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Statistical Investigation of the Saturation Effect of Sunspot on the Ionospheric *foF2*

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Abstract. This study explores the relationship between the sunspot number (SSN) and the ionospheric *foF2*. It is of interest to locate the SSN value at which the *foF2* values are saturated. A regression model is built based on the data of the strictly rise period of the 21st solar cycle recorded by eight ionosonde stations scattering roughly between 40°N and 40°S geomagnetic latitude. Results show that clear saturation features appear around the equatorial anomaly crest region.

1. Introduction

Ionospheric electron densities are produced mainly by solar EUV and X-ray radiations, which are known to have large solar cycle variations. The long continuous, but manually observed data of sunspot numbers are often used to study the ionosphere response to the solar activity. Gopala Rao and Samibasiva Rao (1969) find for the same sunspot number, that *foF2* might show different values during the rise and fall periods of the solar cycle. Scientists observe that *foF2* shows a good linear relationship for low values of the sunspot numbers, but *foF2* seems to show a saturation effect at high sunspot numbers (for example see, Lakshmi *et al.*, 1988). Jones and Obitts (1970) fit a parabolic law to the solar-cycle vaiation of *foF2* and find more saturation at the higher solar epochs. However, it remains the problem of determining the the sunspot number where the *foF2* values are saturated. By examining the relationship between the monthly median noon *foF2* values and smoothed sunspot R_{12} in equinoxes and summer months, respectively, in the rise period of the 21st solar cycle, Kane (1992) further points out that this 'hysteresis' effect is small at low and high latitudes but substantial at middle latitudes. Although the separated analyses of *foF2* rcoreded in equinoxes and summer months could remove the seasonal effects, Kane's study suffers with limited data and still does not provide any information about the smoothed sunspot number at which *foF2* is saturated.

In this paper, to avoid the seasonal variation, we employ a 12-month running mean procedure, which has been used to derive the well know R_{12}, on measurements of the monthly median noon *foF2* values to obtain the associated smoothed *foF2* values, denoted by $foF2_{12}$. Moreover, according to the International Radio Ionosphere IRI90 (Bilitza, 1990), the noon time *foF2* at Chung-Li station during spring equinox is simulated and given in Fig.1. The result indicates that the solar cycle variation of

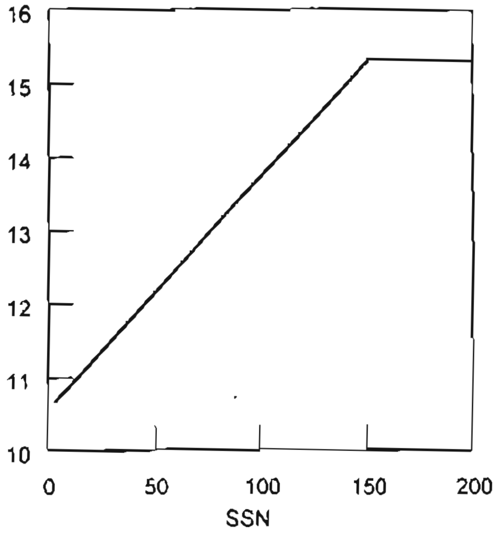

Fig.1. The simulated noon time *foF2* at Chung-Li station during spring equinox.

foF2 can be well represented by a pair of lines. In fact, in IRI90, the saturation of *foF2* for very high solar activities is enforced by keeping *foF2* constant above $R_{12} = 150$. Therefore, we suggest to fit a two-segmented linear regression model into the $foF2_{12}$ - R_{12} data for the saturation effect. This model does not only describe the relationship between the $foF2_{12}$ and R_{12}, but also determines the value of R_{12} at which the $foF2_{12}$ values are saturated.

2. Method and data analysis

A two-segmented linear regression model with unknown change-point c, is a regression model

$$y_t = f(x_t, c, \beta) + e_t, \quad \text{for } t = 1, 2, \ldots, n,$$

where the response function $f(x_t, c, \beta)$ has the following form:

$$f(x, c, \beta) = f_1(x, \beta_1), \text{ if } x \le c,$$
$$= f_2(x, \beta_2), \text{ if } x \ge c,$$

the $f_i(x, \beta_i)$'s being linear functions of x involving a vector of unknown parameters, β_i's. The error terms e_t are assumed to be independent Gaussian variables with mean zero and finite variance σ^2.

In our application, the response variable, y, is the $foF2_{12}$, the smoothed monthly median noon *foF2* value. We examine its relationship with x, the smoothed sunspot number R_{12}, which ranges from 20 to 150 in the strictly rise period of the 21st solar cycle (January 1977 - June 1979) and are presented in Fig. 2. The *foF2* values analyzed in this paper were recorded by eight ionosonde stations (geomagnetic latitudes), including Karaganda (40.3°N), Maui (21.0°N), Chung-Li (13.8°N), Manila (3.5°N), Huancayo (0.7°S), Tahiti (15.3°S), Townsville (28.6°S) and Canberra (43.8°S). Therefore, there are n = 30 pairs of observations in each ionosonde station. The plots of $foF2_{12}$ vs R_{12} for the eight ionosonde stations are given in Fig. 3.

For describing the saturation effect in $foF2_{12}$, the reparameterized model is considered

$$f(x; \theta) = \theta_1 + \theta_2(\theta_3 - x)\, I(\theta_3 - x),$$

where $I(z) = 1$, if $z \ge 0$, and $I(z) = 0$, otherwise. Note that θ_3 is the change point, θ_1 represents the expected y value when $x = \theta_3$ and $-\theta_2$ is the slope or changing rate of the first segmented linear regression model. The least-squares estimates of $\theta = (\theta_1, \theta_2, \theta_3)'$, denoted by $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)'$, minimizes

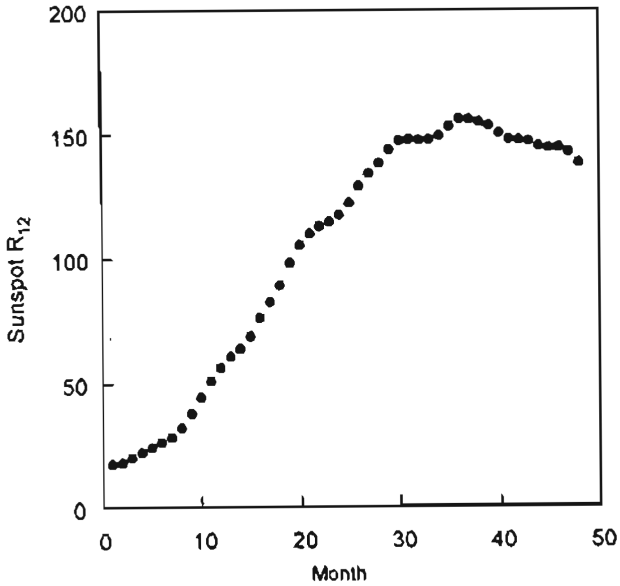

Fig. 2. The 21st solar cycle. (1977/1-1980/12)

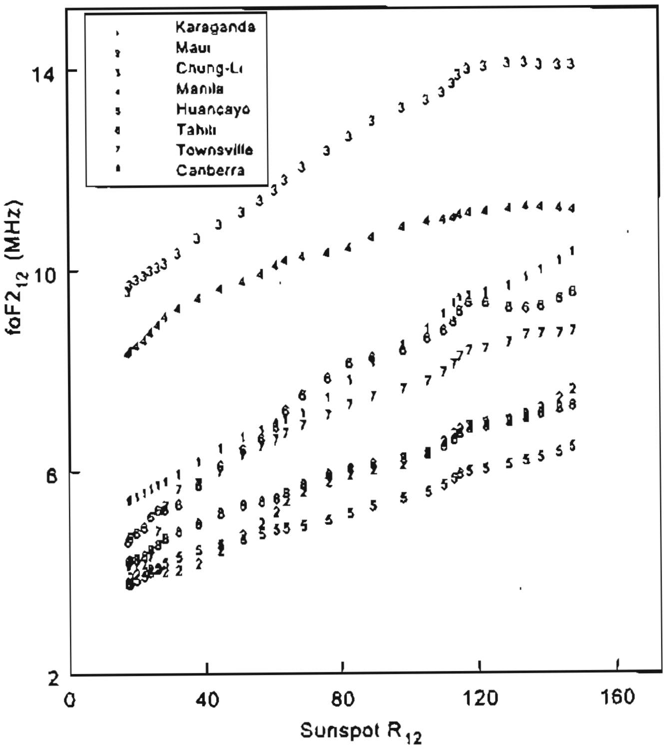

Fig. 3. Plots of $foF2_{12}$ vs Sunspot R_{12} (1977/1-1979/6).

$$Q(\theta) = \sum_{t=1}^{n} \{y_t - f(x_t; \theta)\}^2$$

and can be found using the modified Gauss-Newton algorithm as described in Gallant and Fuller (1973). Since the possible saturated effect occurs for the large values of R_{12}, we fit the $foF2_{12}$ - R_{12} data for the value of θ_3 between 100 and 140. The corresponding residual sum of squares is then obtained as $RSS(SL) = Q(\hat{\theta})$ and the fitted two-segmented linear regression model is $\hat{y} = \hat{\theta}_1 + \hat{\theta}_2\hat{\theta}_3 - \hat{\theta}_2 x$ for $x \le \hat{\theta}_3$, and $\hat{y} = \hat{\theta}_1$ for $x > \hat{\theta}_3$.

To test the goodness-of-fit for the two-segmented linear regression model, a simple linear regression model is also fitted into the $foF2_{12}$ - R_{12} data and the related residual sum of squares is obtained as RSS(L). Note that, under the Gaussian distribution, the distribution of the statistic

$$F = \{27RSS(L)\}/\{28RSS(SL)\}$$

is the F distribution with degrees of freedom 27 and 28, denoted by F(27, 28) and the upper 1st percentile of F(27, 28), is about 2.49. Therefore, if $F \ge 2.49$, then, at the significance level 0.01, the two-segmented linear regression model is claimed to be better than the simple linear regression model for fitting the $foF2_{12}$ - R_{12} data. In other words, a saturation effect in $foF2_{12}$ is declared when R_{12} is equal to or larger than $\hat{\theta}_3$. The F values for the eight ionosonde stations are reported in Table 1.

3. Results and discussion

The saturation effect in the smoothed *foF2* values is observed from Table 1 to be significant at level 0.01 for Tahiti and Chung-Li stations. The fitted two-segmented linear regression models for both the stations are then presented in Table 2. It can be seen that the saturated $foF2_{12}$ values are 14.14 MHz at Chung-Li and 9.42 MHz at Tahiti and both occur when the smoothed sunspot values are about 118 and 120, respectively. Plots of $foF2_{12}$ vs R_{12} together with the associated fitted two-segmented linear regression models at both Chung-Li and Tahiti stations are then given in Fig. 4.

Station	Geom. Lat.	RSS(L)	RSS(SL)	F
Karaganda	40.3°N	38.85	49.40	0.76
Maui	21.0°N	52.94	74.66	0.68
Chung-Li	13.8°N	211.41	45.02	4.53*
Manila	3.5°N	200.16	90.56	2.13
Huancayo	0.7°S	15.98	20.20	0.76
Tahiti	15.3°S	329.36	76.09	4.17*
Townsville	28.6°S	418.68	292.95	1.38
Canberra	43.8°S	37.29	41.27	0.87

*: significant at level 0.01

Table 1. The goodness-of-fit F values for two-segmented linear regression models.

It has been well understood by ionospheric scientists that due to the fountain effect, *foF2* reaches its minimum on the magnetic equator and attains its maximum on each side (around 12°N and 12°S, geomagnetic latitude), which is termed as the equatorial anomaly (see, for example, Davies, 1990). The results of the eight ionosonde stations displyed

Station	Geom. Lat.	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$
Chung-Li	13.8°N	14.14	- 0.043	119.53
		$\hat{y} = 9.04 + 0.043\,x$, $= 14.14$,		if $x \le 119.53$ if $x > 119.53$
Tahiti	15.3°S	9.42	- 0.045	117.75
		$\hat{y} = 4.08 + 0.045\,x$, $= 9.42$,		if $x \le 117.75$ if $x > 117.75$

Table 2. The fitted two-segmented linear regression models.

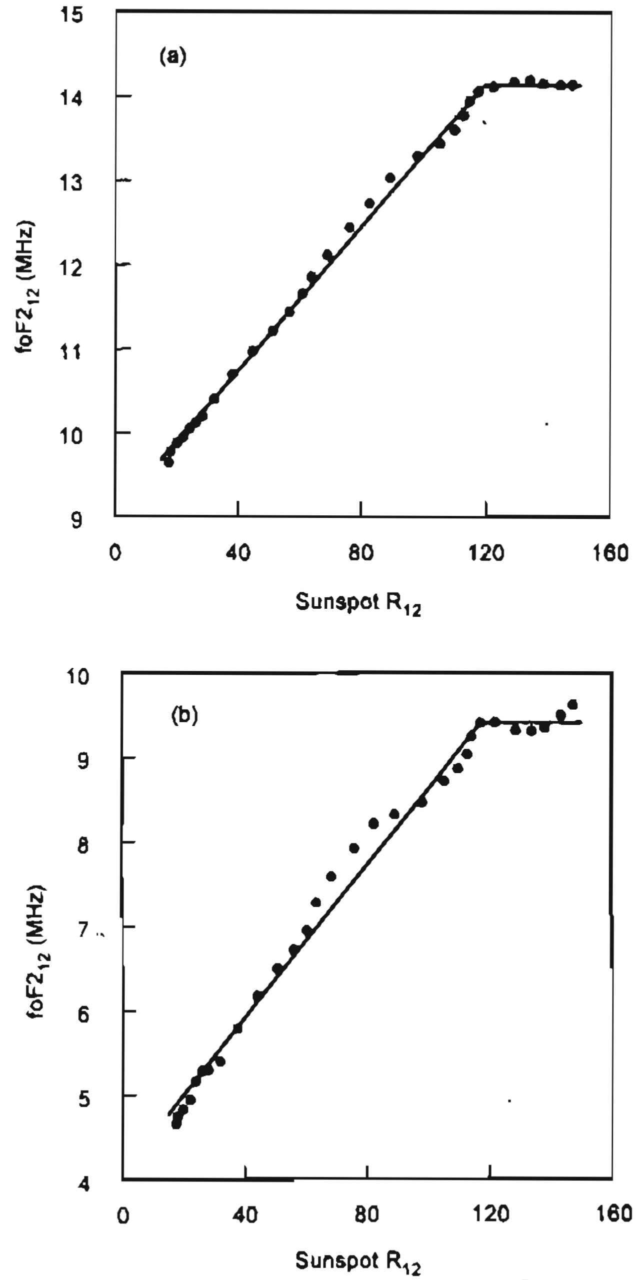

Fig. 4. Two-segmented linear regression models for
(a) Chung Li (Geom. Lat. 13.8°N)
(b) Tahiti (Geom. Lat. 15.3°S)

in Table 1 show that the stations (Chung-Li, Manila, Tahiti, and Townsville) around the equatorial crest regions indeed yield greater F values. However, it can be seen that the two largest values appearing at the Chung-Li (13.8°N, geomagnetic) and Tahiti (15.3°S, geomagnetic) stations are near the north and south equatorial anomaly crests, respectively. The saturation features in *foF2* values indicate that the ambient capacity of the ionospheric electron density may have a certain limitation. Since *foF2* values in the equatorial anomaly regions can significantly enhance when SSN increases, the profound saturation features should be clearly observed at these anomaly regions.

Kane (1992) examined data obtained from three ionospheric stations and concluded that the saturation effect is small at low and high latitudes but substantial at middle latitudes. Based on data recorded by eight stations scattering roughly between 40°N and 40°S geomagnetic latitudes, we found that the F value of Townsville (28.6°S, geomagnetic) in Table 1 is, in fact, not significant. Therefore, the saturation feature may not occur at middle latitude. In contrast, the F value of the Manila station (3.5°N, geomagnetic), 2.13, is only slightly less than the upper 1st percentile of F(27, 28) 2.49, which implies that a possible saturation feature can be observed at very low latitudes.

It is interesting to note that while the saturated values of *foF2* 14.14 MHz at Chung-Li and 9.42 MHz at Tahiti are very different, the two stations yield nearly identical change points, R_{12} = 119.53~117.75 and produce almost the same changing rates for the first segmented linear regression model dy/dx = 0.043~0.045.

4 Conclusion

A statistical model is introduced to describe the relationship between the smoothed sunspot number R_{12} and $foF2_{12}$, the smoothed monthly median noon *foF2* value. The model can further precisely determine the value of R_{12} (change point) at which the $foF2_{12}$ is saturated. It is found that the most profound saturation features in *foF2* occur at the equatorial anomaly crest regions. Finally, the nearly identical values in the changing rates and change point are possibly related to the causal mechanisms of the ionospheric saturation effects, which suggests that a further study seems worthwhile.

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