu_L, \sigma_{Y^*}^2(1-\rho^2) \right)$$

Proposed method

Interpretation of our model:

$$Y^* = \boldsymbol{\beta}_0'\mathbf{X} + \gamma_0 T + \varepsilon, \quad \varepsilon \sim f_\varepsilon$$

NOTE:

If $\gamma_0 \neq 0$ then the Lai & Ying model do not hold in general

$$Y^* \neq \boldsymbol{\beta}_0'\mathbf{X} + \varepsilon_0,$$

Indeed, the nonlinear model

$$f_{Y^*}(y \mid \mathbf{X}) = \int f_{Y^*}(y \mid t, \mathbf{X}) f_T(t \mid \mathbf{X}) dt = \int f_\varepsilon(y - \boldsymbol{\beta}_0'\mathbf{X} - \gamma_0 t) f_T(t \mid \mathbf{X}) dt$$

Modeling this part:

Provocatively utilize truncation information
in statistical modeling

Estimation procedure

Setting:

- Model : $\begin{cases} Y^* = \boldsymbol{\beta}_0'\mathbf{X} + \gamma_0 T + \varepsilon \\ (T, C, \mathbf{X}) \perp \varepsilon \end{cases}$ .... (B)
- Left-truncated & right-censored data :

$$\{(T_i, Y_i, \Delta_i, \mathbf{X}_i); (i = 1, \ldots, n)\} \text{ subject to } T_i \le Y_i$$

Interest:

1) Joint estimation of $(\boldsymbol{\beta}_0', \gamma_0)$

2) Estimation of $S_\varepsilon(t) = \Pr(\varepsilon > t)$

Estimating equations for

a) $\boldsymbol{\beta}_0$ ➔ Inverting the log-rank test statistics

b) γ_0 ➔ Inverting the quasi-independence test statistics

Estimation procedure

Residual transformation:

$$\begin{cases} \varepsilon_i^Y(\boldsymbol{\beta},\gamma) = Y_i - \boldsymbol{\beta}'\mathbf{X}_i - \gamma T_i & \text{residual log-surival} \\ \varepsilon_i^T(\boldsymbol{\beta},\gamma) = T_i - \boldsymbol{\beta}'\mathbf{X}_i - \gamma T_i & \text{residual log-truncation} \end{cases}$$

a) Log-rank estimating equation:

By assumption (B), $H_0: Y^* - \boldsymbol{\beta}_0'\mathbf{X} - \gamma_0 T \perp \mathbf{X}$ is true.

$$\mathbf{S}_n^{Logrank}(\boldsymbol{\beta},\gamma)$$

$$= -\sum_{i<j}(\mathbf{X}_i - \mathbf{X}_j)\operatorname{sgn}\{(\varepsilon_i^Y(\boldsymbol{\beta},\gamma) - \varepsilon_j^Y(\boldsymbol{\beta},\gamma)\} I\{\breve{\varepsilon}_{ij}^T(\boldsymbol{\beta},\gamma) \le \tilde{\varepsilon}_{ij}^Y(\boldsymbol{\beta},\gamma)\} O_{ij}(\boldsymbol{\beta},\gamma)$$

b) Quasi-independence estimating equation:

By assumption (B), $H_0: Y^* - \boldsymbol{\beta}_0'\mathbf{X} - \gamma_0 T \perp T - \boldsymbol{\beta}_0'\mathbf{X} - \gamma_0 T$ is true

$$S_n^{Kendall}(\boldsymbol{\beta},\gamma)$$

$$= \sum_{i<j}\operatorname{sgn}\{(\varepsilon_i^T(\boldsymbol{\beta},\gamma) - \varepsilon_j^T(\boldsymbol{\beta},\gamma))(\varepsilon_i^Y(\boldsymbol{\beta},\gamma) - \varepsilon_j^Y(\boldsymbol{\beta},\gamma))\} I\{\breve{\varepsilon}_{ij}^T(\boldsymbol{\beta},\gamma) \le \tilde{\varepsilon}_{ij}^Y(\boldsymbol{\beta},\gamma)\} O_{ij}(\boldsymbol{\beta},\gamma).$$

Martin & Betensky type statistic (2005 JASA)

Estimation procedure

Numerical solution :

$$\begin{pmatrix} \hat{\boldsymbol{\beta}} \\ \hat{\gamma} \end{pmatrix} : \begin{cases} \mathbf{0} = \mathbf{S}_n^{Logrank}(\boldsymbol{\beta}, \gamma) \\ 0 = S_n^{Kendall}(\boldsymbol{\beta}, \gamma) \end{cases}$$

⬅ Non-monotonic step functions

Use the simplex algorithm (Nelder and Mead, 1965) to find a minimum of

$$M(\boldsymbol{\beta}, \gamma) = \{ \| \mathbf{S}_n^{Logrank}(\boldsymbol{\beta}, \gamma) \|_2 + | S_n^{Kendall}(\boldsymbol{\beta}, \gamma) |^2 \} / n^2$$

Implementation available as "R optim routine".

➔ Not easy to choose good initial in automatic way.

(one of question raised by referees)

NOTE: Newton-Raphson, bisection method, and linear programing (Jin, Lin and Wei, 2003) do not work:

Theorem 2 (Asymptotic normality) :

$$n^{1/2}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0, \hat{\gamma} - \gamma_0) \quad \to \quad N(\mathbf{0}, \mathbf{A}_0{}^{-1}\mathbf{B}_0\mathbf{A}_0{}^{-1})$$

Difficulty : $\mathbf{A}_0 \equiv \partial\Phi(F; \boldsymbol{\beta}, \gamma) / \partial(\boldsymbol{\beta}, \gamma) |_{\boldsymbol{\beta}_0, \gamma_0}$

not differentiable if plug-in $F = F_n$

Kernel-based empirical variance estimator :

$$\hat{\mathbf{A}}_0 = [\hat{\mathbf{A}}_0^{(1)}(\hat{\boldsymbol{\beta}}, \hat{\gamma}; b_1), \cdots, \hat{\mathbf{A}}_0^{(p+1)}(\hat{\boldsymbol{\beta}}, \hat{\gamma}; b_{p+1})]$$

$$\hat{\mathbf{A}}_0^{(k)}(\boldsymbol{\beta}, \gamma; b_k) = \int_{u \neq 0} \frac{1}{u} \Phi(F_n; \beta_1, \ldots, \beta_k + u, \ldots, \beta_p, \gamma) \frac{1}{b_k} K\left(\frac{u}{b_k}\right) du, \quad k = 1, \ldots, p$$

$$\hat{\mathbf{B}}_0 = \sum_{j=1}^{n} \phi_{F_n}(T_j, Y_j, \delta_j, \mathbf{X}_j; \hat{\boldsymbol{\beta}}, \hat{\gamma}) \phi_{F_n}(T_j, Y_j, \delta_j, \mathbf{X}_j; \hat{\boldsymbol{\beta}}, \hat{\gamma})' / n$$

- Optimal bandwidth

$$MSE\{\,\hat{\mathbf{A}}_0^{(k)}(\hat{\boldsymbol\beta},\hat\gamma;b_k)\,\}=Var\{\,\hat{\mathbf{A}}_0^{(k)}(\hat{\boldsymbol\beta},\hat\gamma;b_k)\,\}+Bias[\hat{\mathbf{A}}_0^{(k)}(\boldsymbol\beta,\gamma;b_k)]^2$$

$$\approx\frac{1}{nb_k}E\left[\frac{\partial}{\partial\beta_k}\phi_F\{\,(T_j,Y_j,\delta_j,\mathbf{X}_j);\boldsymbol\beta_0,\gamma_0\,\}\right]^2+\frac{b_k^4}{36}\left[\frac{\partial^3}{\partial\beta_k^3}\Phi(\,F;\boldsymbol\beta_0,\gamma_0\,)\right]^2\mu_2(K)^2\,,$$

The bandwidth that minimizes the preceding expression becomes

$$b_k^{opt}=\left(9E\left[\frac{\partial}{\partial\beta_k}\phi_F\{\,(T_j,Y_j,\delta_j,\mathbf{X}_j);\boldsymbol\beta_0,\gamma_0\,\}\right]^2\left[\frac{\partial^3}{\partial\beta_k^3}\Phi(\,F;\boldsymbol\beta_0,\gamma_0\,)\right]^{-2}\mu_2(K)^{-2}\right)^{1/5}\frac{1}{n^{1/5}}$$

- In practice, use Silverman's reference bandwidth (Sheather, 2004) under the normal kernel

$$\hat{b}_k=0.5\min(\,S_k,\,\mathrm{IQR}_k\,/1.34\,)n^{-1/5}$$

Estimation procedure

<u>Estimation of error distribution:</u>

Target: $S_\varepsilon(t) = \Pr(\varepsilon > t) = \prod_{u \le t}\left\{1 - \frac{\Pr(\varepsilon = u)}{\Pr(\varepsilon \ge u)}\right\}$

$$\begin{cases} \varepsilon_{\mathrm{i}} \approx \varepsilon_i^Y(\hat{\boldsymbol{\beta}}, \hat{\gamma}) = Y_i - \hat{\boldsymbol{\beta}}'\mathbf{X}_i - \hat{\gamma}T_i & \text{residual log-surival} \\ \varepsilon_i^T(\hat{\boldsymbol{\beta}}, \hat{\gamma}) = T_i - \hat{\boldsymbol{\beta}}'\mathbf{X}_i - \hat{\gamma}T_i & \text{residual log-truncation} \end{cases}$$

➔ $\{\, (\, \varepsilon_i^T(\hat{\boldsymbol{\beta}}, \hat{\gamma}), \varepsilon_i^Y(\hat{\boldsymbol{\beta}}, \hat{\gamma}), \Delta_i\,);\, i = 1, \ldots, n\, \}$ subject to $\varepsilon_i^T(\hat{\boldsymbol{\beta}}, \hat{\gamma}) \le \varepsilon_i^Y(\hat{\boldsymbol{\beta}}, \hat{\gamma})$

: Left-truncated and right-censored data for ε

Product-limit estimator for with non-iid samples

$$\hat{S}_\varepsilon(t; \hat{\boldsymbol{\beta}}, \hat{\gamma}) = \prod_{u \le t}\left\{1 - \frac{\sum_j I(\varepsilon_j^Y(\hat{\boldsymbol{\beta}}, \hat{\gamma}) = u, \Delta_i = 1)}{\sum_j I(\varepsilon_j^T(\hat{\boldsymbol{\beta}}, \hat{\gamma}) \le u \le \varepsilon_j^Y(\hat{\boldsymbol{\beta}}, \hat{\gamma}))}\right\}$$

Theorem 3 *Under Assumptions I, II and V, the product-limit estimator*

$\hat{S}_\varepsilon(t; \hat{\boldsymbol{\beta}}, \hat{\gamma})$ *converges in probability to* $S_\varepsilon(t)$, *uniformly over* $t \in [a, b]$.

- Asymptotic normality is remain to be done

Simulation

Model : $\begin{bmatrix} Y^* \\ T \end{bmatrix} \sim N\left(\begin{bmatrix} \beta_0 X \\ -1 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right), \quad C \sim N(1,1), \quad X \sim U(0,1)$

This induces the linear regression model

$$Y^* = \beta_0 X + \gamma_0 T + \varepsilon, \qquad \varepsilon \sim N(\gamma_0, 1 - \gamma_0{}^2)$$

$$\text{, where } \rho = \gamma_0 \quad \text{and} \quad S_\varepsilon(t) = 1 - \Phi\left\{ \frac{t - \gamma_0}{(1 - \gamma_0^2)^{1/2}} \right\}$$

Parameter configurations:

	1	2	3	4	5	6
(β_0, γ_0)	(0, -0.5)	(0, 0)	(0, 0.5)	(1, -0.5)	(1, 0)	(1, 0.5)
$\Pr(T \le Y^*)$	0.72	0.76	0.84	0.80	0.85	0.84
$\Pr(C < Y^* \mid T \le Y^*)$	0.30	0.27	0.23	0.41	0.39	0.34

Simulation

1. Generate data

$(T_i, Y_i, \Delta_i, \mathbf{X}_i)$, subject to $T_i \le Y_i$, for $i = 1, \ldots, n$

from model :

$$Y^* = \beta_0 X + \gamma_0 T + \varepsilon, \quad \text{where } \rho = \gamma_0 \quad \text{and} \quad \varepsilon \sim N(\gamma_0, 1 - {\gamma_0}^2)$$

2. Estimate parameters

$$(\hat{\beta}, \hat{\gamma}) = \arg\min_{(\beta,\gamma)} M(\beta, \gamma):$$

using R optim with the true initial value (β_0, γ_0)

$$\hat{S}_\varepsilon(t; \hat{\boldsymbol{\beta}}, \hat{\gamma})$$

3. Variance estimation & confidence interval

$$[\ \hat{\beta}_k - z_{\alpha/2} se(\hat{\beta}_k),\ \hat{\beta}_k + z_{\alpha/2} se(\hat{\beta}_k)\ ]$$

Table 1 Simulation results for the proposed estimator $(\hat{\beta}, \hat{\gamma})$ †.

		Estimation of β_0				Estimation of γ_0			
(β_0, γ_0)	n	Bias($\hat{\beta}$)	SD($\hat{\beta}$)	E{$se(\hat{\beta})$}	95%Cov	Bias($\hat{\gamma}$)	SD($\hat{\gamma}$)	E{$se(\hat{\gamma})$}	95%Cov
(0, -0.5)	150	0.003	0.333	0.352	0.926	-0.018	0.169	0.165	0.932
	300	-0.007	0.234	0.233	0.944	-0.002	0.112	0.112	0.944
(0, 0)	150	0.005	0.392	0.419	0.946	-0.008	0.173	0.169	0.942
	300	-0.002	0.275	0.287	0.954	0.002	0.113	0.116	0.946
(0, 0.5)	150	0.000	0.320	0.370	0.964	-0.001	0.124	0.132	0.968
	300	-0.006	0.238	0.253	0.958	0.000	0.085	0.087	0.962
(1, -0.5)	150	0.010	0.320	0.359	0.954	-0.021	0.163	0.155	0.950
	300	-0.003	0.234	0.246	0.952	-0.005	0.116	0.108	0.940
(1, 0)	150	0.041	0.411	0.430	0.954	-0.015	0.159	0.160	0.966
	300	0.012	0.284	0.289	0.950	-0.008	0.105	0.108	0.958
(1, 0.5)	150	0.012	0.331	0.355	0.954	0.007	0.115	0.127	0.970
	300	0.002	0.240	0.243	0.958	0.003	0.080	0.082	0.964

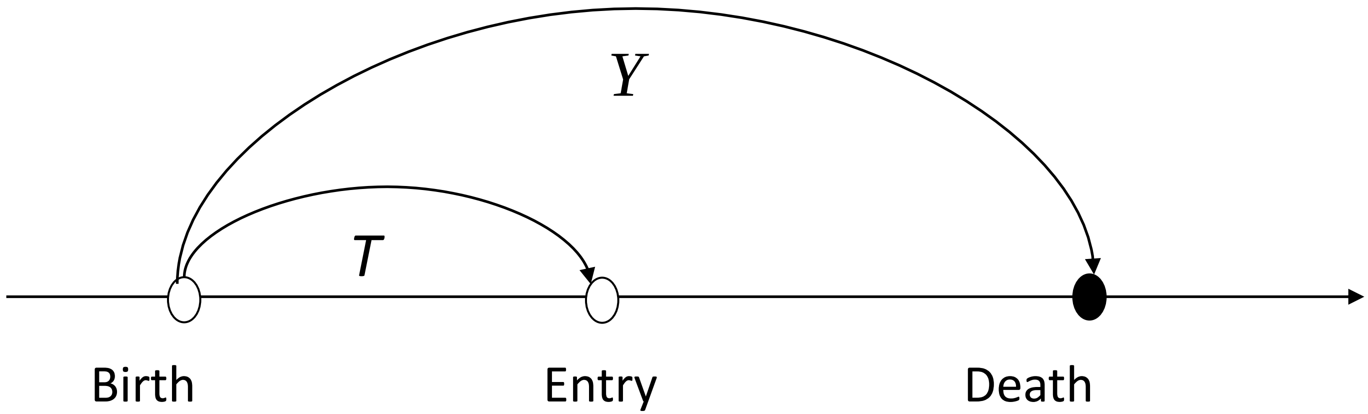
Table 4 Simulation results for the proposed estimator $\hat{S}_{\varepsilon}(t; \hat{\beta}, \hat{\gamma})$ †.

		$S_{\varepsilon}(t) = 0.25$		$S_{\varepsilon}(t) = 0.5$		$S_{\varepsilon}(t) = 0.75$	
(β_0, γ_0)	n	Bias	SD	Bias	SD	Bias	SD
(0, -0.5)	150	-0.076	0.090	-0.008	0.146	0.055	0.121
	300	-0.075	0.065	0.002	0.103	0.073	0.076
(0, 0)	150	0.006	0.109	0.000	0.146	-0.012	0.137
	300	0.003	0.072	0.003	0.098	-0.001	0.090
(0, 0.5)	150	0.044	0.100	0.004	0.129	-0.040	0.131
	300	0.041	0.076	0.000	0.098	-0.042	0.101
(1, -0.5)	150	-0.079	0.087	-0.010	0.149	0.056	0.120
	300	-0.078	0.063	0.000	0.109	0.071	0.083
(1, 0)	150	-0.005	0.102	-0.014	0.141	-0.020	0.131
	300	-0.002	0.071	-0.007	0.096	-0.009	0.089
(1, 0.5)	150	0.043	0.100	0.003	0.118	-0.039	0.113
	300	0.043	0.072	0.002	0.087	-0.040	0.082

Table 3 Simulation results for comparing the proposed estimator with the estimator of Lai and Ying (1991)†.

	(β_0, γ_0)	n	Proposed method E($\hat{\beta}$)	Proposed method Bias($\hat{\beta}$)	Proposed method SD($\hat{\beta}$)	Lai & Ying (1991) E($\hat{\beta}$)	Lai & Ying (1991) Bias($\hat{\beta}$)	Lai & Ying (1991) SD($\hat{\beta}$)
	(-1, -0.5)	150	-0.966	0.034	0.327	-0.779	0.221	0.296
		300	-0.988	0.012	0.241	-0.795	0.205	0.212
Independent truncation	(-1, 0)	150	-1.007	-0.007	0.420	-1.026	-0.026	0.443
		300	-1.003	-0.003	0.322	-1.008	-0.008	0.307
	(-1, 0.5)	150	-1.002	-0.002	0.372	-1.554	-0.554	0.659
		300	-1.004	-0.004	0.292	-1.526	-0.526	0.441
	(1, -0.5)	150	1.010	0.010	0.320	0.829	-0.171	0.305
		300	0.997	-0.003	0.234	0.820	-0.180	0.217
Independent truncation	(1, 0)	150	1.041	0.041	0.411	1.027	0.027	0.411
		300	1.012	0.012	0.284	1.007	0.007	0.281
	(1, 0.5)	150	1.012	0.012	0.331	1.373	0.373	0.506
		300	1.002	0.002	0.240	1.363	0.363	0.345

Data analysis



Channing House data (Hyde, 1980)

Available information for Individuals (n=462)

- *T* : Entry age
- *Y** : Age at death or censoring
- *X*: Gender (*X* = 1 for male; *X* = *0* for female)

Fit the proposed model:

$$Y^* = \beta_0 X + \gamma_0 T + \varepsilon, \quad \text{where } \varepsilon \text{ is unspecified}$$

Data analysis

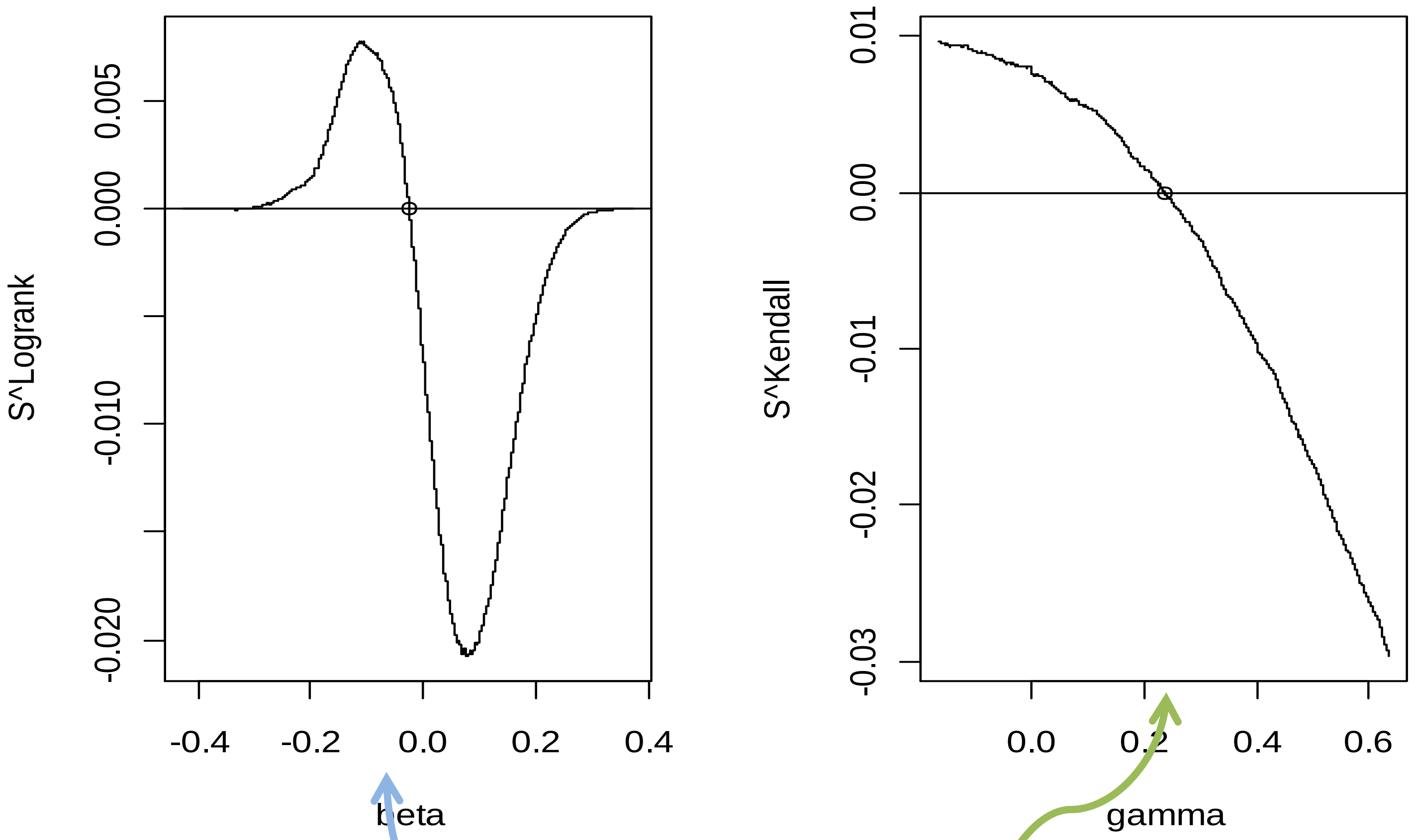


Fig. 3. Plots of $S_n^{Logrank}(\beta, \hat{\gamma})$ and $S_n^{Kendall}(\hat{\beta}, \gamma)$ based on the Channing house data. The numerical solutions $\hat{\beta} = \text{-}0.026$ and $\hat{\gamma} = 0.236$ obtained from the grid search algorithm are indicated by "o".

Data analysis

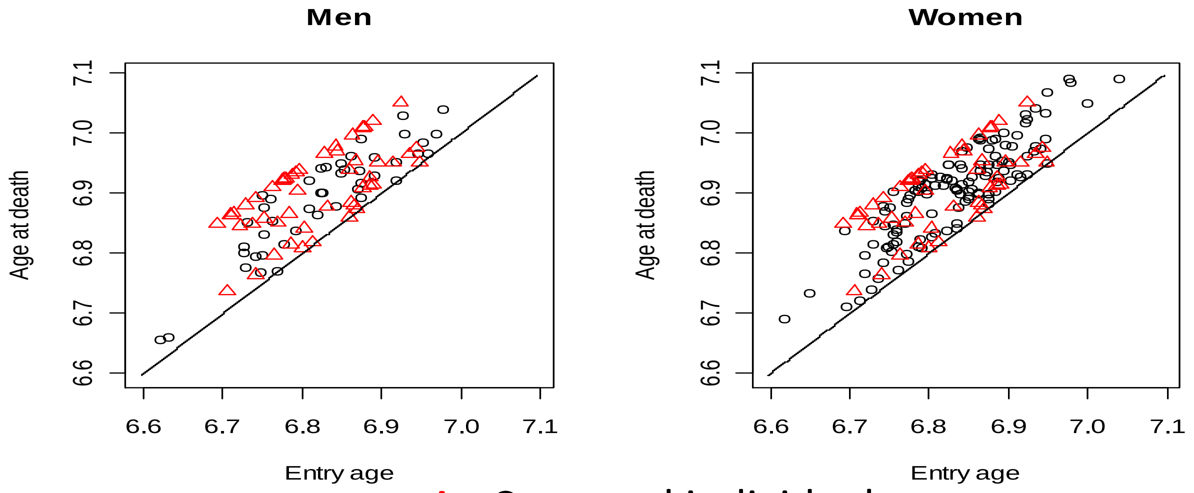
Interpretation:

$$Y^* \quad = \quad -0.026 \times X \qquad + 0.236 \times T \ + \ \varepsilon$$

Age at death   Gender   Age at entry

$$\begin{cases} \hat{\beta} = -0.026 & \text{.... 95\% CI} \quad (\text{-}0.067,\ 0.014) \\ \hat{\gamma} = 0.236 & \text{....95\% CI} \quad (0.014,\ 0.459) \end{cases}$$

$\therefore$ Late entry to Channing house prolong the survival



Δ: Censored individual

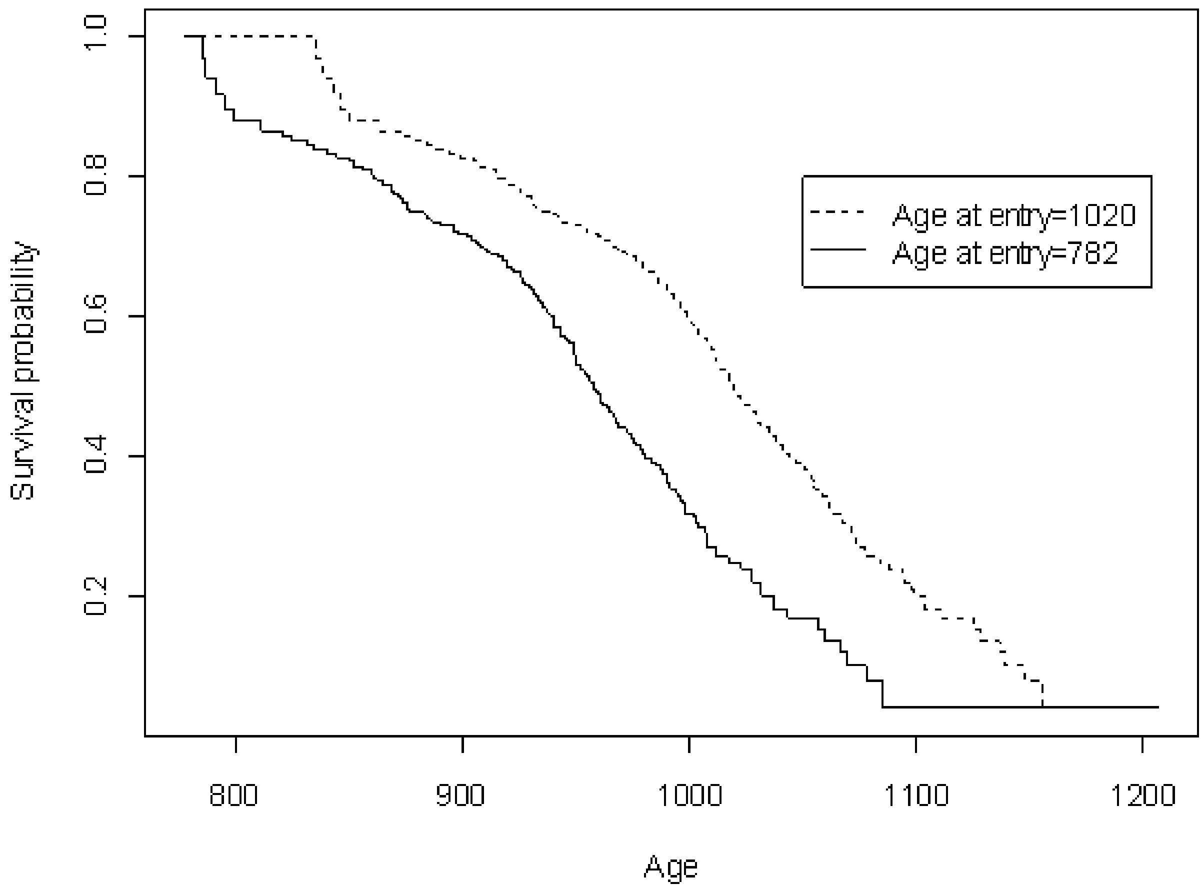
O: Died individual

Data analysis

Subject specific survival :

Survival for the two individual:

- ID#1: Entry age = 782 (month), sex = male
- ID#2: Entry age =1020 (month), sex = male



Conclusion

- We propose a semi-parametric AFT model which utilizes *both covariates and truncation variables* to model lifetimes

  ˇAFT of Lai & Ying (1991) can only utilize covariate as regressors

- We relax the independent truncation assumption

  in the Lai & Ying (1991)'s AFT method

- In Channing house data:

  The entry age (truncation) is informative for survival

  Early entry → shorter survival

  Late entry → Longer survival

  ← Significant

  Male → Shorter survival than female (non-significant)

Thank you for your kind attention